On the numerical simulation of population dynamics with density-dependent migrations and the Allee effects

To cite this article: H N Sweilam et al 2008 J. Phys.: Conf. Ser. 96 012008

View the article online for updates and enhancements.

Related content
- Von Bertalanffy's dynamics under a polynomial correction: Allee effect and big bang bifurcation
  J. Leonel Rocha, A.K. Taha and D. Fournier-Prunaret
- Spatiotemporal stability and sensitivity analysis of a Holling type-II prey predator system with Allee effect
  G Basava Kumar and M N Srinivas
- Nonlinear focusing Manakov systems by variational iteration method and Adomian decomposition method
  N H Sweilam, M M Khader and R F Al-Bar

Recent citations
- A One Step Optimal Homotopy Analysis Method for Propagation of Harmonic Waves in Nonlinear Generalized Magnetothermoelasticity with Two Relaxation Times under Influence of Rotation
  S. M. Abo-Dahab et al
- Approximate analysis of population dynamics with density-dependent migrations and the Allee effects
  Syed Mohyud-Din et al
- Homotopy Perturbation Method and Variational Iteration Method for Harmonic Waves Propagation in Nonlinear Magnetothermoelasticity with Rotation
  Khaled A. Gepreel et al
On the numerical simulation of population dynamics with density-dependent migrations and the Allee effects

HN Sweilam\(^1\), MM Khader\(^2\) and FR Al-Bar\(^3\)

\(^1\)Department of Mathematics and Physics, Qatar University, P.O. Box 2713, Doha-Qatar

\(^2\) Department of Mathematics, Faculty of Science, Benha University, Benha-Egypt.

\(^3\) Department of Mathematics, Faculty of Science, Umm Al-Qura University, K.S.A.

Abstract. In this paper, the variational iteration method (VIM) and the Adomian decomposition method (ADM) are presented for the numerical simulation of the population dynamics model with density-dependent migrations and the Allee effects. The convergence of ADM is proved for the model problem. The results obtained by these methods are compared to the exact solution. It is found that these methods are always converges to the right solutions with high accuracy. Furthermore, VIM needs relative less computational work than ADM.

1. Introduction

Recently much attention has been devoted to various numerical methods which do not require discretization of space-time variables or linearization of the nonlinear differential equations, among which the variational iteration method (see [2], [6], [9]-[13], [20]-[23] and the reference cited therein) and the Adomian decomposition method (see [1], [3], [5], [7], [8], [14] and the reference cited therein) are widely used for this purpose. Many authors pointed out that the variational iteration method has merits over other methods and can overcome the difficulties arising in calculation of Adomian’s polynomials in Adomian decomposition method (see [16], [17], [19] and the references therein). The aim of this paper is to develop VIM and ADM to simulate the solutions of the model of population dynamics with density-dependent migrations and the Allee effect [4], [18]. This model can be described by the transient non-linear advection-diffusion-reaction equation of the form:

\[
\frac{\partial U}{\partial T} = -\frac{\partial}{\partial X} [\Theta(U)U - D \frac{\partial^2 U}{\partial X^2}] + F(U)U \quad X \in \Omega, \quad T > 0. \tag{1}
\]

The unknown field \(U = U(X,T)\) is the population density in \(\Omega \subset \mathbb{R}\) and \(T\). \(U\) changes in space and time due to the non-linear velocity field \(\Theta = \Theta(U)\), the diffusion \(D\) and the intrinsic growth rate \(F(U)\), which includes all local processes (such as birth, death and predation/harvesting).

