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Greenwald density limit and power balance in tokamaks

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Abstract. The critical density limit in tokamaks is investigated. It is shown that the equality of the input power and power radiated by impurities corresponds to the Greenwald limit. In Ohmic tokamak plasmas the auxiliary heating may increase the density limit, as it has been shown in experiments. The radiated power threshold for plasmas with heavy impurities, observed in experiments, is derived. Radiation produced by heavy impurities is spread practically uniformly along the plasma radius in contrast to the radiation of light impurities. The effective heating power is decreased by radiation losses and, as a consequence, becomes lower than the threshold for the H-L transition. If the input power is close to the radiated one, the disruption occurs.

1. Introduction

Greenwald density limit is known for a long time. (See, for example, the review paper [1]). In most experiments, plasma density in tokamaks cannot exceed the Greenwald upper density limit

$\bar{n}_{20} \leq n_G = \frac{I_p}{\pi a^2}$ using as units 10^{20} m^{-3} , MA, m. Many theoretical papers are devoted to the problem.

Some of them are based on the description of plasma inside the separatrix [2]. Only Ohmic tokamak plasmas are described in [2]. Note that the Greenwald limit describes plasmas in tokamaks with divertors as well as tokamaks with limiters. Other papers are based on the processes in the SOL and divertor [3] and cannot be applied to the limiter tokamak plasmas. The SOL and divertor physics will not be discussed here.

The theory described in [2] and papers cited therein supposes that in the edge plasma the tearing mode develops. It was shown that the island's size rises with the plasma density and saturates in time. However it is not clear why the disruption takes place when the plasma density achieves the Greenwald limit.

It is shown in Section 2 of the present paper that the Greenwald density limit is a consequence of the energy balance. For heavy impurities, like tungsten, the radiation losses are distributed throughout the plasma volume. The radiation losses in tungsten-seeded plasmas are practically uniform inside the separatrix. The difference between the input and radiated power determines the transport processes in tokamaks. The L-H transition also is determined by this difference. It may drop below the L-H threshold. As a consequence, the H-L transition can take place. In the L-regime the input power may

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not be sufficient to support the discharge. As a result, the current quench may accompany the H-L transition.

The paper is organized as follows. The Ohmic heated plasma in tokamaks with carbon wall is described in Section 2. The auxiliary heated plasmas are investigated in Section 3. Sections 4 is devoted to tungsten-seeded plasmas. The main results are summarized in Section 5.

2. Ohmic heating

The thermal balance in tokamak edge plasma may be qualitatively described using the slab geometry (see [4]). Here, the convective term is ignored. Thus, the power balance takes a form

$$\frac{d}{dr} \kappa \frac{dT}{dr} = \frac{J^2}{\sigma} - nn_i L(T) . \quad (1)$$

The first term in the right hand side in (1) determines the Ohmic heating. The second one corresponds to the radiative cooling. The function L depends on the temperature only [5], κ is the thermal conductivity, n and n_i are the plasma and impurity densities, respectively, J is the toroidal current density, σ is the electrical conductivity. As it is well known, the electrical conductivity has the form

$$\sigma = 1.96 \frac{0.9 \cdot 10^{13}}{(\lambda / 10) Z_{ef}} T^{3/2} = \sigma_0 \left(\frac{T}{T_0} \right)^{3/2} . \quad (2)$$

Here the temperature is expressed in eV, $T_0 = 1$ eV.

The impurity density as well as the plasma density are supposed to be the constants; $n_i = \alpha n$; $\alpha \ll 1$. The boundary conditions yield

$$\left. \frac{dT}{dr} \right|_{r=0} = 0, \quad T(a) = 0. \quad (3)$$

If the thermal flux at the periphery is equal to zero, $dT/dr|_{r=a} = 0$, the total input power is radiated completely. This condition defines the density limit.

Multiplying (1) by dT/dr and integrating over r from 0 to a , where a is the tokamak plasma's minor radius, one can get

$$\int_{T_{min}}^{T(0)} \frac{J^2}{\sigma} dT = \alpha n_c^2 \int_{T_{min}}^{T(0)} L(T) dT . \quad (4)$$

Here n_c is the critical density. The temperature T_{min} is related to the temperature of the ionization threshold and is equal to few eV. With good accuracy, one can put $T_{min} \approx 1$ eV.

