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Study on chaotic behaviors of RCLSJ model Josephson junctions

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Abstract. Chaotic behaviors of the dc-biased resistively-capacitively-inductively shunted Josephson junctions are studied numerically. The existence of the chaos is proved by the spectrum and strange attractor. We also find out the route to chaos is intermittence. The parameter space in which chaos exits is obtained, and different features of the chaos in different parameter range are also given.

1. Introduction

As a highly nonlinear device, the Josephson junction and its related circuits show complex chaotic behavior. With the advantage of ultra low noise, low power consumption and high operating frequency, the Josephson junction and related circuits are widely studied for many potential applications. While sometimes the chaos is useful, such as utilisations in secret communication, artificial neural networks, etc[1-5], in other times, it is harmful, for example in voltage standard[6] and terahertz source in which the chaos should be avoided. Otherwise, Josephson junctions and related circuits can be used as an ideal object for the research of chaos as well. In1995, Whan C B et al[7] reported the experimentation and simulation research on Josephson junction with parallel resistance outside. They studied the parallel resistance with nonzero inductance and reached consistent results by experimentation and simulation. Basing on this research, the RCLSJ model are introduced[8-9] and proved its rationality in most conditions, especially suitable for the situations of very high frequency. The chaotic behavior in the dc-biased junction has become a focus for the application of chaos. In 2001, Syamal Kumar Dana[10] et al compared different models of Josephson junctions, and studied the application on high-frequency signal generator in communication system. In 2006, Xiao-Song Yang[11] provided rigorous confirmation on the existence of chaotic behavior in RCLSJ model by theoretical method and simulations. In the same year, Ahmad M. Harb[12] reported that the chaos in RCLSJ model was effectively controlled with a nonlinear inverted controller designed by them. In 2007, Ahmet Ucar[13] reported synchronization of chaos in two coupled Josephson junction. They use master-slave method to bring the chaos in slave system into step with its in master system.

However, we have not seen that the research of the parameter range and the route to chaos, which are of great importance in further analyzing RCLSJ model and its chaotic behavior, was reported. In this paper, we used numerical method to simulate the RCLSJ model and found the system trapping in chaos through intermittent route. Moreover, we provided the chaotic parameters range over \( I_1-\beta_l, I_1-\beta_c, \beta_c-\beta_l \) space.

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2. RCLSJ model[8]

It is well known that the general and traditional method to analysis nonlinear system is to use numerical method to find solution of the differential equation set[14-15]. However, the complexity and amount of work increase quickly as the accuracy improves. In this paper, we take advantage of the software PSpice[16] to simulate the Josephson junction to investigate more dynamic characteristics of variables with high degree of accuracy. We introduced the model of Josephson junction in PSpice and justified its qualification for the simulation of Josephson junctions and related circuits. The circuit to be studied, shown in Fig.1, consists of an ideal Josephson junction shunted by a resistor of R, a capacitor of C and an inductor of L. Rs is the series resistor of L.

\[ R(V) \]

Fig.1. Schematic diagram of the simulation circuit

Without taking the thermal noise into account, the governing equations of junction dynamics are as follows:

\[
\begin{align*}
C \frac{dV}{dt} + \frac{V}{R(V)} + I_s \sin(\varphi) + I_s &= I \\
\hbar \frac{d\varphi}{dt} &= V \\
L \frac{dI}{dt} + I_s R_s &= V
\end{align*}
\]

(1)

Where \( I_s \) is the current flowing through L, \( I_c \) is the critical current of the junction. The equations can also be present in form as following:

\[
\frac{dx}{dr} = \bar{F}(x)
\]

(2)

where

\[
\begin{bmatrix}
\varphi \mod 2\pi \\
V/I_c R_s \\
I_s / I_c \\
x_1 - x_3
\end{bmatrix},
\begin{bmatrix}
x_2 \\
\left[ i - gx_2 - \sin(x_1) - x_3 \right] / \beta_c
\end{bmatrix}
\]

\[
\begin{bmatrix}
\left( x_2 - x_3 \right) / \beta_c
\end{bmatrix}
\]

\[
\begin{bmatrix}
\beta_c = 2 \pi R_s C / \Phi_0 \\
\beta_c = 2 \pi L R_s C / \Phi_0 \\
i = I / I_c \\
g = R_s / R(V)
\end{bmatrix}
\]

We use numerical method to find solution of the differential equation set (1) and (2). The critical current of \( I_c \) is set to 0.1mA in simulation, \( R(V), 100\Omega \), \( R_s, 1.5\Omega \), \( C, 1pF \), \( L, 8.6pH \). Based on these settings, the inductive parameter \( \beta_i \) is 2.6, and the hysteresis parameter \( \beta_c \) is 0.68, which is similar to the parameters given in reference [12].

