

PAPER • OPEN ACCESS

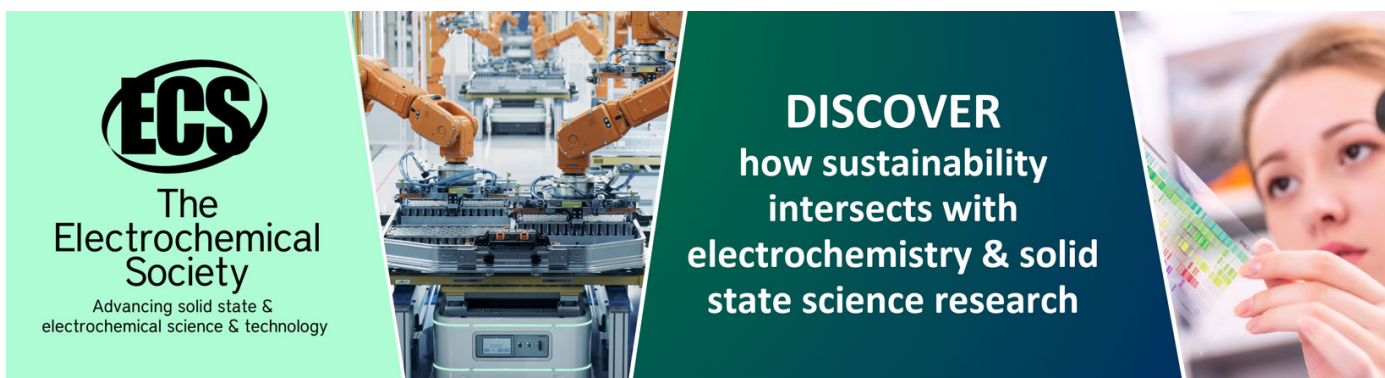
Intense quasiperiodic beam dynamics in accelerating system: mathematical model and optimization method

To cite this article: I D Rubtsova and D A Ovsyannikov 2017 *J. Phys.: Conf. Ser.* **941** 012092

View the [article online](#) for updates and enhancements.

You may also like

- [Effect of bismuth substitution for lead in CsPbBr₃ perovskite](#)
M V Elizárov, O V Lozkina, D I Yeletz et al.
- [Forward-backward correlations between intensive observables](#)
V Kovalenko and V Vechernin
- [Intense quasiperiodic beam dynamics investigation in accelerating system](#)
D A Ovsyannikov, I D Rubtsova and N V Lomonosova



ECS
The
Electrochemical
Society
Advancing solid state &
electrochemical science & technology

DISCOVER
how sustainability
intersects with
electrochemistry & solid
state science research

Intense quasiperiodic beam dynamics in accelerating system: mathematical model and optimization method

I D Rubtsova¹ and D A Ovsyannikov¹

¹St. Petersburg State University, 7/9 Universitetskaya nab., St. Petersburg, 199034, Russia

E-mail: i.ribtsova@spbu.ru, d.a.ovsyannikov@spbu.ru

Abstract. The paper is devoted to quasiperiodic intense beam dynamics in accelerating system. Particle interaction accounting is conducted with the use of well-known cloud-in-cell method. This numerical method is formalized, and mathematical model of space charge field intensity is suggested. Coulomb field intensity is presented in the form of integral over the domain of particle phase states. This permits beam evolution to be described by integro-differential equations. Controlled process quality is characterized by integral functional values. Beam dynamics optimization problem is formulated as trajectory ensemble control problem for dynamic system. Analytical expression of quality functional variation is obtained. It makes possible directed methods using for beam dynamics optimization.

1. Introduction

This paper is devoted to intense quasiperiodic beam dynamics modeling and optimization. Beam evolution mathematical model contains integral representation of beam space charge field.

It should be mentioned that the problems of modeling of self-consistent charged particle distributions are urgent and widely discussed by the researchers [1-11]. This paper deals with analytical representation of space charge field. Such an approach is applied by many authors e.g. [1,4-13]; it expands the potentialities of beam dynamics analysis and optimization. Axially symmetric beam Coulomb field is often expressed in the form of Fourier-Bessel series [6-8,11].

In this paper the version of clouds-in-cells method is used for space charge field determination [7]. This convenient for computation and time-saving method is usually considered to be exclusively numerical. In the article the formalization of this method is performed and the analytical expression of Coulomb field intensity is obtained in the form of integral over particle phase states domain.

