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Crossover in mesoscopic conductance fluctuations in a quasi-one-dimensional system

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Abstract. We study the conductance fluctuations in the crossover between the ballistic and diffusive regimes of phase coherent transport. For a quasi-1D disordered system, the correlation function of the conductance at different frequencies is calculated beyond the diffusion approximation. The result obtained establishes the interrelation between conductance fluctuations in the crossover regime and the characteristics of the disordered system. The frequency dependence of different contributions to the correlation function is analyzed at subdiffusion length scales.

1. Introduction

Previous studies of mesoscopic conductance fluctuations (see, e.g., Refs. [1–11]) were mostly focused on the diffusive regime of wave transport where length L of the sample is much larger than transport mean free path l_{tr} . In this regime, the effect of "universal" conductance fluctuations (UCF) is observed [1, 4]. The variance of the dimensionless conductance appears to be of the order of unity and, to a large extent, does not depend on the size of the sample. Not much is known as regards the crossover from ballistic to diffusive regime and, correspondingly, the fluctuations at subdiffusion length scales, $L \leq l_{tr}$. The currently available results concerned with the crossover regime are based mostly on direct numerical modelling [12–15]. Analytical results within the random matrix theory (RMT) approach [4, 16–18] were obtained only for waves identical in frequency.

In this paper we present a result of diagrammatic calculations for the conductance correlation function of a quasi-1D system (a waveguide with bulk disorder). The correlation function is expressed explicitly in terms of the cross-section of scattering by inhomogeneities of the medium and the propagators that obey the standard transport equation. In the large-length limit, our result transforms with no divergencies to the well-known diffusion formula [6, 8]. The crossover between the quasi-ballistic and diffusive regimes is studied for a system of point-like centers. Our calculations within the two-stream version of the discrete-ordinate method generalize the well-known RMT formula for the conductance variance [16–18] to the case of waves different in frequency.

2. Results of diagrammatic calculations

We consider transmission of classical waves through a disordered waveguide of length L . The dimensionless conductance (or transmittance) of the waveguide can be defined as the sum of



transmission coefficients T_{ab} connecting incoming and outgoing modes a and b , respectively (see, e.g., Refs. [4–6])

$$G = \sum_{a,b} T_{ab}. \quad (1)$$

The transmission coefficients T_{ab} depend on spatial configuration of the scattering centers and vary from sample to sample. Under conditions of weak localization ($G \gg 1$), the value of $\langle G \rangle$ averaged over an ensemble of disordered samples is governed, within the standard impurity technique, by the sum of ladder diagrams. The correlation function of conductance fluctuations at different frequencies $C(\Delta\omega) = \langle \delta G(\omega_0 + \Delta\omega/2) \delta G(\omega_0 - \Delta\omega/2) \rangle$ ($\Delta\omega$ is the shift of incident waves in frequency, ω_0 is the carrying frequency) can be expressed in terms of the ensemble-average fourth moment of a wave field and represented as expansion in orders of interference between ladders. Each interference event between the ladders contains the Hikami vertex [19]. The correlation function is governed by diagrams containing two vertices (see Fig.1). In the presence of time-reversal symmetry, the diagrams shown explicitly in Fig.1 should be supplemented by those that are obtained by interchanging initial and final states in one pair of conjugated wave fields. These diagrams contain the maximally crossed internal graphs (or cooperons) instead of the ladders.

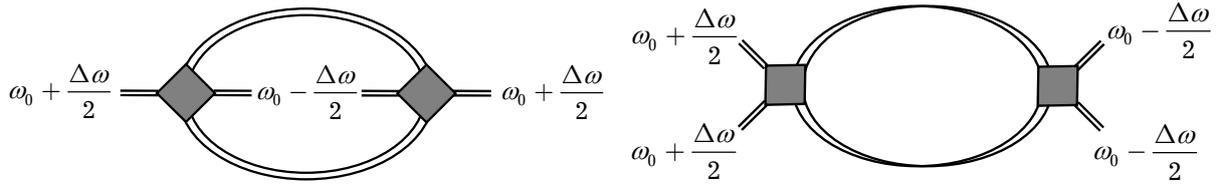


Figure 1. Diagrams contributing to the conductance correlation function. The paired lines correspond to ladder graphs. The shaded boxes are the Hikami vertex [19].