3. Numerical results and discussions
3.1. The chaotic behavior and the route to chaos

Fig. 2 shows the I-V curve of the RCLSJ model with the parameters set above. It is obvious that the I-V curve is not smooth but with some peaks in it. Though we re-calculate the I-V curve with improved accuracy, the I-V curve is still not smooth which gives a prediction of chaos. With the bias current kept 0.12mA, the simulated voltage waveform across the Josephson junction is aperiodic as showed in Fig. 3. And in the Fourier analysis, the chaotic spectrum exhibits the dominant peak at 90GHz but superimposes to it there is a broadband spectrum, a typical signature of chaos. To further illustrate the existence of chaos, Fig. 4 shows the projection of strange attractor onto the inductor current- junction voltage plane. The trajectory does not repeat in the phase space which shows the existing of chaos.
In order to investigate the route to chaos, we use different dc-biased current to simulate. The solution is periodical when the current is less than 0.105mA. As it slowly increase to 0.11mA, the system trap into chaos. In Fig.5, Fig.6 and Fig.7, the voltage waveforms across Josephson junction at dc-biased current of 0.106mA, 0.108mA and 0.110mA, respectively, are presented. The circuit exists clear periodical state at Fig.5 and Fig.6. However, from the figures, we can see that as the dc-biased current increases to about 0.105mA, the periodical state is suddenly interrupted by burst chaos but soon restores to the previous state again. The burst chaos occurs at frequent intervals as the biased current increased which finally leads the circuit into chaotic state as shown in Fig.7. From the comparison based on Fig.5 to Fig.7, we can draw the conclusion that the approach of circuits system to chaos is intermittence.

**Fig.5** The voltage waveform across the josephson junction when the dc-biased current is 0.106mA

**Fig.6** The voltage waveform across the Josephson junction when the dc-biased current is 0.108mA

**Fig.7** The voltage waveform across the josephson junction when the dc-biased current is 0.110mA
3.2. The parameter range over which chaotic behavior occurs and its characters

Keeping other parameters constant, we only change the amplitude of dc-biased current to simulate. The chaos has been observed in the range from 0.11mA to 0.122mA and the range from 0.178mA to 0.180mA. We select an acceptable sampling dc-biased current of 0.179mA in the second range to simulate the voltage waveform across the junction. The aperiodic oscillation of voltage form and broad-band spectrum in the Fourier analysis arouses which predicts the chaos dynamics. To further illustrate the existence of chaos, we drew the phase diagram of current across inductance to analyze. The randomness of the voltage begins to attenuate and the spectrum approaches discrete peaks which reflect the fading of chaos.

As is seen from Eq.1 and Eq.2, it is apparent that the dynamical behavior of the dc-biased junction is governed by parameters: C, L and I. Our research spans values of these parameters to investigate the range existing chaos. In Fig.8, we fix $\beta_c$ at 0.68 and span $\beta_l$ together with dc-biased current of I to investigate the range of chaos spots. To set $\beta_c$ and $\beta_l$ at fixed values, we can observe in Fig.8 that the current range which existed chaos did not distribute continuously. In Fig.9, we have fixed an acceptable sampling $\beta_l$ at 2.6. With changing of the McCumber parameter of $\beta_c$, we made the graph of the chaos range at different dc-biased current. In Fig.9, the current range of chaos is not continuously which consistent with the result in Fig.8. In the plane of $\beta_c$-$\beta_l$ shown in Fig.10, the chaos is less likely to appear when either the L or C parameters are too large or too small.

Fig.8 Locations in the I1-$\beta_l$ plane at with chaos has been observed

Fig.9 Locations in the I1-$\beta_c$ plane at which chaos has been observed
4. Conclusions
We have studied the chaos in Josephson junction with RCL SJ model and discovered the route to chaos is intermittence. Furthermore, we investigated the parameter range which chaos exists. At the fixed McCumber parameter and inductance, the chaotic state and periodic state alternate with the change in dc-biased current. We have observed that the chaos only exists with particular capacitance and inductance values shown in Fig.8 to Fig.10, investigating the locations of chaos in the $I_1-\beta_c$, $I_1-\beta_l$ and $\beta_c-\beta_l$ plane which enable us to avoid or utilize it at suitable parameters as it does in voltage standard terahertz source. The results are valuable for the research of nonlinear characters of Josephson junction and for better application of the junction.

Reference