The integral space charge field representation makes it possible to introduce integro-differential beam dynamics model [1,4,6,9,10,13] and to propose mathematical optimization method. This method is based on the approach formulated in [1,12,14,15] and successfully applied by a number of researchers for investigation of different problems of beam dynamics optimization [10,13,16-19].

In the article presented we consider beam evolution to be a complex of bunch center motion and the motions of particles of bunch. Beam quasiperiodicity is taken into account. Controlled dynamic process is described by a system of integro-differential equations. Beam quality criterion is presented in the form of integral functional defined at beam trajectories. Beam dynamics optimization problem is formulated as trajectory ensemble control problem. An analytical representation of quality functional variation is obtained; it provides the possibility of directed optimization methods applying.



2. Beam dynamics equations

Consider a model of intense beam longitudinal dynamics in some accelerating system. We will investigate the processes at beam quasiperiod (i.e. one bunch evolution) and determine space charge field under the assumption of beam spatial periodicity.

Particle beam is supposed to be axially symmetric. A bunch is presented by N model particles moving in conducting tube of radius a . Model particles are supposed to be “thick” disks (disks-clouds) of radius R and thickness 2Δ (for resting particle).

Let us introduce independent variable $\tau = ct$ (where c is the speed of light, t is the time) and cylindrical coordinates r, θ, z ; may Oz axis be aligned with channel axis. Let particle phase state be presented by vector (z, p) , where z is longitudinal coordinate, p is reduced impulse.

Controlled beam evolution is considered to be a complex of bunch center motion and model particle motions. Bunch center phase state is characterized by vector (z_c, p_c) . Hence longitudinal beam dynamics is described by the equations:

$$\frac{dz_i}{d\tau} = \frac{p_i}{\sqrt{1+p_i^2}}, \quad \frac{dp_i}{d\tau} = \frac{\zeta}{c^2} E^{(RF)}(\tau, z_i, \mathbf{u}) + \frac{|\zeta|}{c^2} E^{(int)}(\tau, z_i, p_i, p_c), \quad (1)$$

$$\frac{dz_c}{d\tau} = \frac{p_c}{\sqrt{1+p_c^2}}, \quad \frac{dp_c}{d\tau} = \frac{\zeta}{Nc^2} \sum_{i=1}^N E^{(RF)}(\tau, z_i, \mathbf{u}) \quad (2)$$

with initial conditions

$$z_i(0) = z_{0i}, \quad p_i(0) = p_{0i}, \quad i = \overline{1, N}, \quad z_c(0) = \frac{1}{N} \sum_{i=1}^N z_{0i}, \quad p_c(0) = \frac{1}{N} \sum_{i=1}^N p_{0i}. \quad (3)$$

Here i is model particle number, (z_i, p_i) is i -th particle phase vector; ζ is particle specific charge; $E^{(RF)}$, $E^{(int)}$ are the longitudinal intensity components characterizing respectively the effect of RF field and space charge field on the disk-cloud as a whole; $\mathbf{u}$ is program control vector function.

Note that model particles are considered to be positively charged in space charge field accounting; $E^{(int)}$ is Coulomb field intensity created by positively charged particles.

3. Particle interaction account

Let some control function $\mathbf{u}$ and τ value of independent variable be fixed. Let us describe i -th model particle form-factor in laboratory reference frame by piecewise continuous even function $\chi(z - z_i, p_i)$ taking nonzero values only at interval $(-d_i, d_i)$; $2d_i = 2\Delta/\sqrt{1+p_i^2}$ is cloud size.

Spatial quasiperiod length in laboratory reference frame is $2H = p_c \lambda / \sqrt{1+p_c^2}$, where λ is RF field first harmonic wavelength.

3.1. Phase domain partitioning into elements

Let us enclose the set of model particles phase states in some rectangle G and partition the rectangle into the elements $G_s, s = \overline{1, S}$ by any way. Suppose the elements to be connected compact subsets of positive measure. Assume q_{is} to be the charge value contributed by i -th model particle to s -th element. Charge values of elements are $q_s = \sum_{i=1}^N q_{is}$, $s = \overline{1, S}$. Let us choose the points $(\xi_s, \eta_s) \in G_s$, $s = \overline{1, S}$ arbitrarily.

