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Chaos and Its Control Problems for Josephson Junction Circuit System

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Abstract. A notch filter is constructed to control the chaos of a Josephson junction circuit system. The chaos of the system is controlled by applying first-order differential of state variable as feedback.

1. Introduction
The research of the chaos has rapidly developed since Lorenz found the first chaos attractor, Lorenz system [1]. During the past four decades, a large number of studies have shown that chaotic phenomena are observed in many physical systems that possess non-linearity. And the new characters of the chaotic system are found continuously. Nonlinear system is found in the many areas such as biologics, physics and chemistry etc. Under the certain condition, chaotic motions will appear in the nonlinear system. Chaos is harmful in some nonlinear systems. So it will be controlled or avoided. The first paper of the chaos controlling is published by Hubler [2] in 1989. In 1990, Ott, Grebogi and Yorke put forward the theory of the chaos controlling, OGY [3]. It produced the wide response. Pecora and Carroll brought forward the thinking of the chaos synchronization [4] in the same year. Ditto and Roy finished the experiment of the chaos controlling [5]. During the past decades, the many methods of the chaos controlling are found such as the improvement of the OGY [6], the continuous controlling method [7], engineering controlling method [8] and so on.
A Josephson Junction Integrated Circuit (JJIC) has characteristics of low power dissipation and of high speed switching. The JJIC has been fabricated by many researchers [9-10]. In the paper, the complex dynamics characters of an equivalent circuit of Josephson junction circuit, RSJ model, is studied. And the chaos is controlled by two methods. Their advantages are discussed.

2. The RSJ model of Josephon junction circuit and its route to chaos
An equivalent circuit of Josephson junction circuit, RSJ model, is depicted in Figure 1. $I(t)$ is current supplied by outer power source. $R$ is resistance of junction. $C$ is capacitance of junction and $I_C(t)$ is the maximum Josephson current when temperature is $T$ without noise. Because super-current and normal current pass through Josephson junction, resistance effect and capacity effect must be considered.

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Where $I_s$ is current passing through ideal junction, $I_n$ is normal current, $I_d$ is displacement current and $I_d$ is direct-current of current passing through detour. In which $I_s = I_c \sin \varphi$, here $\varphi$ is phase difference between both ends of wave function. If the tunnel effect of resistance is considered

$$I_n = \frac{(1 + \mu \cos \varphi)V}{R}$$

The equation (1) can be depicted

$$I_{de} = I_c \sin \varphi + \left(1 + \mu \cos \varphi\right)V / R + C \frac{dV}{dt}$$

Relation of phase difference $\varphi$ and voltage $V$ is

$$\frac{d\varphi}{dt} = \frac{2eV(t)}{h}$$

Where $h$ is Planck constant. So equation (3) can be written as

$$I_{de} = \left(\frac{hC}{2e}\right)\frac{d^2\varphi}{dt^2} + \left(\frac{h}{2eR}\right)\left(1 + \mu \cos \varphi\right)\frac{d\varphi}{dt} + I_c \sin \varphi$$

If current contains direct-current and alternating current, the dynamics equation of Josephson junction circuit is

$$I_{de} + I_{ac} \sin \Omega t = \left(\frac{hC}{2e}\right)\frac{d^2\varphi}{dt^2} + \left(\frac{h}{2eR}\right)\left(1 + \mu \cos \varphi\right)\frac{d\varphi}{dt} + I_c \sin \varphi$$

Adopt non-dimensional constant

$$\rho = I_{de} / I_c, \quad \alpha = I_{ac} / I_c, \quad \beta = 2eI_c CR^2 / h$$