The toroidal plasma current density may be estimated as $I_p / (\pi a^2)$, where I_p is the total plasma current. Hence, the left hand side of (3) may be rewritten as

$$\int_{T_{min}}^{T(0)} \frac{J^2}{\sigma} dT \approx \left(\frac{I_p}{\pi a^2} \right)^2 \int_0^{T(0)} \frac{1}{\sigma} dT \approx \left(\frac{I_p}{\pi a^2} \right)^2 \frac{T_{min}^{1/2}}{\sigma_0} . \quad (5)$$

Here, Eq. (2) is used. The temperature T is expressed in eV.

The value $\int_0^{T(0)} L(T) dT$ may be calculated for carbon using [5] if one can neglect the bremsstrahlung. Calculations give the following result:

$$g \equiv \int_0^{T(0)} L(T) dT \approx 7 \cdot 10^{-18} \text{ erg}^2 \text{ cm}^3 \text{ s}^{-1}. \quad (6)$$

The typical impurity concentration may be estimated as $n_i \approx 0.02n$, i.e. $\alpha \approx 0.02$. One can see that the dependence of n_c on α is weak. In order to compare the result with the Greenwald critical density, one has to express the plasma current in MA, and the minor radius in m.

Finally the equation (4) may be transformed to the usual Greenwald's form

$$n_c (10^{20} \text{ m}^{-3}) = 1.3 \frac{I_p (\text{MA})}{\pi a^2 (\text{m})}. \quad (7)$$

3. Auxiliary heating

Auxiliary heating power usually exceeds the Ohmic power significantly in modern tokamaks. If plasma column is heated by the neutral beam as well as by the fusion power, the input power is localized in the plasma center. Hence, the power balance may be presented in the form

$$\frac{d}{dr} \kappa \frac{dT}{dr} - \frac{J^2}{\sigma} + nn_i L(T) = 0. \quad (8)$$

with the boundary conditions

$$\kappa \frac{dT}{dr} \Big|_{r=0} = P_{aux}, \quad T(a) = 0. \quad (9)$$

Here P_{aux} is the energy flux density of auxiliary heating. Again, multiplying (8) by dT/dr and integrating over r , one can get

$$\kappa \left(\frac{dT}{dr} \Big|_{r=0} \right)^2 = \int_{T_{min}}^{T(0)} \frac{J^2}{\sigma} dT - \alpha n_c^2 g. \quad (10)$$

Again replacing the current density by the averaged current density, one can get

$$n_c^2 = \frac{1}{\alpha g} \left(\frac{2\bar{J}^2}{\sigma_0} T_{min}^{1/2} + \frac{P_{aux}^2}{2\kappa} \right). \quad (11)$$

This finally yields:

$$n_c = n_{Gr} \sqrt{1 + \frac{P_{aux}^2}{2\alpha g \kappa n_G^2}}. \quad (12)$$

Here n_G is Greenwald critical density.

Hence, if the auxiliary heating power is comparable with the Ohmic one, the condition (7) should be replaced by (12). The Greenwald density limit may be exceeded, as it has been observed, e.g., in TEXTOR experiments [6]. Nevertheless, the carbon radiative losses have a minimum near the temperature close to 50 eV, which means that the radiation-condensation instability may develop. As a consequence, MARFE appears. However, as it has been shown theoretically [7] as well as in many experiments, the instability may be prevented by the neon injection. On the other hand, the increase of Z_{eff} may produce the L-H transition for tungsten and, as it will be shown below, the disruption.

4. Heavy impurities

Tokamak JET operates with the ITER-like divertor plates covered with tungsten. Experiments show that the total radiation losses from tungsten-seeded plasmas do not exceed 70% of the input power,

$P_{rad} < 0.7P_{aux}$ [8] in contrast to the carbon seeded plasmas. Hence, one can expect some instability to produce the disruption. In particular the tearing instability may develop, as it is shown for Ohmic plasmas by Teng [2] and in the papers cited therein.