Let us approximate the action of the element G_s (with all its periodic images) on i -th model particle by the force F_{is} presenting the action of uniformly charged “thick” disk D_s with radius R , thickness $2h$, charge q_s , center position ξ_s at structure axis and reduced impulse η_s (and all periodic images of this disk) on i -th model particle. Note that $2h = 2H/J$, J is natural number, $10 \leq J \leq N$.

We will describe the action of periodic beam on i -th model particle by the intensity $E_i = \sum_{s=1}^S F_{is}/q$, where q is model particle charge assumed to be positive.

3.2. Space charge field intensity

The intensity expression is obtained on the basis of the formulae presented in [7]. The dependence $2H = 2H(p_c)$ is taken into account. The result is:

$$E_i = \sum_{s=1}^S \frac{q_s}{Q} V(z_i, p_i, p_c, \xi_s, \eta_s), \quad (4)$$

$$V(z, p, p_c, \xi, \eta) = \sum_{k=1}^{\infty} \frac{I_0 \lambda a J_1^2(\mu_k R/a)}{c \varepsilon_0 \gamma h \mu_k^3 J_1^2(\mu_k) \text{sh}(\mu_k \gamma H/a)} \int_{-\infty}^{\infty} C_k(y, p_c, \xi, \eta) \chi(y - z, p) dy, \quad (5)$$

$$C_k(y, p_c, \xi, \eta) = \cosh\left(\frac{\mu_k \gamma}{a} (H - |y - \xi - h|)\right) - \cosh\left(\frac{\mu_k \gamma}{a} (H - |y - \xi + h|)\right) \quad (6)$$

Here $Q = I_0 \lambda / c$ is bunch charge value; I_0 is average beam current; ε_0 is electric constant; $\gamma = \sqrt{1 + \eta^2}$, $J_0(x)$ and $J_1(x)$ are Bessel functions of the first kind of order 0 and 1 correspondingly, $\mu_k, k = 1, 2, \dots$ are the zeros of Bessel function $J_0(x)$. Notice that the formulae (4)-(6) are valid under the assumption $|z_i - \xi_s| \leq H$, $s = \overline{1, S}$; in the case $|z_i - \xi_s| > H$ the coordinate ξ_s should be replaced by its periodic image nearest to z_i .

3.3. Particle phase density and integral Coulomb field representation

Now let us suppose particle distribution to be continuous; particle phase states compose the domain $M_{\tau, u}$. Consider a smooth function $\rho(\tau, z, p)$ satisfying the condition:

$$\int_{G_s} \rho(\tau, \xi, \eta) d\xi d\eta = q_s / Q, \quad s = \overline{1, S} \quad (7)$$

for any mode of rectangle G partitioning. The relation (4) right side can be viewed as integral sum and

$$\lim_{d_{max} \rightarrow 0} \sum_{s=1}^S \frac{q_s}{Q} V(z_i, p_i, p_c, \xi_s, \eta_s) = \int_{M_{\tau, u}} V(z_i, p_i, p_c, \xi, \eta) \rho(\tau, \xi, \eta) d\xi d\eta,$$

where d_{max} is the maximum of element diameters. So mathematical model of Coulomb field may be suggested in the form:

$$E^{(int)}(\tau, z, p, p_c) = \int_{M_{\tau, u}} V(z, p, p_c, \xi, \eta) \rho(\tau, \xi, \eta) d\xi d\eta. \quad (8)$$

3.4. Particular case

Let us partition the rectangle G into identical cells by straight lines parallel to coordinate axes (J cells along Oz axis and L cells along Op axis, $S = JL$). Let $q_{j,l}$, $j = \overline{1, J}$, $l = \overline{1, L}$ be the cell charges and (ξ_j, η_l) , $j = \overline{1, J}$, $l = \overline{1, L}$ be the points chosen in the cells arbitrarily. Besides, let us neglect the difference between cell impulses and assume $\eta_l = p_c$, $l = \overline{1, L}$. In addition, suppose model particles to have the equal thickness $d = \Delta / \sqrt{1 + p_c^2}$ (in laboratory reference frame). In this case integral expression of space charge field intensity takes the form:

$$E^{(int)}(\tau, z, p_c) = \int_{M_{\tau, u}} \hat{V}(z, p_c, \xi) \rho(\tau, \xi, \eta) d\xi d\eta, \quad (9)$$

$$\hat{V}(z, p_c, \xi) = \sum_{k=1}^{\infty} \frac{I_0 \lambda a J_1^2(\mu_k R/a)}{c \varepsilon_0 \gamma_c h \mu_k^3 J_1^2(\mu_k) \text{sh}(\mu_k \gamma_c H/a)} \int_{-\infty}^{\infty} \hat{C}_k(y - \xi, p_c) \chi(y - z, p_c) dy, \quad (10)$$

$$\hat{C}_k(v, p_c) = \cosh((\mu_k \gamma_c / a) (H - |v - h|)) - \cosh((\mu_k \gamma_c / a) (H - |v + h|)) \quad (11)$$