And denoting

$$\omega = \Omega \sqrt{hc / 2eI_c}, \quad \tau = t \sqrt{2eI_c / hC}$$

Equation (6) can be depicted as

$$\frac{d^2\varphi}{d\tau^2} + \frac{1}{\sqrt{\beta}}\left(1 + \mu \cos \varphi\right)\frac{d\varphi}{d\tau} + \sin \varphi = \rho + \alpha \sin \omega \tau$$
Denoting $\varphi = x, d\varphi/d\tau = y, r = 1/\sqrt{\beta}$ . Equation (7) can be rewritten in the standard form of the two-dimensional system

$$\begin{cases}
\dot{x} = y \\
\dot{y} = \rho + \alpha \sin \omega \tau - \sin x - r(1 + \mu \cos x)y
\end{cases}$$

Equation (8) can be rewritten in the standard form of the two-dimensional system

$$\begin{cases}
\dot{x} = y \\
\dot{y} = \rho + \alpha \sin \omega \tau - \sin x - r(1 + \mu \cos x)y
\end{cases}$$

Denoting the right hand of equation (8) equal to 0, the equilibrium points are

$$\begin{cases}
y = 0 \\
x = \arcsin(\rho + \alpha \sin \omega \tau)
\end{cases}$$

Equation (9) can be rewritten in the standard form of the two-dimensional system

$$\begin{cases}
\dot{x} = y \\
\dot{y} = \rho + \alpha \sin \omega \tau - \sin x - r(1 + \mu \cos x)y
\end{cases}$$

Jacobian Matrix of equation (8) is

$$J = \begin{bmatrix}
0 & 1 \\
-\cos x + ry\mu \sin x & -r(1 + \mu \cos x)
\end{bmatrix}$$

The eigenvalue of equation (10) is

$$|\lambda E - J| = \begin{vmatrix}
0 & 1 \\
-\cos x + ry\mu \sin x & -r(1 + \mu \cos x)
\end{vmatrix}$$

Equation (11) is a two-dimensional equation, and it can be solved by numerical method. According to the value of $\lambda$, the type and character of bifurcation can be divided.
When the values of parameters $\rho, \omega, r, \mu$ are given as $0.0, 5, 1, 0.2$, and $\alpha$ is regarded as a bifurcation parameter, the route to chaos of the system can be analyzed by numerical solution. Chaotic motion can be easily observed in the system. If changing the value of bifurcation parameter, $\alpha$, the periodic behaviors will turn into chaotic motion. The motion of the system turns from periodic behaviours to chaotic motion, when changing the value of $\alpha$ from 1.79 to 1.762. Its bifurcation diagram is depicted in Figure 3 and its Lyapunov exponent map is depicted in Figure 4. When value of $\alpha$ is given as 1.79 the motion of the system (8) is periodic as depicted in Figure 2(a). The periodic orbit lose its stability and became doubling bifurcation as in Figure 3 (b) to Figure 3 (e) with the value of $\alpha$ decreasing from 1.79 to 1.7668. Chaotic motion will appear in system when the value of $\alpha$ is less than 1.762, as depicted in Figure 3 (f). Its chaotic motion is proved by its Lyapunov exponent map in Figure 4 (a).

![Figure 3. Bifurcation diagram of the system](image)

![Figure 4. Lyapunov exponent map of the system](image)

3. Two methods of chaos controlling

3.1. Controlling chaos based on the notch filter

The transfer function of linear notch filter is [11]

$$G(s) = \frac{H(s)}{Y(s)} = \frac{P(s^2 + \omega_n^2)}{(s^2 + 2\xi \omega_n s + \omega_n^2)}$$

Where $\omega_n$ is frequency of notch filter, $\xi$ is resistance coefficient, $P$ is feedback gain.

The state space of linear notch filter can be depicted as

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi \omega_n \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} Y$$

$$h = -2\xi \omega_n P \cdot z_1 + Py$$

Adding the output $h(t)$ of notch filter as negative feedback to the second function of system (8), it will be

\[\text{[Equation]}\]
The chaotic motion will appear in the system when the value of $\alpha$ is less than 1.762 as shown in Figure 2 (f). When the values of parameters of controlled system $\xi, \omega_n$ are given as 0.001, $P$ is regarded as a control parameter, the chaotic motion of system (8) can be controlled to periodic orbit by adjusted the value of parameter $P$. The phase plane portraits of the controlled system are depicted in Figure 5.