However, the modern tokamaks with tungsten divertor use the strong auxiliary heating. The auxiliary power usually significantly exceeds the Ohmic one. Hence, the Ohmic power is ignored in this Section. The auxiliary power is supposed to be localized at the center. Again the plasma temperature is described by the equation (8) with the boundary conditions (9). The variation of tungsten radiation function L in the temperature interval $100 \text{ eV} < T < 10 \text{ keV}$ may be approximated by the constant value of $3 \times 10^{-17} \text{ erg cm}^3/\text{s}$. The interval $T < 100 \text{ eV}$ is ignored in the present paper. The boundary condition $T(r=a)=0$ is used. One can suppose the tungsten radiation losses to be uniform. If the radiation losses increase, the H-regime is replaced by the L-regime [8]. It takes place, if the difference $(P_{aux} - P_{rad})V$ becomes less than the power required for the H-L transition:

$$(P_{aux} - P_{rad})V \leq W_{LH} = 2.84 M_i^{-1} B_t^{0.82} \bar{n}_{e20}^{0.58} R a^{0.81}. \quad (13)$$

Here V is the plasma volume, $M_i = 2$, B_t is the toroidal magnetic field expressed in T, \bar{n}_{e20} is the averaged electron density expressed in 10^{20} m^{-3} , R and a are the major and minor radii, respectively. For *JET* ($B_t = 2.8 \text{ T}$, $\bar{n}_{e20} = 0.28$, $R = 3 \text{ m}$, $a = 1 \text{ m}$) $W_{aux} - W_{rad} = 2.90 \text{ MW}$. Here $W_{aux} = P_{aux} V$, and $W_{rad} = P_{rad} V$ are the total input and radiated power, respectively.

The power of the neutral beam injection is equal to 14.9 MW. Hence, the relation of the radiated power to the input power in the H-regime cannot exceed the value 80 %.

In this case the inequality (13) may be rewritten as

$$W_{aux} < 5W_{LH}. \quad (14)$$

If the Ohmic heating may be neglected in comparison with the auxiliary one, the heating equation (8) yields

$$\left[\kappa \left(\frac{dT}{dr} \right)^2 \right]_{r=a} - \left[\kappa \left(\frac{dT}{dr} \right)^2 \right]_{r=0} = 2nn_i \int_0^{T(0)} L \frac{dr}{dT} dT. \quad (15)$$

Hence, the power balance before the L-H transition takes the form

$$2nn_i \int_0^{T(0)} L \kappa dT = P_{input}^2. \quad (16)$$

After the *H-L* transition the heat conductivity rises significantly. The equation (16) is not satisfied. The radiated power exceeds the power input. Hence the disruption must take place.

5. Summary

Nature of the density limit is analyzed in the present paper. It is shown that different events may lead to the disruption. It is shown that the critical density depends on the heating type and may be different for light and heavy impurities.

As it is shown in Section 2, the Greenwald limit may be explained by the balance of the input power and the impurity radiated power in Ohmically heated tokamak plasmas. As it is shown in [2], one can obtain the same result by analyzing the tearing mode. Some additional investigations must be performed to make a choice between different mechanisms.

As experiments show, the Greenwald limit may be exceeded in tokamaks with auxiliary heating localized at the plasma axis. The experimental results may be explained with the model described in Section 2 at least for carbon, as it is shown in Section 3. The critical density is obtained for the experiments with the auxiliary heating power, exceeding significantly the Ohmic one.

The tungsten seeded plasmas are described in Section 4. In contrast to carbon radiation, variation of the tungsten radiation losses with the temperature is weak, at least, for the temperature less than $T = 10$ keV. Hence, the radiation losses may initiate a significant decrease of the input power assimilation inside the separatrix. The radiated power may achieve approximately 80 % of the input power, and the discharge may transit from H- to L-mode. On the other hand, the heat conductivity in the L-regime increases significantly. Hence, thermal losses may exceed the power input. It means that the thermal quench may take place.

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