\(^1\) To whom any correspondence should be addressed.
The model (1) specifies that the spatial distribution is affected by two physical processes, the advection and the isotropic diffusion of Fickian type [4], [18]. Here, we also consider a biological mechanism on the advection process in order to include the case when the species purposely migrates in some particular direction due to some chemical communication. These assumptions yield the following non-linear velocity field

$$\Theta(U) = \Theta_0 + \Theta_1 U.$$  \hfill (2)

In this speed of migration model (2), $\Theta_0$ is the density-independent migration velocity, which is known or might come from a hydrodynamic solver. The model (2) also assumes the existence of a density-dependent migration that varies linearly with the population density, where $\Theta_1$ depends on the species taxis. We assume here, for simplicity, that the fluid is incompressible (div($\Theta_0$) = 0 ) and $\Theta_0$ and $\Theta_1$ and the diffusion coefficient $D$ are constants, yielding

$$\frac{\partial U}{\partial t} + (\Theta_0 + 2\Theta_1 U)\frac{\partial U}{\partial x} = D \frac{\partial^2 U}{\partial x^2} + F(U)U.$$  \hfill (3)

Consider the growth dynamics with Allee effects given by

$$F(U)U = \bar{a} U(U - K_0)(K - U),$$  \hfill (4)

Where $K$ is the carrying capacity and $K_0$ is the measure of the Allee effects. When $K$ is constant, it is convenient to use the dimensionless variable $u = U/K$ so that (4) is re-written as:

$$f(u) = \alpha u(u - \beta)(1 - u),$$  \hfill (5)

where $\beta = K_0/K$ represents the strength of the Allee effects. The strong and the weak Allee effects occur when $0 < \beta < 1$ and $-1 < \beta < 0$, respectively. The parameter $\alpha = \alpha(\beta)$ is a normalization constant which is defined by a maximum growth rate, leading to a family of models. The qualitative results regarding the Allee effects and asymptotic rates of spread are independent from the choice of the normalization constant. With this assumption and using $t = T \alpha K^2$ and $x = X \frac{\sqrt{\alpha K^2}}{D}$, equation (3) can be written in the following dimensionless form:

$$u_t + (\Theta_0 + \Theta_1 u)u_x = u_{xx} - \beta u + (1 + \beta) u^2 - u^3,$$  \hfill (6)

where we used the additional dimensionless parameters $\Theta_0 = \frac{\Theta_0}{K \sqrt{\alpha D}}$ and $\Theta_1 = \frac{2\Theta_1}{\sqrt{\alpha D}}$. Hence, the population densities have been re-scaled so that $u \in [0,1]$ in $t \in [0,T_{\text{final}}]$. Travelling wave solutions are considered so that the (6) is solved in an unbounded domain with the following conditions at infinity: For (the species is at its carrying capacity); for $x \to -\infty \Rightarrow u = 0$ (the species is absent), some initial condition. Under these boundary conditions, one can find in [4] and the references sited therein, the asymptotic stability analysis of the travelling wave for the scaled diffusion–reaction equation.
The existence of wave fronts $u(x,t) = U(x - ct)$ was derived, relying on the properties of $g$. Here, $g(u) = -\beta u + (1 + \beta) u^2 - u^3$ and has at least two distinct zeros, $g_1 = 1$ and $g_0 = 0$; if there exists a strong Allee effect, there still is another zero between $g_0$ and $g_1$ at which the per capita growth rate is positive. For more details on this model, see [4] and [18].

2. Implementation of VIM

In this section, VIM will apply to the following nonlinear partial differential equation of the form:

$$u_t + (\theta_0 + \theta_1 u)u_x = u_{xx} - \beta u + (1 + \beta)u^2 - u^3,$$

subject to the initial condition $u(x,0) = f(x)$. First, we construct the correction functional:

$$u_{n+1} = u_n + \int_0^t \lambda(\tau) \left[ u_n + \left( \theta_0 + \theta_1 \hat{u}_n \right) \hat{u}_{nx} - \hat{u}_{nxx} + \beta \hat{u}_n - (1 + \beta)\hat{u}_n^2 + \hat{u}_n^3 \right] d\tau,$$

where $\lambda$ is a general Lagrange multiplier, $\hat{u}_n$, $\hat{u}_{nx}$, $\hat{u}_{nxx}$ denote restricted variations, i.e.

