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# Depolarization of an Ultrashort Pulse in a Disordered Ensemble of Mie Particles

**E E Gorodnichev, S V Ivliev, A I Kuzovlev and D B Rogozkin**

National Research Nuclear University MEPhI, Kashirskoe Shosse 31, 115409 Moscow, Russia

E-mail: gorodn@theor.mephi.ru

**Abstract.** We study propagation of an ultrashort pulse of polarized light through a turbid medium with the Reynolds-McCormick phase function. Within the basic mode approach to the vector radiative transfer equation, the temporal profile of the degree of polarization is calculated analytically with the use of the small-angle approximation. The degree of polarization is shown to be described by the self-similar dependence on some combination of the transport scattering coefficient, the temporal delay and the sample thickness. Our results are in excellent agreement with the data of numerical simulations carried out previously for aqueous suspension of polystyrene microspheres.

## 1. Introduction

Over past two decades, the propagation of light through highly scattering media has attracted great attention due to its potential applications to optical tomography [1].

An ultrashort pulse of light propagating in a scattering medium experiences multiple scattering events, resulting in broadening of its temporal profile and depolarization. Early arrival photons of the transmitted pulse propagate along nearly straight lines. These photons are the least depolarized. Therefore the polarization can be used for gating the early arrival component of the transmitted pulse with the gate of the order of a polarization decay time (in accordance with Refs. [1–3] the degree of polarization decays over the initial  $100ps$  after the arrival of the ballistic photons).

Depolarization of polarized pulses transmitted through a thick turbid slab was studied both experimentally [2–5] and numerically with Monte Carlo [6] and DISORT [7] codes. The results [2–7] pertain to turbid media composed of Mie-particles (polystyrene microspheres suspended in water). As follows from Refs. [2–7], the transmitted light retains its initial polarization over a small temporal interval following the arrival of the ballistic photons,  $\Delta = ct - z \ll z/2$  ( $c$  is the velocity of light in the medium,  $z$  is the slab thickness). The photons arriving with delays  $\Delta > z$  are completely depolarized.

An analytical treatment of the depolarization process was carried out within the small-angle diffusion (or Fokker-Planck) approximation [8] which holds provided that the phase function falls off rapidly as the scattering angle increases. Such an approximation fails for many realistic situations where the phase function decreases rather slowly.

It what follows the depolarization of an ultrashort pulse of polarized light propagating through a turbid medium is considered. Within the basic mode approximation [9–11], we develop a theoretical approach to calculating the temporal profile of the degree of polarization. The



calculations are carried out within the small-angle approximation in application to the Reynolds-McCormick phase function [12] which is frequently used for modeling the single-scattering by Mie-particles. The degree of polarization is shown to depend only on a certain combination of the delay  $\Delta$ , the slab thickness  $z$  and the transport scattering coefficient  $\sigma_{tr}$ . Our results are in excellent agreement with the data of numerical simulations [6].

## 2. Basic mode approximation

To calculate the temporal profile of the degree of polarization, we take advantage of the nonstationary vector radiative transfer equation. The normal incidence of a  $\delta$ -pulse of polarized light on the surface is considered. The beam width is assumed to be greater than the beam transverse spread in the medium. For highly forward single-scattering, we can neglect the off-diagonal elements of the scattering matrix [9, 10]. Then, the vector radiative transfer equation decomposes into three independent equations for the basic modes [9–11, 13].

The intensity satisfies the scalar transfer equation

$$\left\{ \frac{1}{c} \frac{\partial}{\partial t} + \boldsymbol{\Omega} \frac{\partial}{\partial \mathbf{r}} + \sigma_{tot} \right\} I(\mathbf{r}, \boldsymbol{\Omega}, t) = \sigma \int d\boldsymbol{\Omega}' a_1(\boldsymbol{\Omega}\boldsymbol{\Omega}') I(\mathbf{r}, \boldsymbol{\Omega}', t) \quad (1)$$

where  $\sigma_{tot} = \sigma + \sigma_a$  is the total extinction coefficient;  $\sigma$  and  $\sigma_a$  are the scattering and absorption coefficients of the medium, respectively, the vectors  $\boldsymbol{\Omega}$  and  $\boldsymbol{\Omega}'$  denote the directions of photon propagation,  $a_1(\boldsymbol{\Omega}\boldsymbol{\Omega}')$  is the phase function of single-scattering. For the unit incident flux, the boundary condition for Eq.(1) is written as

