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Synchronization between two different chaos systems using active control

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Abstract. In this paper, the problem of chaos synchronization using active control is investigated. The systems under consideration have different structures and dynamical behaviours, two chaos systems are synchronized using a proper nonlinear controller. Here Lyapunov function is developed to confirm the stability of chaos synchronization. Also a numerical example is presented to show the effectiveness and efficiency of the proposed synchronization scheme.

1. Introduction
In recent years, there has been intensive interest in the research of synchronizing chaotic dynamical systems[1-8]. In the 1990, Pecora and Carroll addressed the synchronization of chaotic systems using drive-response scheme. The idea of synchronization is to use the output of the master system to control the slave system so that the output of the slave system follows the output of the master system asymptotically. Many methods for handling chaos control and synchronization have been developed, such as PC method, OGY method, active control, adaptive method, impulsive control, fuzzy sliding-mode control and coupling control, etc. The afore mentioned methods and many other existing synchronization methods mainly concern the synchronization of two identical chaotic systems. In real-life applications, however, it is hardly the case that the structure of drive and response chaos systems can be assumed to be identical. In fact, in systems such as laser array, biological systems to cognitive processes, it is hardly the case that every component can be assumed to be identical. Moreover, the system’s parameters are inevitably perturbed by external inartificial factors and cannot be exactly known in priori. Therefore, synchronization of two different chaos systems is more essential and useful in real-life applications.

Recently, the chaos synchronization of different chaos systems is studied in ref.[9-15]. Ref.[9] presents chaos synchronization between two different chaos systems by nonlinear control laws. Ref.[12,13] proposes an adaptive control method to synchronize two chaos systems with different structure and unknown parameters. Based on Lyapunov stability theory, an adaptive synchronization controller is designed, also analytic expression of the controller and the adaptive laws of parameters

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are developed. Ref[14] studies the synchronization problem between the hyperchaotic Rossler system and the hyperchaotic Chen systems. An adaptive control law is derived for the case when the parameters of the drive and response systems to be synchronized are fully uncertain. In this paper, we apply control theory to synchronize two different chaos systems. A class of nonlinear control scheme is applied to solve the synchronization problems. Then, stability of the synchronization between drive and response system is proved by linear control theory (Lyapunov function).

This paper is arranged as follows. The synchronization scheme is designed in section 2, the Genesio system and the Lorenz system are studied. Section 3 numerical simulations are provided to demonstrate the synchronization scheme. Finally the conclusion is given in Section 4.

2. The synchronization scheme

We consider the drive chaos system in the form of

\[ \dot{x} = Ax(t) + Bf(x(t)) \]  

(1)

And the response system is assumed by

\[ \dot{x}_r = A_r x_r(t) + B_r g(x_r(t)) + u \]  

(2)

\( x, x_r \) are the state vectors, \( A, B, A_r, B_r \) are the system parameters. Let \( e = x_r - x \) is the synchronization error, our goal is to design controller \( u \) such that the trajectory of the response system (2) with initial conditions \( x_{r0} \) can approach the drive system (1) with initial condition \( x_0 \). The synchronization is describe as follow:

\[ \lim_{t \to \infty} \left\| e(t) \right\| = \lim_{t \to \infty} \left\| x_r(t, x_{r0}) - x(t, x_0) \right\| = 0 \]  

(3)

Where \( \left\| \right\| \) is the Euclidean norm.

The synchronization systems includes the Genesio system and the Lorenz system. The Genesio system is master system(or drive system) and the Lorenz system with controller is slave system(or response system).

The master system (Genesio system) is:

\[ \begin{align*}
\dot{x} &= y \\
\dot{y} &= z \\
\dot{z} &= -ax - by - cz + x^2
\end{align*} \]  

(4)

The slave system(Lorenz system) is:

\[ \begin{align*}
\dot{x}_i &= -a_1 x_i + a_1 y_i + u_1(t) \\
\dot{y}_i &= -x_i z_i + c_1 x_i - y_i + u_2(t) \\
\dot{z}_i &= x_i y_i - b_1 z_i + u_3(t)
\end{align*} \]  

(5)

The error dynamical system as follow:

\[ \begin{align*}
\dot{e} &= g(x_i, y_i, z_i, A_1, B_1) + u - f(x, y, z, A, B) \\
&= \begin{pmatrix}
a_1(y_i - x_i) + u_1 - y \\
c_1 x_i - x_i z_i - y_i + u_2 - z \\
x_i y_i - b_1 z_i + u_3 - ax - by - cz + x^2
\end{pmatrix}
\]  

(6)

The \( a, b, c, a_1, b_1, c_1 \) are the parameters of equations (4) and (5), they have definite values in this work. The two systems are chaos if the parameters’ values as follows:
To realize the synchronization between two different chaos systems, we design the nonlinear controller and inject the controller to the slave system.