For great number N of propagating modes ($N = k_0^2 A / 4\pi$, k_0 is the wavenumber, A is the area of the waveguide cross-section) the summation over modes can be replaced by integration over directions $\mathbf{\Omega}$ of wave propagation (see, e.g., Ref. [6]),

$$\sum_a \dots = \int \frac{A d\mathbf{q}_a}{(2\pi)^2} \dots = \frac{k_0^2 A}{(2\pi)^2} \int d\mathbf{\Omega}_a |\mu_a| \dots, \quad (2)$$

where \mathbf{q}_a is the transverse momentum ($q_a < k_0$), $\mu_a = \Omega_{az}$, the z -axis is directed along the waveguide. Hemispheres $\Omega_{az} > 0$ and $\Omega_{az} < 0$ correspond to the waves that propagate in the forward and backward directions, respectively. The average conductance is expressed in terms of the average intensity,

$$\langle G \rangle = \frac{N}{\pi} \int d\mathbf{\Omega}_a d\mathbf{\Omega}_b |\mu_a| |\mu_b| I_{ab}(z_f = L | z_i = 0), \quad (3)$$

where propagator $I_{ab}(z|z') = I(z, \mathbf{\Omega}_a|z', \mathbf{\Omega}_b)$ denotes the intensity at depth z in direction $\mathbf{\Omega}_a$ from a source placed at depth z' and emitting waves in direction $\mathbf{\Omega}_b$. Intensity $I_{ab}(z|z')$ is subject to the transport equation [20]

$$\left(\mu_a \frac{\partial}{\partial z} + n\sigma_{\text{tot}} \right) I_{ab}(z|z') = \delta(z - z') \delta(\mathbf{\Omega}_a - \mathbf{\Omega}_b) + \int d\mathbf{\Omega}_c \sigma_{ac} I_{cb}(z|z'), \quad (4)$$

where $\sigma_{ac} = n d\sigma(\mathbf{\Omega}_a \mathbf{\Omega}_c)/d\Omega$, n is the number of scattering centres per unit volume, $d\sigma/d\Omega$ is the differential scattering cross-section, $\sigma_{\text{tot}} = \sigma + \sigma_a$ is the total cross-section of interaction, σ and σ_a are the cross-sections of elastic scattering and absorption, respectively.

The diagrams shown in Fig.1 can be evaluated by a straightforward manner (see, e.g., Refs. [21, 22]). In what follows, we take into account the ladders incorporating an arbitrary number of scattering events, among them the graphs without any scattering. These latter describe nonscattered waves. Contrary to calculations, performed within the diffusion approximation [6, 8], we need not introduce particular diagrams containing only one internal ladder propagator and the six-point Hikami vertex. Such diagrams are already contained among the diagrams depicted in Fig.1. They correspond to the pair of nonscattered waves in either of two internal ladders.

Our diagrammatic calculations are similar to those of Ref. [22] where the conductance variance was found. Extending results [22] to the case of the waves differing in frequency, we derive the following expression for the correlation function:

$$C(\Delta\omega) = C^A(\Delta\omega) + C^B(\Delta\omega) + C^C(\Delta\omega), \quad (5)$$

$$C^A(\Delta\omega) = \int_0^L dz dz' \int d\mathbf{\Omega}_a d\mathbf{\Omega}_b d\mathbf{\Omega}_c d\mathbf{\Omega}_d \sigma_{ab} \sigma_{cd} \times \quad (6)$$