$$I(z = 0, \boldsymbol{\Omega}, t) = \delta(t) \delta(\boldsymbol{\Omega} - \boldsymbol{\Omega}_0) \quad (2)$$

where  $\boldsymbol{\Omega}_0$  is the internal normal to the surface. The basic modes of linear and circular polarizations,  $W(\mathbf{r}, \boldsymbol{\Omega}, t)$  and  $V(\mathbf{r}, \boldsymbol{\Omega}, t)$ , are subject to the following transfer equations:

$$\left\{ \frac{1}{c} \frac{\partial}{\partial t} + \boldsymbol{\Omega} \frac{\partial}{\partial \mathbf{r}} + \sigma_{tot} \right\} W(\mathbf{r}, \boldsymbol{\Omega}, t) = \sigma \int d\boldsymbol{\Omega}' \left( \frac{a_1(\boldsymbol{\Omega}\boldsymbol{\Omega}') + a_2(\boldsymbol{\Omega}\boldsymbol{\Omega}')}{2} \right) e^{2i(\chi_+ - \psi)} W(\mathbf{r}, \boldsymbol{\Omega}', t) \quad (3)$$

$$\left\{ \frac{1}{c} \frac{\partial}{\partial t} + \boldsymbol{\Omega} \frac{\partial}{\partial \mathbf{r}} + \sigma_{tot} \right\} V(\mathbf{r}, \boldsymbol{\Omega}, t) = \sigma \int d\boldsymbol{\Omega}' a_2(\boldsymbol{\Omega}\boldsymbol{\Omega}') V(\mathbf{r}, \boldsymbol{\Omega}', t) \quad (4)$$

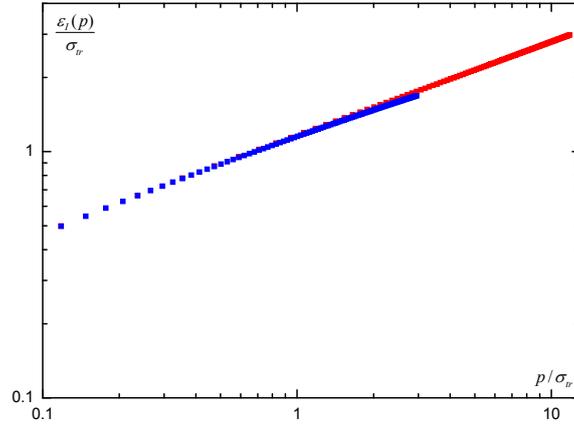
where  $a_2(\boldsymbol{\Omega}\boldsymbol{\Omega}')$  is the second diagonal element of the scattering matrix [10, 11], the definition of the angle  $\chi_+$  is given in Refs. [10, 11],  $\psi = \varphi - \varphi'$  is the difference between the azimuth angles of the vectors  $\boldsymbol{\Omega}$  and  $\boldsymbol{\Omega}'$ . For polarized light, the boundary conditions to Eqs.(3) and (4) are similar to Eq.(2).

## 3. Small-angle multiple scattering

The small-angle approximation is valid provided that the mean square of the multiple-scattering angle  $\theta$  at depth  $z$  and time instant  $t$  satisfies inequality  $\langle \theta^2 \rangle_{z,t} \ll 1$ . According to Refs. [14, 15]  $\langle \theta^2 \rangle_{z,t} \sim \Delta/z$  and, therefore, the early arrival component of the transmitted pulse,  $\Delta \ll z$  can universally be treated within the small-angle approximation.

To describe the scattering of light by large particles, we take advantage of the two-parameter Reynolds-McCormick phase function [12]

$$a_1(\cos \gamma) = \frac{\alpha - 2}{2\pi} \frac{g(1 - g^2)^{\alpha - 2}}{(1 + g)^{\alpha - 2} - (1 - g)^{\alpha - 2}} \frac{1}{(1 + g^2 - 2g \cos \gamma)^{\alpha/2}}, \quad (5)$$



**Figure 1.** Eigenvalue  $\varepsilon_I(p)$  as a function of variable  $p/\sigma_{tr}$ . Results of numerical calculations with the Mie theory for aqueous suspension of polystyrene microspheres (red and blue symbols correspond to diameters of  $0.993 \mu m$  and  $0.300 \mu m$ , respectively).

where  $\alpha$  and  $g$  are the parameters. For  $\alpha = 3$ , Eq.(5) coincides with the well-known Henyey-Greenstein phase function. In scattering through small angles the Reynolds-McCormick phase function falls off with increasing angle  $\gamma$  in accordance with a power law  $a_1 \sim 1/\gamma^\alpha$ ,  $2 \leq \alpha \leq 4$ .