We design the active control function, which makes the corresponding Lyapunov function’s derivative less than zero. \( (u_{a1}, u_{a2}, u_{a3}) \) are first part of the controller \( (u_1, u_2, u_3) \). The function as follows:

\[
\begin{align*}
  u_{a1} &= -a_1y + a_1x + y \\
  u_{a2} &= x_iz_1 - c_1x_1 + y + z \\
  u_{a3} &= -x_1y_1 + b_1z - ax - by - cz + x^2
\end{align*}
\]  

(7)

The parameters \( \tau_{dx}, \tau_{dy}, \tau_{dz} \) represent the controller’s activate time. The nonlinear controller \( (u_1, u_2, u_3) \) is designed as follow:

\[
\begin{align*}
  u_1 &= \begin{cases} 
  0 & t < \tau_{dx} \\
  u_{a1} + a_1e_1 - a_1e_2 - \Delta e_1 & t > \tau_{dx}
  \end{cases} \\
  u_2 &= \begin{cases} 
  0 & t < \tau_{dy} \\
  u_{a2} + e_2 - \Delta e_1 & t > \tau_{dy}
  \end{cases} \\
  u_3 &= \begin{cases} 
  0 & t < \tau_{dz} \\
  u_{a3} + b_1e_3 - \Delta e_3 & t > \tau_{dz}
  \end{cases}
\end{align*}
\]  

(8)

The error dynamical system (6) is described as follow:

\[
\begin{align*}
  \dot{e}_x &= -\Delta e_x, \quad \dot{e}_y = -\Delta e_y, \quad \dot{e}_z = -\Delta e_z \\
  \dot{e}_x &= y_1 - y, \quad \dot{e}_y = z_1 - z
\end{align*}
\]  

(9)

The parameter \( \Delta \) is the negative real part of the eigenvalues of the system that describes the difference between the master and the controlled slave system. For any \( \Delta > 0 \), the system will have all of the eigenvalues with negative real parts \( -\Delta \), and the whole system will be stable. So this parameter controls the rate at which the controller is activated.

To verify the synchronization system’s stability, we construct a Lyapunov function with the error variation and its derivative function as the following function:

\[
\begin{align*}
  V(e_x, e_y, e_z) &= \frac{1}{2}(e_x^2 + e_y^2 + e_z^2) \\
  \dot{V}(e_x, e_y, e_z) &= e_x \dot{e}_x + e_y \dot{e}_y + e_z \dot{e}_z \\
  &= -\Delta e_x^2 - \Delta e_y^2 - \Delta e_z^2 < 0
\end{align*}
\]  

(10)

Therefore the state error system is asymptotically stable. That is to say:

\[
e_x \to 0, e_y \to 0, e_z \to 0
\]

Hence the synchronization can be realized. With the nonlinear controller the slave system can synchronize the master system.
3. Numerical simulation
In this section, we give the scheme’s simulation. Genesio system is the master system and Lorenz system is the slave system. And the parameters of the two systems are:

\[ a = 6, b = 2.92, c = 1.2, a_1 = 10, b_1 = 8 / 3, c_1 = 28 \]

To ensure the two systems’ chaos behavior, the initial values are:

\[ x = 3, y = -4, z = 2, x_1 = -4, y_1 = 5, z_1 = 3 \]

Thus the initial errors are –7, 9, 1. And the controller’s activate parameters as follows:

\[ \tau_{dx} = 10, \tau_{dy} = 20, \tau_{dz} = 30 \]

The simulation results are illustrated in fig.1. The fig.1(a) displays the signals error of \( x \) and \( x_1 \), while fig.1(b) displays \( y \) and \( y_1 \), fig1(c) displays \( z \) and \( z_1 \). We find that the slave system traces the master system and finally their dynamical behaviors become the same.

Finally, we explore the effect of external noise. Because the chaos system depends sensitively on a tiny perturbation of the trajectory, we add white noise signal in the master system. The white noise signal \( \eta_x \sim N(0,0.06) \) is added in the x channel and \( \eta_y \sim N(0,0.014) \) is added in the y channel.

With the same parameters, initial values and activate time, the numerical simulations demonstrate as fig.2. It is found easily that the noise does not destroy the two systems’ synchronization.

4. Conclusion
In this article, we design a scheme to synchronize two different chaos systems. Also we construct a Lyapunov function to verify the synchronization stability. It is found that this scheme is effective in synchronizing other chaos systems. This scheme’s shortcoming is that the nonlinear controller’s structure is a bit complex to realize.
Fig. 2. Synchronization error under the noise (time: sec)

References