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Robust $H_{\infty}$ Fuzzy Control of a Class of Fuzzy Bilinear Systems with Time-Delay

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Abstract. This paper presents robust $H_{\infty}$ fuzzy controllers for a class of T-S fuzzy bilinear systems (FBSs) with time-delay. First, the parallel distributed compensation (PDC) method is adopted to design a fuzzy controller which ensures the robust asymptotic stability of the FBS with time-delay and guarantees an $H_{\infty}$ norm bound constraint on disturbance attenuation. Based on the Schur complement and some variable transformation, the stability conditions of the overall fuzzy control system are formulated by linear matrix inequalities (LMIs). Finally, the validity and effectiveness of the proposed schemes are demonstrated by the simulation.

1. Introduction

The bilinear models have been utilized to describe many physical systems and dynamical processes in engineering fields [1] for last decades. The corresponding control problems of bilinear systems have been also attracted in many studies [1-3]. Recently, the fuzzy decision logic has been successfully applied to controller designs for nonlinear systems [3-4], [6], [10]. Among various fuzzy modeling methods, the well-known Takagi-Sugeno (T-S) fuzzy model [3-4], [10] is recognized as a popular and powerful tool in approximating a complex nonlinear system.

Time-delay phenomenon is commonly existed in dynamic systems due to measurement, transport lags, transmission and computational delays. Thus the stabilization problem has also been investigated for nonlinear systems with time-delay, as in [4-5], [8-10]. Besides stabilization problem, another important issue for a controlled system is its robustness and this becomes a key problem in the study of uncertain nonlinear control systems and their controller designs. Robustness in fuzzy model-based control has been extensively studied in the past [3-4], [10]. For example, the fuzzy control design for T-S fuzzy systems with interval time-varying delay robustness performance is examined in [10].

This paper is organized as follows. In Section 2 the T-S FBS with time-delay is established and its fuzzy controller is also designed. Applying Schur complement, the stabilization conditions can be transformed into the forms of LMI in Section 3. A numerical simulation is illustrated in Section 4. Finally, conclusions are given in Section 5.

2. System description and controller design

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In this section, we introduce the T-S FBS with time-delay and then develop the fuzzy controller. The T-S fuzzy model is described by IF-THEN rules and is used to present the T-S FBS with time-delay. The \( i \) th rule of the T-S FBS with time-delay is represented by the following form:

**Plant Rule:**

\[
i \text{Plant Rule:} \quad g_i(s) \sum_{i=1}^{L} F_1(s(t)) \quad \text{and} \quad \ldots \quad \text{and} \quad g_i(s) < 0\]

\[
z(t) = C_i x(t) + D_{i1} \omega(t) \]

\[
x(t) = \phi(t), \quad t \in [-\tau_0, 0], \quad i = 1, L, r \]

where \( s_j \in R^n \), \( j = 1, 2, L \), \( g_j \) are known premise variables that may be functions of the state variables, external disturbances, and/or time [10], \( F_g \) is the fuzzy set, \( A_i, A_{di}, B_i, N_i, N_{di}, D_{i1}, C_i \) and \( D_{i2} \) have compatible dimensions, and \( \phi(t) \) is the initial condition. The time-delay \( \tau(t) \) may be unknown but is assumed to be smooth function of time as considered in [5],

\[
\text{IF } s_i(t) \text{ is } F_{1i} \text{ and } \ldots \text{ and } s_i(t) \text{ is } F_{gi} \text{ THEN } x(t) = \phi(t), \quad \tau(t) \leq \tau_0.
\]

By fuzzy blending, the overall fuzzy model is inferred as (3),

\[
\text{IF } s_i(t) \text{ is } F_{1i} \text{ and } \ldots \text{ and } s_i(t) \text{ is } F_{gi} \text{ THEN } x(t) = \phi(t), \quad \tau(t) \leq \tau_0.
\]