Within the small-angle approximation, Eq.(1) takes the form [15]

$$\left\{ \frac{\partial}{\partial z} + \frac{\theta^2}{2} \frac{\partial}{\partial \Delta} \right\} \tilde{I}(z, \theta, \Delta) = \frac{\sigma_{tr}}{4\pi} 2^{\alpha-2} (4 - \alpha) \int d\theta' \frac{\tilde{I}(z, \theta, \Delta) - \tilde{I}(z, \theta', \Delta)}{|\theta - \theta'|^\alpha} \quad (6)$$

where  $\tilde{I}(z, \theta, \Delta) = c^{-1} \exp(-\sigma_a ct) I(z, \theta, \Delta)$ ,  $\theta$  is the angle between vectors  $\mathbf{\Omega}$  and  $\mathbf{\Omega}_0$ , and  $\Delta = ct - z$  is the difference between path length  $ct$  and depth  $z$ ,  $\sigma_{tr} = \sigma(1 - \langle \cos \gamma \rangle)$  is the transport scattering coefficient, and  $\langle \cos \gamma \rangle$  is the mean cosine of single scattering.

Using the Bessel transform with respect to the angular variable  $\theta$  and the Laplace transform with respect to  $\Delta$ , we can reduce Eq.(6) to an eigenvalue problem for the equation that resembles the stationary Schrödinger equation [15]. For relatively small delays,  $\Delta \ll z$ , only the minimum eigenvalue and the corresponding eigenfunction contributes to the intensity of the transmitted pulse. Therefore  $\tilde{I}(z, \theta, \Delta)$  can be presented in the form [15]

$$\tilde{I}(z, \theta, \Delta) = \int_{-i\infty}^{i\infty} \frac{dp}{2\pi i} \exp(p\Delta - \varepsilon_I(p)z) \int_0^{i\infty} \frac{ldl}{2\pi} J_0(l\theta) \Phi_I(l, p) \int_0^{i\infty} \frac{l'dl'}{2\pi} \Phi_I(l', p) \quad (7)$$

where  $\varepsilon_I(p)$  and  $\Phi_I(l, p)$  are determined from the equation

$$\left( -\frac{p}{2}\Delta + \sigma(1 - a_1(l)) \right) \Phi_I(l, p) = \varepsilon_I(p) \Phi_I(l, p) \quad (8)$$

The function  $a_1(l)$  appearing in Eq.(8) is the Bessel transform of the small-angle phase function  $a_1(\mathbf{\Omega}\mathbf{\Omega}') \approx a_1(|\theta - \theta'|)$ . For the Reynolds-McCormick phase function

$$\sigma(1 - a_1(l)) = \frac{\sigma_{tr}}{2} \left( \frac{4 - \alpha}{\alpha - 2} \right) \frac{\Gamma(2 - \alpha/2)}{\Gamma(\alpha/2)} \cdot l^{\alpha-2} \quad (9)$$

In the case of  $\alpha = 3$  which corresponds to the Henyey-Greenstein phase function,  $\sigma(1 - a_1(l)) = \sigma_{tr}l$ .

From Eqs.(8), (9) it follows that the eigenvalue  $\varepsilon_I(p)$  can be written as [15]

$$\varepsilon_I(p) = c_\alpha \sigma_{tr}^{2/\alpha} p^{1-2/\alpha} \quad (10)$$

where  $c_\alpha$  is the numerical coefficient which depends only on the parameter  $\alpha$ . Our numerical calculations carried out for polystyrene microspheres of diameters  $0.3 \mu m$  and  $0.993 \mu m$  at the wavelength  $\lambda = 0.633 \mu m$  (see Fig.1) are approximated by the relations  $\varepsilon_I(p)/\sigma_{tr} = 1.15(p/\sigma_{tr})^{0.357}$ ,  $\alpha = 3.11$  and  $\varepsilon_I(p)/\sigma_{tr} = 1.62(p/\sigma_{tr})^{0.381}$ ,  $\alpha = 3.23$ , respectively.

#### 4. Degree of polarization

For the basic modes of linear and circular polarizations, solutions of Eqs.(3) and (4) can also be presented in the form that is similar to Eq.(7). The corresponding solutions differ from Eq.(7) only by the specific values of the eigenvalues,  $\varepsilon_W(p)$  and  $\varepsilon_V(p)$ , and the eigenfunctions,  $\Phi_W(l, p)$  and  $\Phi_V(l, p)$ .