Here $2h = 2H/J$ is the cell size along Oz axis; $\gamma_c = \sqrt{1 + p_c^2}$; the condition $|z - \xi| \leq H$ is imposed in view of the assumption of beam periodicity (similarly to the formulae (4)-(6)).

Computational formula (4) is now represented as follows:

$$E_i = \sum_{j=1}^J Q_j / Q \hat{V}(z_i, p_c, \xi_j), \quad (12)$$

where $Q_j = (\sum_{l=1}^L q_{j,l})$, $j = \overline{1, J}$. Now it is enough to introduce the grid on variable z only, and Q_j , $j = \overline{1, J}$ are the charges of grid cells. The formulae (12),(10)-(11) correspond to Coulomb field calculation with the use of clouds-in-cells method based on spatial quasiperiod partitioning into the cells [7,8].

4. Integro-differential beam dynamics model

Let us generalize beam dynamics model (1)-(3). Primarily, let us take into account integral representation (8) of space charge field.

Besides, we have an approximate equality

$$\int_{M_{\tau,u}} \frac{\zeta}{c^2} E^{(RF)}(\tau, \xi, \mathbf{u}) \rho(\tau, \xi, \eta) d\xi d\eta \approx \frac{\zeta}{Nc^2} \sum_{i=1}^N E^{(RF)}(\tau, z_i, \mathbf{u}) \quad (13)$$

presenting Monte Carlo method of calculation of the integral in left-hand side. The approximation accuracy increases with increasing model particles number. Consequently, the integral in relation (13) can be viewed as mathematical model of averaged RF force in right-hand side.

Consider dynamic controlled process (beam evolution) described by the equations:

$$\frac{d\mathbf{x}}{d\tau} = \mathbf{f}(\tau, \mathbf{x}, \mathbf{x}_c, \mathbf{u}) = \mathbf{f}_1(\tau, \mathbf{x}, \mathbf{u}) + \int_{M_{\tau,u}} \mathbf{f}_2(\tau, \mathbf{x}, \mathbf{x}_c, \mathbf{y}_\tau) \rho(\tau, \mathbf{y}_\tau) d\mathbf{y}_\tau, \quad (14)$$

$$\frac{d\mathbf{x}_c}{d\tau} = \mathbf{f}_c(\tau, \mathbf{x}_c, \mathbf{u}) = \mathbf{f}_{c1}(\tau, \mathbf{x}_c) + \int_{M_{\tau,u}} \mathbf{f}_{c2}(\tau, \mathbf{y}_\tau, \mathbf{u}) \rho(\tau, \mathbf{y}_\tau) d\mathbf{y}_\tau, \quad (15)$$

$$\frac{\partial \rho(\tau, \mathbf{x})}{\partial \tau} + \frac{\partial \rho(\tau, \mathbf{x})}{\partial \mathbf{x}} \mathbf{f}(\tau, \mathbf{x}, \mathbf{x}_c, \mathbf{u}) + \rho(\tau, \mathbf{x}) \operatorname{div}_{\mathbf{x}} \mathbf{f}(\tau, \mathbf{x}, \mathbf{x}_c, \mathbf{u}) = 0 \quad (16)$$

with initial conditions

$$\mathbf{x}(0) = \mathbf{x}_0 \in M_0, \quad \mathbf{x}_c(0) = \int_{M_0} \mathbf{x}_0 \rho_0(\mathbf{x}_0) d\mathbf{x}_0, \quad (17)$$

$$\rho(0, \mathbf{x}(0)) = \rho_0(\mathbf{x}_0), \quad \mathbf{x}_0 \in M_0. \quad (18)$$