$$\left[(I_a^f - I_b^f)^2 I_{ac}(\Delta\omega) I_{bd}(-\Delta\omega) (I_c^i - I_d^i)^2 + (I_a^f - I_b^f)(I_{-a}^i - I_{-b}^i) I_{ac}(\Delta\omega) I_{bd}(-\Delta\omega) (I_{-c}^f - I_{-d}^f)(I_c^i - I_d^i) \right]$$

$$C^B(\Delta\omega) = \int_0^L dz dz' \int d\mathbf{\Omega}_a d\mathbf{\Omega}_b d\mathbf{\Omega}_c d\mathbf{\Omega}_d \sigma_{ab} \sigma_{cd} \left[(I_a^f - I_b^f) I_b^i + I_{-b}^f (I_{-a}^i - I_{-b}^i) \right] \times \quad (7)$$

$$\text{Re} \{ (I_{ac}(\Delta\omega) - I_{ad}(\Delta\omega))(I_{-a-c}(\Delta\omega) - I_{-b-c}(\Delta\omega)) \} \left[(I_c^f - I_d^f) I_d^i + I_{-d}^f (I_{-c}^i - I_{-d}^i) \right]$$

$$C^C(\Delta\omega) = \int_0^L dz \int d\mathbf{\Omega}_a d\mathbf{\Omega}_b \sigma_{ab} \left[(I_a^f - I_b^f) I_b^i + I_{-b}^f (I_{-a}^i - I_{-b}^i) \right]^2 \text{Re} \{ I_{ab}(z|z, \Delta\omega) \} \quad (8)$$

where internal propagator $I_{ab}(\Delta\omega) = I_{ab}(z|z', \Delta\omega)$ obeys the transport equation that is obtained from Eq.(4) by substitution of complex absorption coefficient ($n\sigma_a + i\Delta\omega/c$) for $n\sigma_a$ (c is the velocity of waves). The incoming and outgoing propagators are defined as:

$$I_a^i(z) = \int d\mathbf{\Omega}_b |\mu_b| I_{ab}(z|z_i = 0, \Delta\omega = 0), \quad I_a^f(z) = \int d\mathbf{\Omega}_b |\mu_b| I_{ba}(z_f = L|z, \Delta\omega = 0). \quad (9)$$

The change of sign in a subscript of any propagator entering into Eq.(5) implies the reverse of the direction (i.e. substitution of $-\mathbf{\Omega}_a$ for $\mathbf{\Omega}_a$ into the propagator). The incoming and outgoing propagators appearing in Eq.(5) with subscripts $\pm a$, $\pm b$ are functions of z , while those with subscripts $\pm c$, $\pm d$ are functions of z' .

The terms that contain products of the incoming and outgoing propagators with subscripts of opposite sign (e.g., $[(I_a^f - I_b^f) I_b^i][\dots][I_{-d}^f (I_{-c}^i - I_{-d}^i)]$) correspond to the cooperon contribution which includes the maximally crossed internal graphs and, in the low-order scattering limit, the graphs that describe the waves propagating in opposite directions. Equations (6)-(8) has the presented form provided that the time-reversal symmetry is not violated.

Note that the third term in Eq.(5), $C^C(\Delta\omega)$, corresponds to the contribution which, within the diffusion approach [6, 8], is given by the six-point Hikami vertex.

Equation (5) generalizes the result of the diffusion approximation [6, 8], much as the transport equation generalizes the equation of diffusion. Equation (5) establishes the interrelation between the conductance fluctuations and the characteristics of scattering centers of the disordered system, and enables us to study evolution of the fluctuations in going from the quasi-ballistic propagation to the diffusive regime.