**Control Rule:**

\[
i \text{Control Rule:} \quad g_F(s) \sum_{i=1}^{L} F_1(s(t)) \quad \text{and} \quad \ldots \quad \text{and} \quad g_F(s) < 0\]

\[
u(t) = \rho D_i x(t)^T / \sqrt{1 + x^T D_i x}, \quad j = 1, 2, L, r.
\]

The overall fuzzy control law can be represented by

\[
u(t) = \sum_{j=1}^{L} h_j \rho D_i x(t)^T / \sqrt{1 + x^T D_i x}, \quad \text{where} \quad \rho \in R^{+}\n\]

\[
in R^{+}\n\]

The control objective is to design a T-S fuzzy controller (5) to stabilize the T-S FBS with time-delay (3). Substituting (4) into (3), one can get the closed-loop system,

\[
\text{IF } s_i(t) \text{ is } F_{1i} \text{ and } \ldots \text{ and } s_i(t) \text{ is } F_{gi} \text{ THEN } x(t) = \phi(t), \quad \tau(t) \leq \tau_0.
\]

**Control Rule:**

\[
i \text{Control Rule:} \quad g_F(s) \sum_{i=1}^{L} F_1(s(t)) \quad \text{and} \quad \ldots \quad \text{and} \quad g_F(s) < 0\]

\[
u(t) = \rho D_i x(t)^T / \sqrt{1 + x^T D_i x}, \quad \text{where} \quad \rho \in R^{+}\n\]

\[
in R^{+}\n\]

Then the robust fuzzy control problem to be addressed in this paper can be formulated as follows.

Given a fuzzy system with time-delay (3) and a scalar \( \gamma > 0 \), determine a fuzzy controller such that

R1) The closed-loop system (6) is robustly asymptotically stable when \( \omega(t) = 0 \).

R2) under the zero initial condition, the controlled output \( z(t) \) satisfies

\[
[|z(t)|]_\infty < \gamma [\| \omega(t) \|]\]

for all nonzero \( \omega(t) \in l_2^\infty \).
3. Main results

The main result on the asymptotic stability of the T-S bilinear fuzzy controlled time-delay system is propounded in the following theorem. Before proceeding to the main theorem, we give the following results, which will be used in the proof of the theorem.

**Lemma 1** [3,10]: Given any matrices $X$, $Y$ and $Z$ with appropriate dimensions such that $Z > 0$. Then, one has $X^T Y + YX^T \leq X^T Z X + Y^T Z^{-1} Y$.

**Theorem 1**: For the T-S FBS with time-delay (1), there exists a fuzzy feedback controller (5) such that the closed-loop system (6) is robustly asymptotically stable and (7) is satisfied if there exist symmetric and positive definite matrices $P$ and $S$, a scalar $\rho$, some vectors $\delta D_i$ and some scalars $\varepsilon_i$, $i,j = 1,L, r$, satisfying the following LMIs (8) and (9).

$$
\begin{bmatrix}
\Theta_i & * & * & * & * \\
D_{ij}^T P & -\gamma^2 I & * & * & * \\
\rho P & 0 & -\varepsilon_i I & * & * \\
\delta A_{ix}^T P & 0 & 0 & -S & * \\
\delta N_{ix}^T P & 0 & 0 & 0 & -S \\

C_i & D_{ij} & 0 & 0 & 0 & -I
\end{bmatrix} < 0, \quad (1 \leq i \leq r)
$$

(8)

$$
\begin{bmatrix}
\Theta_i \\
(D_i + D_{ij})^T P & -2\gamma^2 I
\end{bmatrix} < 0, \quad (1 \leq i < j \leq r)
$$

(9)

where $\gamma = (\gamma^2 + 1)^{1/2}$, $\Theta_i = A_i^T P + PA_i + \varepsilon_i (N_i^T N_i + D_{ij}^T B_i^T B_i D_j) + S/(1-\alpha)$, $\Theta_y = (A_y + A_y)^T P + P(A_y + A_y) + \varepsilon_y (N_y^T N_y + D_{ij}^T B_i^T B_i D_j) + \varepsilon_y (N_y^T N_y + D_{ij}^T B_i^T B_i D_j) + 2S/(1-\alpha)$, $\delta P = \rho P \delta PA_{ii} \delta PN_{ii} \delta PA_{ij} \delta PN_{ij} (C_i + C_j)^T$, $LMI_1 = \Theta_i$, $LMI_2 = \rho P \delta PA_{ii} \delta PN_{ii} \delta PA_{ij} \delta PN_{ij} (C_i + C_j)^T$, $LMI_3 = \text{diag}[-\varepsilon_i, \varepsilon_y, -S, -S, -S, -S, -2I]$,