Here $\tau \in [0, T]$ is independent variable; T is a constant; $\mathbf{x}, \mathbf{x}_c$ are phase vectors of systems (14) and (15) respectively; $\mathbf{u}(\tau)$ is a control; vector functions $\mathbf{f}_1, \mathbf{f}_{c1}, \mathbf{f}_{c2}$ are determined by the method of external fields modeling; vector function $\mathbf{f}_2$ is determined by the method of particle interaction account; $\rho(\tau, \mathbf{x})$ is phase density defined on system (14) trajectories; M_0 is initial domain in phase space of the system (14); $\rho_0(\mathbf{x})$ is initial phase density; $M_{\tau,u} = \{\mathbf{x}_\tau = \mathbf{x}(\tau, \mathbf{x}_0, \mathbf{u}) : \mathbf{x}_0 \in M_0\}$. All the functions in the equations (14)-(16) are supposed to be rather smooth to use mathematical optimization methods [1]. Control $\mathbf{u}(\tau)$ is supposed to be piecewise-continuous vector-function taking values in a compact.

Assuming $\mathbf{x} = (x_1, x_2)^* = (z, p)^*$, $\mathbf{x}_c = (x_{c1}, x_{c2})^* = (z_c, p_c)^*$ (where $*$ denotes transposition),

$$\mathbf{f}_1(\tau, \mathbf{x}, \mathbf{u}) = \begin{pmatrix} x_2 / \sqrt{1 + x_2^2} \\ (\zeta / c^2) E^{(RF)}(\tau, x_1, \mathbf{u}) \end{pmatrix}, \quad \mathbf{f}_2(\tau, \mathbf{x}, \mathbf{x}_c, \mathbf{y}) = \begin{pmatrix} 0 \\ (|\zeta| / c^2) \hat{V}(x_1, x_{c2}, y_1) \end{pmatrix},$$

$$\mathbf{f}_{c1}(\tau, \mathbf{x}_c) = \begin{pmatrix} x_{c2} / \sqrt{1 + x_{c2}^2} \\ 0 \end{pmatrix}, \quad \mathbf{f}_{c2}(\tau, \mathbf{y}, \mathbf{u}) = \begin{pmatrix} 0 \\ (\zeta / c^2) E^{(RF)}(\tau, y_1, \mathbf{u}) \end{pmatrix},$$

one can obtain the system (1)-(2), where $E^{(int)}$ is presented in terms of integral (9) and averaged value of external force is replaced by integral in accordance with (13). In addition, initial bunch center phase state is presented in integral form (see (3), (17)).

Following the approach [1,4], it is not hard to prove that smooth function $\rho(\tau, z, p)$ satisfying the condition (7) for any mode of rectangle G partitioning at any $\tau \in [0, T]$, satisfies the equation (16) with initial condition (18).

5. Trajectory ensemble control problem

Quality criterion of dynamic control process (14)-(16) is presented by integral functional

$$I(\mathbf{u}) = \int_0^T \int_{M_{\tau,u}} \Phi(\tau, \mathbf{x}_\tau, \mathbf{x}_c(\tau)) \rho(\tau, \mathbf{x}_\tau) d\mathbf{x}_\tau d\tau + \int_0^T \Phi_c(\tau, \mathbf{x}_c(\tau)) d\tau + \int_{M_{T,u}} g(T, \mathbf{x}_T, \mathbf{x}_c(T)) \rho(T, \mathbf{x}_T) d\mathbf{x}_T + g_c(T, \mathbf{x}_c(T)). \quad (19)$$

All integrands and free term in right-hand side of formula (19) are supposed to be smooth functions.

Choosing the proper functions in quality criterion (19), one can formulate the problem of beam dynamics optimization as the problem of trajectory ensemble control of dynamic process (14)-(16).

Let us consider the problem of quality criterion (19) minimization with respect to control $\mathbf{u}$.

6. Quality functional variation

Quality functional (18) variation is obtained on the basis of the approach [1]:

$$\delta I(\mathbf{u}, \Delta \mathbf{u}) = - \int_0^T \int_{M_{\tau,u}} (\Psi^*(\tau, \mathbf{x}_\tau, \mathbf{x}_c) \Delta \mathbf{u} f_1(\tau, \mathbf{x}_\tau, \mathbf{u}) + \Lambda^*(\tau, \mathbf{x}_c) \Delta \mathbf{u} f_{c2}(\tau, \mathbf{x}_\tau, \mathbf{u})) \rho(\tau, \mathbf{x}_\tau) d\mathbf{x}_\tau d\tau. \quad (20)$$