In the case of highly forward multiple scattering, the values of  $\varepsilon_W(p)$  and  $\varepsilon_V(p)$  turn out to be close to  $\varepsilon_I(p)$  provided that the value of  $p$  is rather great,  $p > \sigma_{tr}$ . This follows directly from our numerical calculations of  $\varepsilon_I(p)$ ,  $\varepsilon_W(p)$  and  $\varepsilon_V(p)$  with the use of the corresponding characteristic equations [11] both for the Reynolds-McCormick phase function and for large Mie-particles (polystyrene and silica microspheres in water). Therefore we can calculate the values of  $\varepsilon_W(p)$  and  $\varepsilon_V(p)$  analytically with a perturbation theory taking the solution of the scalar transfer equation (8) as the first approximation. Within such an approach, the difference between the values of  $\varepsilon_I(p)$ ,  $\varepsilon_W(p)$  and  $\varepsilon_V(p)$  can be written in the form

$$\delta\varepsilon_{W,V}(p) = \sigma \int_0^\infty \frac{ldl}{2\pi} \Phi_I(l, p) \Delta a_{W,V}(l) \Phi_I(l, p) \quad (11)$$

where  $\Delta a_{W,V}(l)$  is the difference between  $a_1(l)$  and the Bessel transforms of the "effective" phase functions appearing in Eqs.(3) and (4).

The small-angle transfer equation for the basic mode of linear polarization differs from Eq.(8) only by the "perturbation" term  $\sigma \Delta a_{W,V}(l)$  which has the form [9, 10]

$$\sigma \Delta a_W \approx \sigma \Delta a_W^{geom} + \sigma \Delta a_W^{dyn} = \frac{\sigma}{2} \left[ \frac{1}{l} \frac{\partial}{\partial l} \left[ \frac{\partial a_1(l)}{\partial l} \frac{\partial}{\partial l} \right] \right] + \frac{\sigma}{2} (a_1(l) - a_2(l)) \quad (12)$$

The first term in Eq.(12) is responsible for the "geometrical" depolarization due to the Rytov effect. The second term describes the "dynamical" depolarization. With the standard perturbation theory [16], the value of  $\varepsilon_W(p)$  can be written as  $\varepsilon_W = \varepsilon_I + \delta\varepsilon_W^{geom} + \delta\varepsilon_W^{dyn}$ . The "geometrical" contribution to  $\varepsilon_W$  is determined by

$$\delta\varepsilon_W^{geom}(p) = \frac{\sigma}{2} \int_0^\infty \frac{ldl}{2\pi} \Phi_I(l, p) \left[ \frac{1}{l} \frac{\partial}{\partial l} \left[ \frac{\partial a_1(l)}{\partial l} \frac{\partial}{\partial l} \right] \right] \Phi_I(l, p) = \frac{\Gamma(2 - \alpha/2)}{2} \sigma_{tr} \left( \frac{\varepsilon_I(p)}{p} \right)^{3-\alpha/2} \quad (13)$$

When deriving Eq.(13), we use the eigenfunction  $\Phi_I(l, p)$  approximated by [15]

$$\Phi_I(l, p) = \sqrt{\frac{4\pi\varepsilon_I(p)}{p}} \exp\left(-\frac{l^2\varepsilon_I(p)}{2p}\right) \quad (14)$$

The "dynamical" contribution to  $\varepsilon_W$  depends on ratio  $\varepsilon_I(p)/p$  in a similar way,

$$\delta\varepsilon_W^{dyn}(p) = \frac{\sigma}{2} \int_0^\infty \frac{ldl}{2\pi} \Phi_I(l, p) [a_1(l) - a_2(l)] \Phi_I(l, p) = \frac{\sigma_{tr}(4 - \alpha)}{2^\alpha} \cdot \left( \frac{\varepsilon_I(p)}{p} \right)^{3-\alpha/2} \quad (15)$$

For the basic mode of circular polarization, the difference  $\Delta a_V$  is only due to the "dynamical" mechanism of depolarization. The value of  $\delta\varepsilon_V$  turns out to be two times greater than  $\delta\varepsilon_W^{dyn}$ .

