A Markov jump theory based connectivity model of mobile ad hoc networks

To cite this article: D Li et al 2008 J. Phys.: Conf. Ser. 96 012012

View the article online for updates and enhancements.

Related content

- Hierarchical auto-configuration addressing in mobile ad hoc networks (HAAM)
P. Ram Srikumar and S. Sumathy

- AD HOC Networks for the Autonomous Car
Davidescu Ron and Eugen Negrus

- An Investigation into Node Strength Connectivity Correlation
Shi Jian-Juh, Wang Yong-Li and He Da-Ren
A Markov Jump Theory Based Connectivity Model of Mobile Ad Hoc Networks

Demin Li\textsuperscript{*}, Jie Zhou\textsuperscript{2}, Jiacun Wang\textsuperscript{3}

1. College of Information Science and Technology, Donghua University, Songjiang District, Shanghai 201620, China

2. School of Science, Donghua University, Songjiang District, Shanghai 201620, China

3. Department of Software Engineering, Monmouth University, West Long Branch, NJ 07762, USA,

E-mails: deminli@dhu.edu.cn; zhoujie@dhu.edu.cn; jwang@monmouth.edu

Abstract. Mobile node connectivity has been one of the fundamental issues in a mobile ad hoc network for mobile nodes can spread in an arbitrary manner. In this paper, we model the connectivity using Markov jump theory, which generalizes Papavassiliou and Zhu's results (IEEE Communications Letters 9, 2005, 337). Properties of the connectivity model are discussed. In particular, the conditions, which will cause an ad hoc network connectivity exponentially decreasing and eventually no nodes in the network will be able to communicate with each other, are provided.

1. Introduction

Mobile ad hoc networks (MANET) are decentralizing, self-organizing, and highly dynamic networks formed by a set of mobile hosts connected through wireless links, without requiring any infrastructure. As they are infrastructure-less networks, each node should also act as a router. If the destination node is not within the transmission range of the source node, the source node communicates with the destination node with the help of the intermediate nodes or routers. Civilian applications include peer-to-peer computing and file sharing, mobile decision making [1, 2], collaborated mobile computing [3], and searching rescue operations. Military applications include battlefields among a fleet of ships, a group of armored vehicles or a large of military aircraft. With the rapid progress of wireless communication and embedded micro-sensing MEMS technologies, those applications make possible and wide.

Since mobile nodes may spread in an arbitrary manner, one of the fundamental issues in a mobile ad hoc network is to maintain connectivity. Papavassiliou et al. [4] proposed an enhanced model based on continuum theory [5], which describes the local linkage evolution processes of wireless sensor networks, considering new links, rewiring of existing links, and deleting of existing links. In wireless

\textsuperscript{*} To whom any correspondence should be addressed.
sensor networks, such events are coupled with the actual physical events such as node movement, network density, power coverage, etc.

The links of nodes in a mobile ad hoc network are always subject to frequent changes, which may result from abrupt phenomena such as node movement, energy decrease of nodes, network density changes, power coverage changes etc. Systems with those characters may be modeled as hybrid ones. Markov Jump Systems (MJS) [6, 7] are an important class of hybrid modeling tools. In a MJS, the dynamics is subject to random switches among a collection of systems sharing the same state vector. For a finite-state MJS, the finite Markov chain governs the transition from one logical state to another. Such systems are particularly useful to model systems that are subject to abrupt random changes in their dynamics, e.g. related to phenomena such as node movement, energy decrease of nodes, network density changes, power coverage changes etc. In this paper, we model the network connectivity using Markov jump theory and generalize the model established in [4].

The remainder of the paper is arranged as follows. The basic connectivity model is formulated and connectivity properties are discussed in section 2 for ad hoc networks with no nodes movement. Section 3 discusses the connectivity modeling and analysis for network with constant speed movement of nodes. Finally, some concluding remarks are given in section 4.

2. Basic Model
In this section, we assume the nodes connectivity change in an ad hoc mobile network is caused by power coverage change or linkage optimization, and all nodes stay still.