$$\delta \hat{u}_n = \delta \hat{u}_{nx} = \delta \hat{u}_{nxx} = 0.$$

Making the above correction functional stationary, we obtain the following stationary condition:

$$\lambda(\tau) = 0, \quad 1 + \lambda(\tau) \big|_{\tau = t} = 0.$$

The Lagrange multiplier, therefore, can be defined in the following form:

$$\lambda(\tau) = -1.$$

Substituting from (10) into (9) results the following iteration formula:

$$u_{n+1} = u_n - \int_0^t \left[ u_n + \left( \theta_0 + \theta_1 u_n \right) u_{nx} - u_{nxx} + \beta u_n - (1 + \beta)u_n^2 + u_n^3 \right] d\tau.$$

Now, if we start with the following initial approximation

$$u(x,0) = \frac{\beta \exp(\lambda_1 \xi_1) + \exp(\lambda_2 \xi_2)}{1 + \exp(\lambda_1 \xi_1) + \exp(\lambda_2 \xi_2)},$$

where, $\xi_i = x + \varphi_i$, $i = 1, 2$; $\lambda_1 = \beta/\sqrt{2}$ and $\lambda_2 = 1/\sqrt{2}$, and $\varphi_1, \varphi_2$ are arbitrary constants. Using the recurrence relation (11), we obtain the first components of the solution in the case ($\theta_0 = \theta_1 = 0$) in the following form:

$$u_0(x,t) = u(x,0).$$
\[ u_1(x,t) = u_0(x,t) - \frac{1}{(1 + \exp(\lambda_1 \xi_1) + \exp(\lambda_2 \xi_2))^3} \left[ t [-\beta \exp(\lambda_1 \xi_1)(-\beta + \lambda_1^2) + \exp(2\lambda_1 \xi_1) \beta + (\beta - \beta^2 + \lambda_1^2) + \exp(2\lambda_1 \xi_1) + \exp(2\lambda_2 \xi_2)(-1 + \beta)(\beta - \beta^2 + (\lambda_1 - \lambda_2)^2) + \exp(2\lambda_2 \xi_2)(-1 - \beta - \lambda_2^2) - \exp(\lambda_1 \xi_1 + 2\lambda_2 \xi_2)(-1 + \beta)(-1 + \beta + \lambda_2^2 - 2\lambda_1 \lambda_2 + \lambda_2^2) + \exp(\lambda_1 \xi_1 + \lambda_2 \xi_2)(-1 + 2\beta)\lambda_2^2 + 2(1 + \beta)\lambda_1 \lambda_2 + (-2 + \beta)\lambda_2^2) \right] \]

and so on. The rest of components of the iterative formula (11) were obtained in the same manner using the Mathematica package. The exact solution of the equation (8) [in the case (\theta_0 = \theta_1 = 0)] under the initial condition (12) is given by:

\[ u(x, t) = \frac{\beta \exp(\lambda_1 \xi_1)}{1 + \exp(\lambda_1 \xi_1) + \exp(\lambda_2 \xi_2)} \]

where, 

\[ \xi_1 = x - \eta_i t + \varphi_i, \quad i = 1, 2; \quad \eta_i = \sqrt{2}(1 + \beta) - 3\lambda_i; \quad \lambda_1 = \beta/\sqrt{2} \quad \text{and} \quad \lambda_2 = 1/\sqrt{2}, \quad \text{and} \quad \varphi_i, \varphi_2 \quad \text{are arbitrary constants. Here, we set} \ \beta = 0.2, \ \varphi_1 = 100 \quad \text{and} \quad \varphi_2 = -100. \]

The error behaviour for different time values are shown in figures 1-4 where the numerical results are obtained by using two terms only from the iterative formula (11). It is evident that the overall errors can be made smaller by adding new terms from the iteration formula.

3. Implementation of ADM

In this section, the ADM will apply to (8) and (12), so we rewrite (8) in the following form:

\[ L_t u = u_{xx} - \theta_0 u_x - \beta u + N(u) \]

where \[ L_t = \frac{\partial}{\partial t} \] is linear operator, \[ N(u) = -\theta_1 u_x + (1 + \beta) u^2 - u^3 \] is nonlinear operator.