Here $\Delta \mathbf{u}$ is control $\mathbf{u}$ variation; $\Delta \mathbf{u}$ designates the increment of any function with respect to argument $\mathbf{u}$ only; vector functions $\Psi(\tau, \mathbf{x}, \mathbf{x}_c)$ and $\Lambda(\tau, \mathbf{x}_c)$ satisfy on the trajectories of dynamic process (14)-(16) the auxiliary system of integro-differential equations

$$\begin{aligned} \frac{d\Psi(\tau, \mathbf{x}, \mathbf{x}_c)}{d\tau} &= \left(\frac{\partial \Phi(\tau, \mathbf{x}, \mathbf{x}_c)}{\partial \mathbf{x}} \right)^* - \left(\frac{\partial f(\tau, \mathbf{x}, \mathbf{x}_c, \mathbf{u})}{\partial \mathbf{x}} \right)^* \Psi(\tau, \mathbf{x}, \mathbf{x}_c) - \\ &- \int_{M_{\tau,u}} \left(\frac{\partial f_2(\tau, \mathbf{y}_\tau, \mathbf{x}_c, \mathbf{x})}{\partial \mathbf{x}} \right)^* \Psi(\tau, \mathbf{y}_\tau, \mathbf{x}_c) \rho(\tau, \mathbf{y}_\tau) d\mathbf{y}_\tau - \\ &- \left(\int_{M_{\tau,u}} \left(\frac{\partial f_{c2}(\tau, \mathbf{x}_\tau, \mathbf{u})}{\partial \mathbf{x}} \right)^* \rho(\tau, \mathbf{x}_\tau) d\mathbf{x}_\tau \right) \Lambda(\tau, \mathbf{x}_c), \\ \frac{d\Lambda(\tau, \mathbf{x}_c)}{d\tau} &= \left(\frac{\partial \Phi(\tau, \mathbf{x}, \mathbf{x}_c)}{\partial \mathbf{x}_c} \right)^* + \left(\frac{\partial \Phi_c(\tau, \mathbf{x}_c)}{\partial \mathbf{x}_c} \right)^* - \left(\frac{\partial f_{c1}(\tau, \mathbf{x}_c)}{\partial \mathbf{x}_c} \right)^* \Lambda(\tau, \mathbf{x}_c) - \\ &- \left(\int_{M_{\tau,u}} \left(\frac{\partial f_2(\tau, \mathbf{x}, \mathbf{x}_c, \mathbf{y}_\tau)}{\partial \mathbf{x}_c} \right)^* \rho(\tau, \mathbf{y}_\tau) d\mathbf{y}_\tau \right) \Psi(\tau, \mathbf{x}, \mathbf{x}_c). \end{aligned}$$

with the following conditions at $\tau = T$:

$$\begin{aligned} \Psi(T, \mathbf{x}(T), \mathbf{x}_c(T)) &= - \left(\frac{\partial g(T, \mathbf{x}(T), \mathbf{x}_c(T))}{\partial \mathbf{x}} \right)^*, \\ \Lambda(T, \mathbf{x}_c(T)) &= - \left(\frac{\partial g(T, \mathbf{x}(T), \mathbf{x}_c(T))}{\partial \mathbf{x}_c} \right)^* - \left(\frac{\partial g_c(T, \mathbf{x}_c(T))}{\partial \mathbf{x}_c} \right)^*. \end{aligned}$$

The analytical representation (20) of quality criterion variation provides the possibility of directed optimization methods application in beam dynamics optimization problems. It may be beneficial to combine the gradient optimization with random search [20].

References

- [1] Ovsyannikov D A 1990 *Modeling and Optimization of Charged Particle Beam Dynamics* (Leningrad: Leningrad State University Press) p 312
- [2] Drivotin O I and Ovsyannikov D A 2009 Self-consistent distributions for charged particle beam in magnetic field *Int. J. Mod. Phys. A* **24** (5) pp 816-42
- [3] Bondarenko T V Polozov S M and Sumbaev A P 2016 Numerical simulation of the beam loading effect at the LUE-200 accelerator *Physics of Particles and Nuclei Letters* **13** (7), pp 919-22