2.1. Basic Connectivity Model
Suppose there are N mobile nodes in an ad hoc network. The connectivity model presented in [4] is as follows:

Addition of $m_1$ new links with probability $p$:

$$\frac{dx_k(t)}{dt} = pm_1 \frac{1}{N} + pm_1 Q_1(x_k(t))$$

(2.1)

The first term of (2.1) represents the random selection of the starting point of a link, while the second term corresponds to the end point selection where $Q_1(x_k(t))$ denotes the probability that a node $k$ associated with $x_k(t)$ links at time $t$ is selected.

Rewiring of $m_2$ links with probability $q$:

$$\frac{dx_k(t)}{dt} = -qm_2 \frac{1}{N} + qm_2 Q_2(x_k(t))$$

(2.2)

The first term of (2.2) corresponds to the decrease of the number of links of the node where the link was removed from, and the second term corresponds to the increasing connectivity of the node that the link is reconnected to and based on probability $Q_2(x_k(t))$ which is defined similarly to $Q_1(x_k(t))$.

Removing $m_3$ existing links with probability $r$:

$$\frac{dx_k(t)}{dt} = -rm_3 A_k - rm_3 Q_3(x_k(t))$$

(2.3)

The first term of (2.3) corresponds to the decreasing connectivity because other nodes connected with this node select to remove their links with this node, while second term corresponds to the case that the node itself selects to remove one of its links. $A_k = \sum_{all\ link \ k} Q_3(x_i(t))/x_i(t)$ and $Q_3(x_i(t))$ is defined similarly to $Q_1(x_i(t))$.

With probability $1 - p - q - r$, no changes occur.
It is assumed that every state must repeat several times before the switching by continuum theory in [4]. Considering the random switching properties of the linkage of mobile nodes in ad hoc networks, we get the general model in the form of Markov jump differential equation:

$$\frac{dx_k(t)}{dt} = f_k(x_k(t), t, r(t)) \quad k = 1, 2, ..., N \tag{2.4}$$

Where \( f_k(x_k(t), t, r(t)) \) is the rate at which connections of node \( k \) change; \( r(t) \) is a Markov chain which takes values in a finite set \( S = \{1, 2, ..., M\} \) with generator \( P = (p_{ij})_{M \times M} \) given by

$$P[r(t + \Delta) = j | r(t) = i] = \begin{cases} p_{ij} \Delta + o(\Delta) & \text{if } i \neq j \\ 1 + p_{ii} \Delta + o(\Delta) & \text{if } i = j \end{cases} \tag{2.5}$$

where \( \Delta \geq 0 \) and \( p_{ij} \geq 0 \) is the transition rate from state \( i \) to state \( j \) while \( p_i = p_{ii} = \sum_{j \neq i} p_{ij} \).

Let

$$x(t) = [x_1(t), x_2(t), ..., x_N(t)]^T, \quad \frac{dx(t)}{dt} = [\frac{dx_1(t)}{dt}, \frac{dx_2(t)}{dt}, ..., \frac{dx_N(t)}{dt}]^T$$

$$f(x(t), t, r(t)) = [f_1(x_1(t), t, r(t)), f_2(x_2(t), t, r(t)), ..., f_N(x_N(t), t, r(t))]^T$$

We have

$$\frac{dx}{dt} = f(x(t), t, r(t)) \tag{2.6}$$

Equation (2.6) gives the general connectivity model of a mobile ad hoc network. In Subsection 2.2, we will discuss the properties of model (2.6).

2.2. Connectivity Analysis of Ad Hoc Networks

**Definition 2.1** (M-matrix) A square matrix \( A = (a_{ij})_{N \times N} \) is a nonsingular M-matrix if \( A \) can be expressed in the form \( A = sI - B \) with some \( B \geq 0 \) and \( s > \rho(B) \), where \( I \) is the identity matrix and \( \rho(B) \) is the spectral radius of \( B \).