By taken the inverse operator \[ L_t^{-1} (\cdot) = \int_0^t (\cdot) dt \] of (13), then the solution of (13) can be written in the form

\[ u(x,t) = u(x,0) + L_t^{-1} [u_{xx} - \theta_0 u_x - \beta u + N(u)] \]

The ADM assumes that the unknown solution \( u(x,t) \) can be expressed by an infinite series of the form:

\[ u(x,t) = \sum_{n=0}^{\infty} u_n (x,t) \]

and the nonlinear operator term \( N(u) \) can be decomposed by an infinite series of polynomials, given by:

\[ N(u) = \sum_{n=0}^{\infty} A_n \]

and so on. The rest of components of the iterative formula (11) were obtained in the same manner using the Mathematica package. The exact solution of the equation (8) [in the case (\theta_0 = \theta_1 = 0)] under the initial condition (12) is given by:

\[ u(x, t) = \frac{\beta \exp(\lambda_1 \xi_1)}{1 + \exp(\lambda_1 \xi_1) + \exp(\lambda_2 \xi_2)} \]

where, 

\[ \xi_1 = x - \eta_i t + \varphi_i, \quad i = 1, 2; \quad \eta_i = \sqrt{2}(1 + \beta) - 3\lambda_i; \quad \lambda_1 = \beta/\sqrt{2} \quad \text{and} \quad \lambda_2 = 1/\sqrt{2}, \quad \text{and} \quad \varphi_i, \varphi_2 \quad \text{are arbitrary constants. Here, we set} \ \beta = 0.2, \ \varphi_1 = 100 \quad \text{and} \quad \varphi_2 = -100. \]

The error behaviour for different time values are shown in figures 1-4 where the numerical results are obtained by using two terms only from the iterative formula (11). It is evident that the overall errors can be made smaller by adding new terms from the iteration formula.
the components \( u_n(x,t) \) will be determined recurrently and \( A_n \) are the Adomian’s polynomials of 
\( u_0, u_1, u_2, \ldots \) defined by:

\[
A_n = \frac{1}{n!} \left[ \frac{d^n}{ds^n} N \left( \sum_{i=0}^{n} s^i u_i \right) \right]_{s=0}, \quad n = 0, 1, 2, \ldots
\]  

(17)

Substituting from (15), (16) in (14), we can obtain the subsequent components:

\[
u_0(x,t) = u(x,0), \quad u_{n+1}(x,t) = L_t^{-1} \left( u_{nxx} - \theta_0 u_{nx} - \beta u_n \right) + L_t^{-1} (A_n), \quad n \geq 0.
\]  

(18)

One can use the general form of formula (17) for \( A_n \) as follows:

\[
A_0 = (1 + \beta) u_0^2 - \theta_1 u_1^3 u_0 x, \\
A_1 = 2(1 + \beta) u_0 u_1 - 3 u_2^2 u_0 x - \theta_1 u_1 u_0 x - \theta_0 u_0 u_1 x, \\
A_2 = \frac{1}{2} (-6 u_0^2 u_1 - 6 u_2^2 u_2 + (1 + \beta) (2 u_2^2 + 4 u_0 u_2) - \theta_1 (2 u_2 u_0 x + 2 u_1 u_1 x + 2 u_0 u_2 x)), \\
A_3 = \frac{1}{6} (-6 u_1^3 - 36 u_0 u_1 u_2 - 18 u_3^2 u_0 x + 12(1 + \beta) (u_2 u_1 + u_3 u_3) - 6 \theta_1 (u_3 u_0 x + u_1 u_1 x + u_2 u_2 x + u_3 u_3 x)).
\]

For numerical comparisons purpose, based on the ADM, we constructed the solution \( u(x,t) \) as:

\[
\lim_{n \to \infty} \Phi_n(x,t) = u(x,t), \quad \Phi_n(x,t) = \sum_{m=0}^{n-1} u_m(x,t), \quad n \geq 0.
\]  