- [4] Ovsyannikov D A and Edamenko N S 2013 Modeling of charged particle beam dynamics *Vestnik St. Petersburg University, Ser. 10: Applied Mathematics, Informatics, Control Processes* **2** 61–66
- [5] Kozynchenko V A 2014 The modeling of charged particle interactions in the elliptic beam *Proc. 20th Int. Workshop: Beam Dynamics & Optimization (BDO) (30 June-4 July 2014 St. Petersburg)* (St. Petersburg: IEEE) pp 94-95 DOI: 10.1109/BDO.2014.6890040
- [6] Rubtsova I D 2014 On quasiperiodic beam of interacting particles dynamics modeling *Vestnik St. Petersburg University, Ser. 10: Applied Mathematics, Informatics, Control Processes* **1** 104–19
- [7] Ovsyannikov D A, Rubtsova I D and Kozynchenko V A 2013 *Some Problems of Intense Charged Particle Beams Modeling in Linear Accelerators* (St. Petersburg: VVM publishing office) p 144
- [8] Rubtsova I D and Suddenko E N 2012 Investigation of program and perturbed motions of particles in linear accelerator *Proc. XXIII Russian Particle Accelerator Conf. RuPAC-2012 (24-28 September 2012 St. Petersburg)*(Geneva: JACoW <http://www.JACoW.org>) pp 367-69
- [9] Rubtsova I D 2014 Integral-differential model of quasi-periodic beam longitudinal dynamics *Proc. 20th Int. Workshop: Beam Dynamics & Optimization (BDO) (30 June-4 July 2014 St. Petersburg)* (St. Petersburg: IEEE) p 144
- [10] Rubtsova I D 2016 On modeling and optimization of intense quasiperiodic beam dynamics *Proc. XXV Russian Particle Accelerator Conf. RuPAC-2016 (21-25 November 2016 St. Petersburg)*(Geneva: JACoW <http://www.JACoW.org>) pp 363-66
- [11] Rubtsova I D 2016 Analytical Approach to Quasiperiodic Beam Coulomb Field Modeling, *II Conference on Plasma&Laser Research and Technologies (2016), Journal of Physics: Conference Series* **747** No 1 012074 <http://iopscience.iop.org/1742-6596/747/1/012074>
- [12] Ovsyannikov D A 2012 Mathematical modeling and optimization of beam dynamics in accelerators *Proc. XXIII Russian Particle Accelerator Conf. RuPAC-2012 (24-28 September 2012 St. Petersburg)*(Geneva: JACoW <http://www.JACoW.org>) pp 68-72
- [13] Rubtsova I D 2014 Mathematical optimization model of longitudinal beam dynamics in klystron-type buncher *Proc. XXIV Russian Particle Accelerator Conf. RuPAC-2014 (6-10 October 2014 Obninsk)*(Geneva: JACoW <http://www.JACoW.org>) pp 66-68
- [14] Ovsyannikov D A 1997 Modeling and Optimization Problems of Charged Particle Beam Dynamics *Proc. European Control Conf. ECC'97 (1-4 July 1997 Brussels Belgium)* **4** pp 1463-67
- [15] Ovsyannikov D A, Ovsyannikov A D, Vorogushin M F, Svistunov Yu A and Durkin A P 2006 Beam dynamics optimization: models, methods and applications *Nuclear Instruments and Methods in Physics Research A* **558** (1) 11-19
- [16] Rubtsova I D 2015 Optimization of iterative beam dynamic process *Proc. III Int. Conf. "Stability and Control Processes" in Memory of V.I.Zubov (SCP) (5-9 October 2015 St. Petersburg)* ed L A Petrosyan and A P Zhabko (St. Petersburg: IEEE) pp 198-200 DOI: 10.1109/SCP.2015.7342092
- [17] Zavadsky S V, Ovsyannikov D A and Chung S-L 2009 Parametric Optimization Methods for the Tokamak Plasma Control Problem *Int. J. Mod. Phys. A* **24** No 5 pp.1040–47
- [18] Ovsyannikov A D, Ovsyannikov D A, Altsybeyev V V, Durkin A P and Papkovich V G 2014 Application of Optimization Techniques for RFQ Design *Problems of Nuclear Science and Engineering* **91** No 3 pp 116-19
- [19] Ovsyannikov D A and Altsybeyev V V 2013 Mathematical Optimization Model for Alternating-Phase Focusing (APF) Linac *Problems of Nuclear Science and Engineering* No 4 p 93
- [20] Vladimirova L V 2014 Global Extremum Search on the Basis of Density and Its Mode Estimation *Proc. 20th Int. Workshop: Beam Dynamics & Optimization (BDO) (30 June-4 July 2014 St. Petersburg)* (St. Petersburg: IEEE) p 186