**Definition 2.2** (\( h \)-th moment exponential decreasing) Let \( h, \lambda \) be positive numbers, and \( x(t) \) be a solution of equation (2.6) given the initial value \( x(0) = x_0 \in \mathbb{R}^N \). If \( \limsup_{t \to \infty} \frac{1}{t} \log\mathbb{E}[|x(t; x_0)|^h] = -\lambda \), where the \( | \cdot | \) denotes the Euclidean norm in \( \mathbb{R}^n \), the solution \( x(t; x_0) \) of equation (2.6) is \( h \)-th moment exponential decreasing.

**Lemma 2.1** For every \( i \in S \), there is a constant \( \alpha_i \in \mathbb{R} \) such that \( x^T f(x, t, i) \leq \alpha_i |x|^2 \) for all \( (x(t), t, i) \in \mathbb{R}^N \times R_+ \times S \). For \( 0 < h < 2 \), define a matrix \( A(h) = \text{diag}(-h\alpha_1, ..., -h\alpha_N) - P \). If \( A(h) \) is a nonsingular M-matrix, then the solution of (2.6) is \( h \)-th moment exponential decreasing, where generator \( P = (p_{ij})_{M \times M} \) is defined in (2.5).

**Proof:** It is proved by setting \( g(x, t, i) = 0 \) in theorem 4.3 of [6].

**Lemma 2.2** For every \( i \in S \), there is a constant \( \alpha_i \in \mathbb{R} \) such that \( x^T f(x, t, i) \leq \alpha_i |x|^2 \) for all \( (x(t), t, i) \in \mathbb{R}^N \times R_+ \times S \). For \( h > 2 \), define a matrix \( A(h) = \text{diag}(-h\alpha_1, ..., -h\alpha_N) - P \). If \( A(h) \) is a
nonsingular M-matrix, then the solution of (2.6) is $h$-th moment exponential decreasing, where generator $P = (p_{ij})_{M	imes M}$ is defined in (2.5).

**Proof:** It is proved by setting $g(x,t,i) = 0$ in theorem 4.8 of [6].

**Lemma 2.3** For every $i \in S$, there is a constant $\alpha_i \in R$ such that $x^T f(x,t,i) \leq \alpha_i |x|^2$ for all $(x(t),t,i) \in R^N \times R_+ \times S$. For $h > 0$, define a matrix $A(h) = \text{diag}(-h\alpha_1,...,-h\alpha_N) - P$. If $A(h)$ is a nonsingular M-matrix, then the solution of (2.6) is $h$-th moment exponential decreasing, where generator $P = (p_{ij})_{M	imes M}$ is defined in (2.5).

**Proof:** This lemma is obtained by combining lemma 2.1 and lemma 2.2.

Especially for $M = 4$, every node only has four states for switching, such as new link state, rewired link state, deleted link state and no change state discussed in section 2.1. For the $k$-th mobile nodes, we have:

For $r(t) = 1$, in the new link state

$$P(r(t) = 1) = p_1 = p, f(x_k(t),t,i) = pm_1 \frac{1}{N} + pm_1 Q_1(x_k(t))$$

For $r(t) = 2$, in the rewiring link state

$$P(r(t) = 2) = p_2 = q, f(x_k(t),t,2) = -qm_2 \frac{1}{N} + qm_2 Q_2(x_k(t))$$

For $r(t) = 3$, in the deleting link state

$$P(r(t) = 3) = p_3 = r, f(x_k(t),t,3) = -rm_3 A_k - rm_3 Q_3(x_k(t))$$

For $r(t) = 4$, in the no change state

$$P(r(t) = 4) = p_4 = 1 - p - q - r, f(x_k(t),t,4) = 0$$

We rewrite (2.6) as

$$\frac{dx_k(t)}{dt} = f(x_k,t,i),$$

where $i = 1,2,3,4, k = 1,...,N$.