**Proof**: Omitted.

4. Numerical example

In this section, we apply the proposed method to design a bilinear fuzzy controller for a T-S FBS with time-delay. The T-S FBS with time-delay can be described as follows:

Rule 1: IF $x_i$ is about $-1$

THEN $\dot{x}(t) = A_i x(t) + B_i u(t) + N_i x(t) u(t) + A_{ij} x(t-\tau) + N_{ij} x(t-\tau) u(t) + D_{i1} \omega(t)$

$$
z(t) = C_i x(t) + D_{i2} \omega(t)
$$

(10a)

Rule 2: IF $x_i$ is about 1

THEN $\dot{x}(t) = A_2 x(t) + B_2 u(t) + N_2 x(t) u(t) + A_{2j} x(t-\tau) + N_{2j} x(t-\tau) u(t) + D_{21} \omega(t)$

$$
z(t) = C_2 x(t) + D_{22} \omega(t)
$$

(10b)

where $A_i = \begin{bmatrix} -75.23 & 7.79 \\ 35 & -67 \end{bmatrix}$, $A_2 = \begin{bmatrix} -52.64 & 9.61 \\ 30 & -83 \end{bmatrix}$, $B_1 = B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $N_i = \begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix}$, $N_2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$,

$A_{ij} = \begin{bmatrix} 10 & 0 \\ 5 & 20 \end{bmatrix}$, $A_{2j} = \begin{bmatrix} 10 & 0 \\ 25 & 0 \end{bmatrix}$, $N_{ij} = \begin{bmatrix} 10 & 10 \\ 0 & 10 \end{bmatrix}$, $N_{2j} = \begin{bmatrix} -5 & 0 \\ 0 & 10 \end{bmatrix}$, $D_{11} = D_{12} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $D_{21} = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$, $D_{22} = \begin{bmatrix} 0 \\ 0.3 \end{bmatrix}$, $\tau = 2$. Let $\rho = 0.5$ and choose the controller gain
matrices as $D_1 = [-0.5 \quad -1.0]$ and $D_2 = [-1.0 \quad -0.5]$. Choosing $\gamma = 1$ and applying $\rho$ and all these matrices to inequalities (8) and (9) in Theorem 1, one can figure out the common positive-definite matrices, $P = \begin{bmatrix} 17.4212 & 4.7547 \\ 4.7547 & 15.9928 \end{bmatrix}$ and $S = \begin{bmatrix} 303.0810 & -13.4138 \\ -13.4138 & 313.3527 \end{bmatrix}$. The membership functions of the state $x_i$ is defined as $\mu_1 = (1-x)/2$ and $\mu_2 = (1+x)/2$. Simulation results of applying the fuzzy controller (5) to the T-S FBS with time-delay (10) with initial conditions $x(0) = [-1.3 \quad 1.6]^T$, $x(0) = [1.3 \quad -0.5]^T$, and the external disturbance as $\omega(t) = \begin{bmatrix} 0.5e^{-0.001t} \sin(t) \\ 0.8e^{-0.002t} \cos(t) \end{bmatrix}$ are shown in figure 1, where all these states are regulated to zero about after 7 seconds.

5. Conclusion
This paper designs the robust $H_\infty$ fuzzy controller for a class of T-S FBSs with time-delay. For stabilizing the T-S FBS with time-delay, some sufficient conditions have been derived to guarantee the stability of the overall fuzzy control system via LMIs. Finally, a numerical example has been utilized to illustrate the feasibility and effectiveness of the proposed schemes.

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References