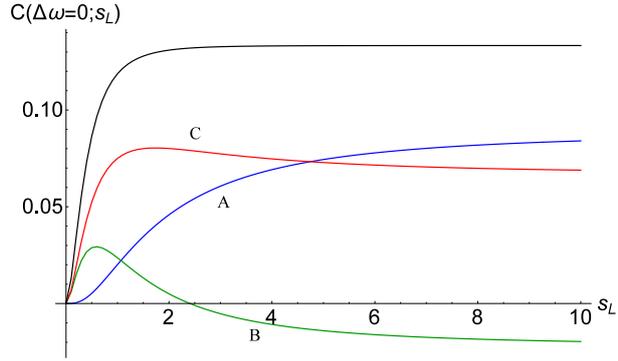


Figure 2. Different contributions to $C(\Delta\omega = 0, s_L)$ as functions of the waveguide length. The upper curve is the sum of all contributions.

3. Crossover between the quasi-ballistic and diffusive regimes

As an illustration of application of Eq.(5) to calculating the correlation function of conductance fluctuations beyond the diffusion approximation, we take advantage of the two-stream version of the discrete-ordinate method [20]. This simplest model enables us to perform integration in Eq.(5) explicitly and to derive analytical results for $C(\Delta\omega)$ which describe the crossover between the quasi-ballistic and diffusive regimes.

Within this approach, each integral over Ω is supposed to be equal to the sum of the values of an integrand quantity at $\Omega_z = \pm\mu_0$,

$$\int d\Omega_a I(z, \Omega_a) = 2\pi I_+(z) + 2\pi I_-(z), \quad (10)$$

where $I_{\pm}(z) = I(z, \Omega_z = \pm\mu_0)$, and $\pm\mu_0$ are the discrete ordinates [20].

For a widely used model of pointlike centers, quantity σ_{ab} in Eqs. (4) and (5) is independent of the directions, and the disorder is characterized only by mean free path l , $\sigma_{ab} = (4\pi l)^{-1}$ (the medium with no absorption is considered, $\sigma_a = 0$). In this case, the C -functions entering into Eq.(5) are expressed in terms of the values of intensity propagators at $\pm\mu_0$ as follows

$$C^A(\Delta\omega) = 2 \left(\frac{\pi}{l}\right)^2 \text{Re} \int_0^L dz dz' [I_{++}(\Delta\omega)I_{--}(-\Delta\omega) + I_{+-}(\Delta\omega)I_{-+}(-\Delta\omega)] \times \quad (11)$$

$$[(I_+^f - I_-^f)^2 (I_+^i - I_-^i)^2 + (I_+^f - I_-^f)(I_+^i - I_-^i)(I_+^f - I_-^f)'(I_+^i - I_-^i)']$$

$$C^B(\Delta\omega) = \left(\frac{\pi}{l}\right)^2 \text{Re} \int_0^L dz dz' [h_+(I_{++}(\Delta\omega) - I_{+-}(\Delta\omega)) + h_-(I_{--}(\Delta\omega) - I_{-+}(\Delta\omega))] \times \quad (12)$$

$$[h_+(I_{--}(\Delta\omega) - I_{+-}(\Delta\omega)) + h_-(I_{++}(\Delta\omega) - I_{-+}(\Delta\omega))]$$

$$C^C(\Delta\omega) = \frac{\pi}{l} \text{Re} \int_0^L dz [h_+^2 I_{+-}(z|z, \Delta\omega) + h_-^2 I_{-+}(z|z, \Delta\omega)], \quad (13)$$

where $h_{\pm} = [\pm (I_+^f - I_-^f)I_{\mp}^i \mp (I_+^i - I_-^i)I_{\pm}^f]$. The propagators $I_{\pm\pm}(\Delta\omega) = I_{\pm\pm}(z|z', \Delta\omega)$ and $I_{\pm}^{i,f} = I_{\pm}^{i,f}(z)$ entering into Eqs. (11)-(13) are determined analytically from the transport equation and given by

$$I_{++}(z|z', \Delta\omega) = \frac{1}{2\pi\mu_0 \sinh(\gamma) \sinh(\xi_L + \gamma)} \begin{cases} \sinh(\xi) \sinh(\xi_L - \xi'), & z < z' \\ \sinh(\xi' + \gamma) \sinh(\xi_L - \xi + \gamma), & z > z' \end{cases} \quad (14)$$

$$I_{-+}(z|z', \Delta\omega) = \frac{1}{2\pi\mu_0 \sinh(\gamma) \sinh(\xi_L + \gamma)} \begin{cases} \sinh(\xi + \gamma) \sinh(\xi_L - \xi'), & z < z' \\ \sinh(\xi' + \gamma) \sinh(\xi_L - \xi), & z > z' \end{cases} \quad (15)$$