**Theorem 2.1** If $A(h) = \text{diag}(-h\alpha_1,...,-h\alpha_N) - P$ is a nonsingular M-matrix, and for every $i \in \{1,2,3,4\}$, there is a constant $\alpha_i \in R$ such that $x^T f(x,t,i) \leq \alpha_i |x|^2$, or

$$\sum_{k=1}^{N} \left[ \alpha_1 x_k^2 - pm_1 x_k Q_1(x_k) - \frac{pm_1}{N} x_k \right] \geq 0$$

$$\sum_{k=1}^{N} \left[ \alpha_2 x_k^2 - qm_2 x_k Q_2(x_k) + \frac{qm_2}{N} x_k \right] \geq 0$$

$$\sum_{k=1}^{N} \left[ \alpha_3 x_k^2 + rm_3 x_k Q_3(x_k) + rm_3 x_k A_k \right] \geq 0$$

(2.8)

then the solution to (2.6), i.e. the linkage vector of $N$ nodes $x(t)$, is $h$-th moment exponential decreasing.

**Proof:** It results from lemma 2.3.
Remark 2.1 If the equation (2.6) is $h$-th moment exponential decreasing, we have the result followed $\limsup_{t \to \infty} -\frac{1}{t} \log(E[x(t; x_0)^h]) = -\lambda$, which means that $x(t) = [x_1(t), x_2(t), \ldots, x_N(t)]^T$ initiated with $x_0 = x(0) = [x_1(0), x_2(0), \ldots, x_N(0)]^T$ will approach to zero vector with the increase of time, in other words, the number of links $x_k(0)$ of the $k$-th node with the other nodes will be decreased to zero. If the conditions are satisfied in theorem 2.1, the network connectivity will decrease exponentially with time and eventually no nodes in the ad hoc networks will be able to communicate with each other. In other words, the nodes in the networks have some possibility to communicate with the others when either $A(h)$ or $f(x,t,i)$ does not satisfy the conditions stated in the theorem.

3. Connectivity Analysis with Nodes Moving at Constant Speed
Assume that all nodes move at the same constant speed $v$ but along random directions. The link generation rate for constant velocity situation is $4\rho v \sin \frac{\theta}{2}$ in [8], where $\rho$ is node density for $N$ nodes, $R$ the transmission range of a node, and $\theta$ a random variable uniformly distributed between 0 and $\pi$. Thus the average link generation rate is given by

$$\lambda_{gen} = \int_0^\pi 4\rho v \sin \frac{\theta}{2} d\theta = \frac{8}{\pi} \rho v$$

Consider the link generation rate is equal to the break rate for movement situation when the node moves with constant velocity, and the link change rate for no movement situation in (2.7), thus the total link change rate of the node $k$

$$\frac{dx_k}{dt} = f(x_k,t,r(t)) + \frac{16}{\pi} \rho v$$

(3.1)

It follows from Theorem 2.1 and Equations (2.7) and (3.1) that:

Theorem 3.1 If $A(h)$ is a nonsingular M-matrix, and for every $i \in S$, there is a constant $\alpha_i \in R$ such that

$$\sum_{k=1}^N [\alpha_1 x_k^2 - pm_1 x_k Q_1(x_k) - \frac{pm_1}{N} x_k - \frac{16}{\pi} \rho v] \geq 0$$

$$\sum_{k=1}^N [\alpha_2 x_k^2 - qm_2 x_k Q_2(x_k) + \frac{qm_2}{N} x_k - \frac{16}{\pi} \rho v] \geq 0$$

$$\sum_{k=1}^N [\alpha_3 x_k^2 + rm_3 x_k Q_3(x_k) + rm_3 x_k A_k - \frac{16}{\pi} \rho v] \geq 0$$

(3.2)

Then the solution to (3.2), or the linkage vector of $N$ nodes $x(t)$ is $h$-th moment exponential decreasing.

4. Conclusion
We built the connectivity model for ad hoc mobile networks using Markov jump theory, which generalizes the model developed in [4]. We provided the conditions that, if satisfied, will cause the network connectivity exponentially decreasing and eventually no nodes in the ad hoc networks will be able to communicate with each other.

It is a challenge to model the nodes linkage in an ad hoc network, especially mobile ad hoc network. We managed to establish the model for the network with nodes moving at a constant speed.

Acknowledgement
This work is partially supported by NSFC under granted number 70271001 and China Postdoctoral Fund under granted number 200203271.

References