(19)

To obtain the components of the solution, we start by substituting the initial condition (12) in (18):

\[
u_0(x,t) = u(x,0),
\]

\[
u_1(x,t) = \frac{1}{(1 + \exp(\lambda_1 \xi_1) + \exp(\lambda_2 \xi_2))^3} \left[ t [\beta \exp(\lambda_1 \xi_1) (-\beta + \lambda_1^2) - \exp(2\lambda_1 \xi_1) \beta (\beta - \lambda_1^2 + \lambda_1^2) \\
- \exp(2\lambda_1 \xi_1 + \lambda_2 \xi_2) (-\beta + \lambda_1 - \lambda_2)^2 + \exp(\lambda_2 \xi_2) \beta - \lambda_2^2 - \exp(2\lambda_2 \xi_2) (-\beta + \lambda_2^2) \\
+ \exp(\lambda_1 \xi_1 + \lambda_2 \xi_2) (-\beta + \lambda_1 - \lambda_2)^2 + \exp(\lambda_1 \xi_1 + \lambda_2 \xi_2) \beta - \lambda_1^2 + \lambda_2^2] \\
- 2(1 + \beta) \lambda_1 \lambda_2 - (-2 + \beta) \lambda_2^2 \right],
\]

and so on the other terms can be obtained in the case \((\theta_0 = \theta_1 = 0)\).

3.1. Convergence Analysis of the ADM

In this section, we will prove the convergence of ADM applied to equation (8). Let us define the Hilbert space \( H = L^2((\alpha(\beta) \times [0,T])) \), as a set of all applications

\[
u : (\alpha, \beta) \times [0,T] \to R \quad \text{with} \quad \int u^2(x,s)ds \, d\tau < +\infty.
\]  

(\alpha, \beta) \times [0,T]
Consider (8) with the notation \( L(u) = \frac{\partial u}{\partial t} \), then we can write (8) in the following operator form:

\[
L(u) = u_{xx} - \theta_0 u_x - \theta_1 u_x - \beta u + (1 + \beta)u^2 - u^3.
\]  

(20)

**Theorem:** (Sufficient conditions of convergence)
The ADM applied to the nonlinear equation (20) is converging towards a particular solution if the following two hypotheses are satisfied:

(H1): \( (L(u) - L(v), u - v) \geq m\|u - v\|^2 \), \( m > 0 \), \( \forall u, v \in H \);

(H2): \( \exists \ C(K) > 0, K > 0 \) such that \( \forall u, v \in H \) with \( \|u\| \leq K \), \( \|v\| \leq K \),

we have \( (L(u) - L(v), w) \leq C(K) \|u - v\| \|w\| \) \( \forall w \in H \).

**Proof:** To verify (H1) for the operator \( L(u) \), we have

\[
L(u) - L(v) = \frac{\partial^2}{\partial x^2} (u - v) - \theta_0 \frac{\partial}{\partial x} (u - v) - \frac{1}{2} \theta_1 \frac{\partial}{\partial x} (u^2 - v^2) - \beta(u - v) + (1 + \beta)(u^2 - v^2) - (u^3 - v^3).
\]

Then we claim:

\[
(L(u) - L(v), u - v) = \left( \frac{\partial^2}{\partial x^2} (u - v), u - v \right) - \theta_0 \left( \frac{\partial}{\partial x} (u - v), u - v \right) - \frac{1}{2} \theta_1 \left( \frac{\partial}{\partial x} (u^2 - v^2), u - v \right) - \\
\beta(u - v, u - v) + (1 + \beta)(u^2 - v^2, u - v) - (u^3 - v^3, u - v).
\]

(21)

Since \( \frac{\partial}{\partial x} \) and \( \frac{\partial^2}{\partial x^2} \) are differential operators in \( H \), then there exist constants \( \delta_1, \delta_2 \):

\[
\left( -\frac{\partial}{\partial x} (u - v), u - v \right) \geq \delta_1 \|u - v\| \|u - v\| = \delta_1 \|u - v\|^2,
\]