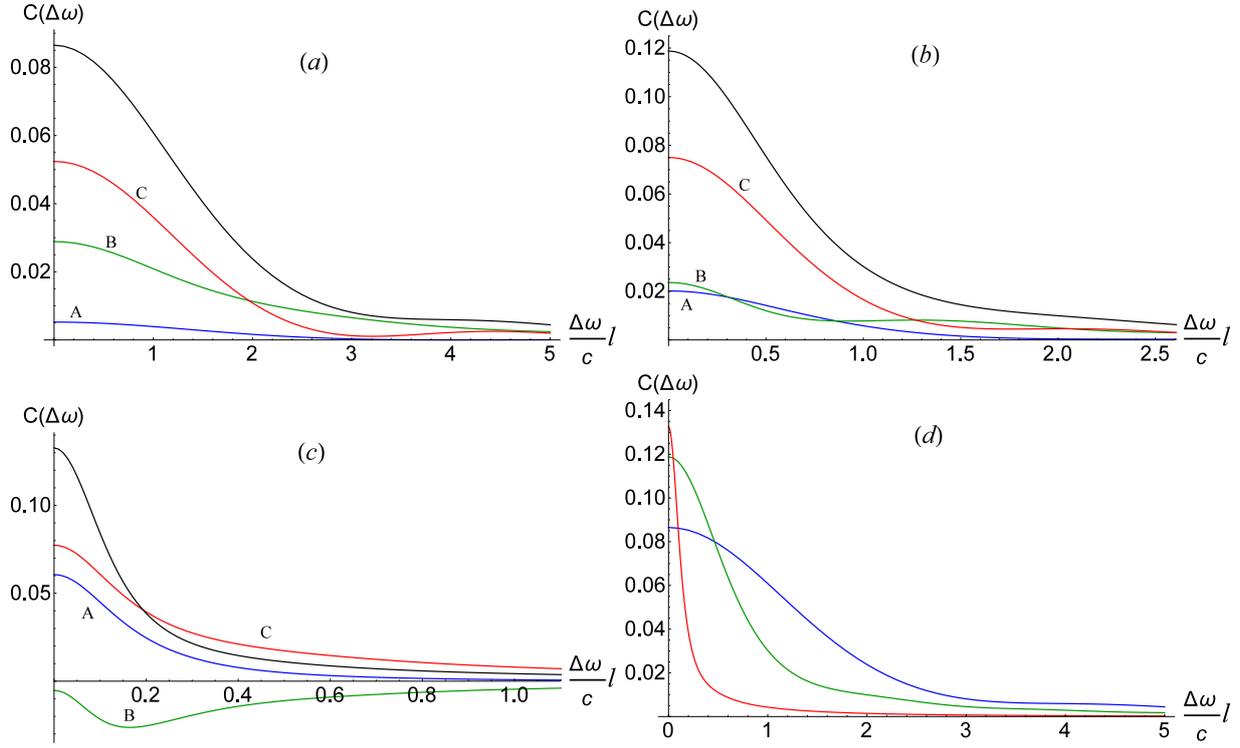


Figure 3. Frequency dependence of the different contributions to the correlation function for various waveguide lengths. (a) $s_L = 0.5$, (b) $s_L = 1$, (c) $s_L = 3$. The upper curves correspond to the sum of all contributions. (d) Correlation function $C(\Delta\omega)$, from upper to lower curves, $s_L = 0.5, 1$ and 3 .