From the simulation process, this method is more effective and applicable for control of chaos to the system. The motions of the controlled system can be controlled to the period-1 orbits when the value of the parameter $P$ is appropriate. The stabilities of the orbits can be analyzed by the Lyapunov function.

3.2. Applying first-order differential of state variable as feedback to realize chaos control

The normal form of state feedback control is

$$
\dot{x} = f(x) + u(x) \tag{16}
$$

Where $x \in \mathbb{R}^n$ is system variable, $f$ is n-dimensional vector field and $u(x)$ is n-dimensional control vector. Denoting the control vector $u(x)[12]$ as

$$
u(x_1, x_2) = \begin{bmatrix} k_1 \\ 0 \end{bmatrix} \tag{17}$$

Here $k$ is control variable. Adding the control vector to the system (8), the state equation of the controlled Josephson junction circuit system is depicted as

$$
\begin{bmatrix}
\dot{x} \\
\dot{y}
\end{bmatrix} = \begin{bmatrix}
\rho + \alpha \sin \omega \tau - \sin x - r(1 + \mu \cos x)y \\
\rho + \alpha \sin \omega \tau - \sin x - r(1 + \mu \cos x)y
\end{bmatrix} + \begin{bmatrix}
k_1 x_1 \\
k_1 x_2
\end{bmatrix} = \begin{bmatrix}
\nu_1 \\
\nu_2
\end{bmatrix} \tag{18}
$$

Jacobian Matrix of equation (18) is

$$
J = \begin{bmatrix}
-k \cos x + k r y \mu \sin x & 1 - k r (1 + \mu \cos x) \\
-\cos x + r y \mu \sin x & -r (1 + \mu \cos x)
\end{bmatrix} \tag{19}
$$

The eigenvalue of equation (19) is

$$
|J - \lambda I| = \begin{bmatrix}
\lambda + k \cos x - k r y \mu \sin x & -1 - k r (1 + \mu \cos x) \\
\cos x - r y \mu \sin x & \lambda + r (1 + \mu \cos x)
\end{bmatrix} \tag{20}
$$

It can be solved by numerical method. According to the value of $\lambda$, the type and character of bifurcation can be divided.
By this method, the chaos of system (8) shown in Figure 2 (f) can be controlled, and the phase plane portraits and bifurcation diagram of the controlled system are depicted in Figure 6 and Figure 7. From Figure 6 and Figure 7, we can see that if the value of the weight, $k$, is chosen as 0.6, the motion of the system is controlled to a period-1 orbit. And if it is decreased to 0.385, the motion of the system is controlled to a period-2 orbit. We get a conclusion that the period of the controlled system is relied on the weight of the feedback. The weight of the feedback is greater; the motion of the system is more stable.

![Phase plane portraits of the controlled system](image1)

**Figure 6.** Phase plane portraits of the controlled system ($x - y$)

![Bifurcation diagram of the controlled system](image2)

**Figure 7.** Bifurcation diagram of the controlled system ($k - x$)

4. **Conclusion**

The dynamic system of the Josephson junction circuit system exhibits a rich variety of nonlinear behaviours as certain parameters varied. Due to the effect of nonlinearity, regular or chaotic motions may occur. In this paper, both analytical and computational methods have been employed to study the dynamical behaviours of the nonlinear system. A notch filter is constructed to control the chaos of the Josephson junction circuit system. The chaos of the system is controlled by applying first-order differential of state variable as feedback, too.

Under the controlling with a notch filter, chaos of the system can be transformed to the periodic motion quickly with the change of the value of the controlling parameter $P$. But the value of $P$ can’t be too much, which makes the orbit can’t converge to the equilibriums. An epitude value of the resistance coefficient $\xi$ is the key element in controlling the system’s chaos to period. In the method of applying first-order differential of state variable as feedback, chaos can be controlled by choosing an epitude value of controlling parameter $k$. It is a simple and effective method.

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