(22)

\[
\left( \frac{\partial^2}{\partial x^2} (u - v), u - v \right) \leq \delta_2 \|u - v\| \|u - v\| \leq \delta_2 \|u - v\|^2,
\]

(23)

and

\[
\left( \frac{\partial}{\partial x} (u^2 - v^2), u - v \right) \leq \delta_1 \|u^2 - v^2\| \|u - v\|,
\]

this according to Schwartz inequality. Now, by using the mean value theorem and the above relation we get:

\[
\left( \frac{\partial}{\partial x} (u^2 - v^2), u - v \right) \leq \delta_1 \left( u^2 - v^2 \right) \|u - v\| = 2\delta_1 \eta^2 \|u - v\|^2,
\]

where, \( u < \eta < v \) and \( \|u\|, \|v\| \leq K \). Therefore, \( \left( -\frac{\partial}{\partial x} (u^2 - v^2), u - v \right) \geq 2\delta_1 K^2 \|u - v\|^2 \)

(24)

Also, we have \( (u - v, u - v) \leq \|u - v\| \|u - v\| = \|u - v\|^2 \)

(25)

\[
(u^2 - v^2, u - v) \leq \left( u^2 - v^2, u - v \right) \leq 2\eta^2 \|u - v\|^2 = 2K \|u - v\|^2.
\]

(26)
\[ (u^3 - v^3, u - v) \leq \|u^3 - v^3\|_\infty \geq 3\eta^3 \|u - v\|^2 = 3K^2 \|u - v\|^2, \]  \hspace{1cm} (27)

substituting from (22)-(27) into (21), we get

\[ (L(u) - L(v), u - v) \geq (\delta_2 + \theta_0, 0) \eta_1 + 0.5 \theta_1 \eta_1 K + \beta + 2(1 + \beta)K + 3K^2 \|u - v\|^2 = m\|u - v\|^2, \]

where, \( m = \delta_2 + \theta_0, 0 \delta_1 + 0.5 \theta_1 \delta_1 K + \beta + 2(1 + \beta)K + 3K^2 \). Hence, we verified (H1).

To verify (H2) for the operator \( L(u) \), we have

\[
(L(u) - L(v), w) = \left( \frac{\partial}{\partial x}^2 (u-v), w \right) - \beta(u-v, w) + (1 + \beta)(u^2 - v^2, w) - (u^3 - v^3, w),
\]

\[
- \theta_0 \left( \frac{\partial}{\partial x} (u^2 - v^2), w \right) - \frac{1}{2} \theta_1 \left( \frac{\partial}{\partial x} (u^2 - v^2), w \right),
\]

\[
(28)
\]

therefore,

\[
(L(u) - L(v), w) \leq (1 + \theta_0, 0 + 0.5 \theta_1 K + \beta + 2(1 + \beta)K + 3K^2 \|u - v\|_\infty \|w\| = C(K)\|u - v\|_\infty \|w\|,
\]

where, \( C(K) = 1 + \theta_0 + 0.5 \theta_1 K + \beta + 2(1 + \beta)K + 3K^2 \). Hence, we verified (H2).

### 4. Special Cases of the Model

**Case I: No Migration:** \( \theta_0 = 0 \) and (6) reduces to:

\[
u_t = u_{xx} - \beta u + (1 + \beta)u^2 - u^3.
\]

(29)

The exact solution of (29) is

\[
u(x, t) = \beta \exp(\lambda_1 \xi_1) + \exp(\lambda_2 \xi_2),
\]

where, \( \xi_i = x - \eta_i t + \varphi_i, i = 1, 2 \); \( \eta_i = \sqrt{2} (1 + \beta) - 3 \lambda_i \); \( \lambda_1 = \beta / \sqrt{2} \) and \( \lambda_2 = 1 / \sqrt{2} \), and \( \varphi_1, \varphi_2 \) are arbitrary constants.