where $\xi = s \sinh(\gamma)$, $\xi_L = s_L \sinh(\gamma)$ and $\sinh(\gamma) = 2\sqrt{i\Delta\omega l(1 + i\Delta\omega l/c)/c}$, $s = z/2\mu_0 l$, $s_L = L/2\mu_0 l$. The propagators $I_{\pm,-}(z|z', \Delta\omega)$ are obtained from Eqs.(14) and (15) with the relations $I_{--}(z|z', \Delta\omega) = I_{++}(z'|z, \Delta\omega)$, $I_{+-}(z|z', \Delta\omega) = I_{-+}(L - z|L - z', \Delta\omega)$. The incoming and outgoing propagators are equal to

$$I_{\pm}^i(z) = \frac{s_L - s + (1 \pm 1)/2}{s_L + 1}, \quad I_{\pm}^f(z) = \frac{s + (1 \pm 1)/2}{s_L + 1}. \quad (16)$$

In Eqs.(11)-(13), the incoming and outgoing propagators marked by prime are functions of z' , otherwise they are functions of z .

The length dependence of the different contributions to the correlation function at $\Delta\omega = 0$ is shown in Fig.2. Their sum is equal to

$$C(\Delta\omega = 0, s_L) = \frac{2}{15} \left(1 - \frac{1 + 6s_L}{(1 + s_L)^6} \right), \quad (17)$$

The obtained value of $C(\Delta\omega = 0, s_L)$ (i.e., the conductance variance $\langle(\delta G)^2\rangle$) coincides with the well-known result of RMT calculations [16–18].

An analytical formula that extends Eq.(17) to a nonzero value of the frequency shift $\Delta\omega$ is too cumbersome to be presented here and, therefore, we illustrate the frequency-dependence of the conductance correlation function in the graphical form. Evolution of the frequency dependence both of the different contributions to the correlation function and of their sum with the waveguide length is illustrated in Fig.3. As follows from Fig.3(d), the frequency profile of $C(\Delta\omega)$ becomes

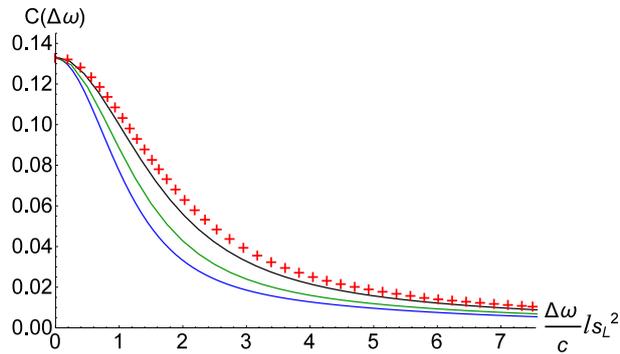


Figure 4. Correlation function of conductance fluctuations versus variable $\Delta\omega ls_L^2/c$ (from upper to lower curve $s_L = 5, 10, 15$). Crosses are the results of the diffusion approximation [6]. The discrete ordinate is chosen to be equal to $\mu_0 = 1/\sqrt{3}$.

narrower as the waveguide length increases. In the diffusive limit ($L \gg l$), the correlation function as a function of the variable $\Delta\omega ls_L^2/c$ tends to the universal law which coincides with the corresponding result of the diffusion approximation [6, 8] (see Fig.4).

4. Conclusions

In conclusion, we have developed a theoretical approach to calculating the correlation function of conductance fluctuations in a quasi-1D system, paying special attention to the subdiffusion length scales. We have presented an analytical result which relates the correlation function to the characteristics of the disordered system. The correlation function has been calculated within the two-stream model. It has been shown that $C(\Delta\omega)$ becomes narrower with increasing the waveguide length L and tends to the diffusion result [6, 8] in the large L limit. The results obtained generalize RMT calculations [4, 16–18] to the case of a nonzero frequency shift, and can be useful for experimental studies of the conductance correlation function [10].

5. Acknowledgments

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