Evaluating the integral in Eq.(7) and the integrals in the corresponding representations for the modes  $W$  and  $V$  by the saddle-point method, we obtain the following expressions for the degree of polarization of linearly and circularly polarized pulses

$$\begin{aligned} P_L &= \frac{W}{I} = \exp\left(-(\beta(\alpha) + \eta(\alpha)) \cdot \sigma_{tr} z \cdot \langle \theta^2 \rangle_{z,t}^{3-\alpha/2}\right), \\ P_C &= \frac{V}{I} = \exp\left(-2\eta(\alpha) \cdot \sigma_{tr} z \cdot \langle \theta^2 \rangle_{z,t}^{3-\alpha/2}\right) \end{aligned} \quad (16)$$

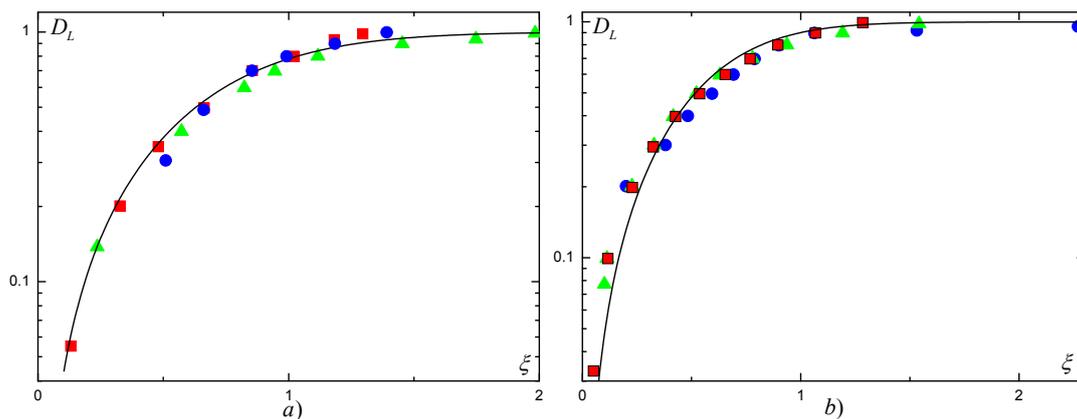
where  $\beta(\alpha) = \Gamma(3 - \alpha/2) / 2^{4-\alpha/2}$ ,  $\eta(\alpha) = (4 - \alpha) / 2^{3+\alpha/2}$  and

$$\langle \theta^2 \rangle_{z,t} = \frac{2\alpha}{\alpha - 2} \left(\frac{\Delta}{z}\right)$$

is the mean square of the multiple-scattering angle  $\theta$  for the Reynolds-McCormick phase function [15]. From Eq.(16) it follows that the degree of polarization depends on one dimensionless variable

$$\xi = \sigma_{tr} z \left(\frac{\Delta}{z}\right)^{\alpha-3/2}. \quad (17)$$

This conclusion is confirmed by comparison of our result (16) with data of Monte Carlo simulations for aqueous suspension of polystyrene microspheres [6] (see Fig. 2). Numerical simulation [6] was carried out for linearly polarized light and for various values of the transport optical thickness  $\sigma_{tr} z$ . Depolarization ratio  $D_L = (1 - P_L) / (1 + P_L)$  was presented in [6] as a function of the normalized delay  $\Delta/z$ . When going from  $\Delta/z$  to the variable  $\xi$  we obtain a virtually universal pattern of data [6]. As follows from Fig. 2 the results of our calculations are in excellent agreement with the data of numerical simulations.



**Figure 2.** Depolarization ratio as a function of variable  $\xi$ . Symbols (■ –  $\sigma_{tr} z = 2$ , ▲ –  $\sigma_{tr} z = 4$ , ● –  $\sigma_{tr} z = 10$ ) are the results of Monte Carlo simulations [6] for aqueous suspension of polystyrene microspheres of diameter  $0.3 \mu m$  (a) and  $0.993 \mu m$  (b). Solid lines are the results of our calculations ( $\sigma_{tr}/\sigma = 0.339$ ,  $\alpha = 3.11$  (a) and  $\sigma_{tr}/\sigma = 0.085$ ,  $\alpha = 3.23$  (b)).

## 5. Conclusions

We have studied the transmission of an ultrashort polarized pulse through highly scattering media. A theoretical model based on the Reynolds-McCormick phase function has been proposed for the analysis of depolarization of light in the medium with large inhomogeneities. In accordance with the model, the polarization state is determined by the intensity and the basic modes of linear and circular polarizations. The values of  $W$  and  $V$  differ from the intensity  $I$  only by the factors that are responsible for additional attenuation of  $W$  and  $V$  with increasing the delay  $\Delta = ct - z$ . The results of our calculations are in excellent agreement with the data of numerical simulations [6] and make it possible to estimate the dependence of the degree of polarization on time and characteristics of the medium.

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