**Case II: Density-Independent Migration:**

In the case that the speed of the species migration does not depend on the population density e.g., when drifting with the wind, the dynamics of the population are described by the following equation:

\[
u_t + \theta_0 u_x = u_{xx} - \beta u + (1 + \beta)u^2 - u^3,
\]

where \( \theta_0 \) is the speed of advection.

Considering traveling wave coordinates, \((x, t) \rightarrow (z, t)\) where \( z = x - \theta_0 t \), so that \( u = \hat{u}(z, t) \), from (30) we obtain

\[
\hat{u}_t = \hat{u}_{xx} - \beta \hat{u} + (1 + \beta)\hat{u}^2 - \hat{u}^3.
\]

(31)
Equation (31) coincides with (29) and thus the exact solution of (29) gives also an exact solution of (31) with the obvious change $x \rightarrow z$.

**Case III: Density-Dependent Migration:**

In this section, we consider the case when the density-independent advection caused by environmental factors is absent and migration takes place due to biological mechanisms which are assumed to be density-dependent. Then $\theta_0 = 0$ and from (6) we arrive at the following equation:

$$u_t + \theta_1 uu_x = u_{xx} - \beta u + (1 + \beta)u^2 - u^3,$$

The exact solution of (32) is given by:

$$u(x, t) = \frac{\exp(\omega_1 \psi_1) + \exp(\omega_2 \psi_2)}{1 + \exp(\omega_1 \psi_1) + \exp(\omega_2 \psi_2)},$$

where, $\psi_1 = x - q_1 t + \epsilon_1$, $q_1 = (1 + \beta)\nu - (3 + \theta_1 \nu)\omega_i$; $i = 1, 2$; $\omega_i = \beta / \nu$, $\omega_2 = 2 / \nu$ such that $\nu = 0.5(\theta_1 + \sqrt{\theta_1^2 + 8})$ and $\varphi_1, \varphi_2$ are arbitrary constants.

**Case IV: General Case:**

In a general case, migrations can take place due to both density-dependent and density-independent factors. The dynamics of a given population are then described by full (6) where now $\theta_0 \neq 0$ and $\theta_1 \neq 0$. The exact solution in this case is given by:

$$u(x, t) = \frac{\exp[\omega_1 (x - (q_1 + \theta_2) t + \epsilon_1)] + \exp[\omega_2 (x - (q_2 + \theta_2) t + \epsilon_2)]}{1 + \exp[\omega_1 (x - (q_1 + \theta_2) t + \epsilon_1)] + \exp[\omega_2 (x - (q_2 + \theta_2) t + \epsilon_2)]},$$

where the notations are the same as in (32).

The figures 1-4, simulate the error between the exact solution and the both methods approximate solution of the above four cases respectively.

**Figure 1:** (Case I) The error at $x_0 = 20$ and $\theta_0 = \theta_1 = 0$.

**Figure 2:** (Case II) The error at $x_0 = 65$ and $\theta_0 = 0.1$, $\theta_1 = 0$. 
5. Conclusion
In this paper, VIM and ADM are applied to solve the model of population dynamics with density-dependent migrations and the Allee effects, the methods need much less computational work compared with traditional methods. We achieved a very good approximation with the actual solution of the model by using two terms of the iteration scheme derived above in the ADM and VIM. It is evident that the overall results come very close to the exact solution even using only few terms of the iteration formula. Errors can be made smaller by taking new terms of the iteration formulas. It is found that these methods are always converges to the right solutions with high accuracy. We found that the variational iteration method can overcome the difficulties arising in calculation of Adomian’s polynomials in Adomian decomposition method. Furthermore, VIM needs relative less computational work than ADM.

References
[16] Lesnic D 2002 Computers and Mathematics with Applications 44 13
[17] Lesnic D 2005 Communications in Nonlinear Science and Numerical Simulation 10 581
[18] Petrovskii S and Bai-Lian Li 2003 Mathematical Biosciences 186 79