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Scattering of kinks in a non-polynomial model

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Abstract. We study a model described by a single real scalar field in the two-dimensional space-time. The model is specified by a potential which is non-polynomial and supports analytical kink-like solutions that are similar to the standard kink-like solutions that appear in the φ^4 model when it develops spontaneous symmetry breaking. We investigate the kink-antikink scattering problem in the non-polynomial model numerically and highlight some specific features, which are not present in the standard case.

1. Introduction

Topological defects are extremely important for theoretical physics. They are used for describing models in cosmology high energy physics, quantum and classical field theory, condensed matter and so on. In this context, the (1 + 1)-dimensional models are of special interest, because the dynamics of many three- and two-dimensional systems can be reduced to the one-dimensional models [1]–[23].

Consider a field-theoretical model in (1 + 1)-dimensional space-time with a real scalar field $\varphi(x, t)$ with the Lagrangian density

$$\mathcal{L} = \frac{1}{2} \left(\frac{\partial\varphi}{\partial t}\right)^2 - \frac{1}{2} \left(\frac{\partial\varphi}{\partial x}\right)^2 - U(\varphi),\tag{1}$$

where the potential $U(\varphi)$ is a non-negative function with two or more degenerate minima (vacua of the model): $\varphi_1^{(0)}, \varphi_2^{(0)}, \ldots, U(\varphi_1^{(0)}) = U(\varphi_2^{(0)}) = \ldots = 0$. The equation of motion for the field φ is

$$\frac{\partial^2 \varphi}{\partial t^2} - \frac{\partial^2 \varphi}{\partial x^2} + \frac{dU}{d\varphi} = 0.$$
(2)

The energy functional corresponding to the Lagrangian (1) reads

$$E[\varphi] = \int_{-\infty}^{\infty} \left[\frac{1}{2} \left(\frac{\partial \varphi}{\partial t} \right)^2 + \frac{1}{2} \left(\frac{\partial \varphi}{\partial x} \right)^2 + U(\varphi) \right] dx.$$
(3)

In this model, kinks and antikinks are static solutions which minimize energy and interpolate between neighboring vacua.

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2. The sinh-deformed φ^4 model

This paper deals with the model which we call the sinh-deformed φ^4 model. The potential $U(\varphi)$ of this model can be obtained from the potential $V(\varphi) = \frac{1}{2}(1-\varphi^2)^2$ of the standard φ^4 model by deformation procedure which was elaborated and applied in [24]–[28]:

$$U(\varphi) = \frac{V(\varphi \to f(\varphi))}{(df/d\varphi)^2},\tag{4}$$

where " $f(\varphi)$ " — the deforming function and " $\varphi \to f(\varphi)$ " means that φ is replaced by $f(\varphi)$. So,

$$U(\varphi) = \frac{1}{2} \operatorname{sech}^2 \varphi \left(1 - \sinh^2 \varphi \right)^2.$$
(5)

This potential has two minima with $\varphi_{\pm} = \pm \operatorname{arsinh} 1$, $U(\varphi_{\pm}) = 0$, figure 1(a). The kink of the new model, $\varphi_{\mathbf{k}}^{(\operatorname{new})}(x)$, is related with kink of the φ^4 model, $\varphi_{\mathbf{k}}^{(\operatorname{old})}(x)$, by the inverse deforming function f^{-1} ,

$$\varphi_{\mathbf{k}}^{(\text{new})}(x) = f^{-1}(\varphi_{\mathbf{k}}^{(\text{old})}(x)).$$
(6)

Using $\varphi_{\mathbf{k}}^{(\text{old})}(x) = \tanh x$, we obtain kink and antikink of the sinh-deformed φ^4 model:

$$\varphi_{\mathbf{k}}(x) = \operatorname{arsinh}(\tanh x), \quad \varphi_{\bar{\mathbf{k}}}(x) = -\operatorname{arsinh}(\tanh x),$$
(7)

see figure 1(b).



Figure 1. Potentials and kinks of the φ^4 (dashed curves) and the sinh-deformed φ^4 (solid curves) models.

3. Kink-antikink scattering

We studied the kink-antikink scattering in the sinh-deformed φ^4 model using the initial configuration

$$\varphi(x,t) = \operatorname{arsinh}\left[\tanh\left(\frac{x+x_0-v_{\rm in}t}{\sqrt{1-v_{\rm in}^2}}\right) \right] - \operatorname{arsinh}\left[\tanh\left(\frac{x-x_0+v_{\rm in}t}{\sqrt{1-v_{\rm in}^2}}\right) \right] - \operatorname{arsinh}1, \quad (8)$$

where x_0 is half-distance between two kinks, and they start moving towards each other with the initial velocities $v_{\rm in}$. We used $2x_0 = 10$ and the following parameters of the numerical scheme: $\delta t = 0.008$, and $\delta x = 0.01$

We found the critical value of the initial velocity $v_{\rm cr} \approx 0.4639$, which separates two different regimes of the kinks scattering. At $v_{\rm in} > v_{\rm cr}$ the kinks escape to infinities after one collision.



(a) A bound state of two oscillons at $v_{\rm in} = 0.44190$

(b) An escape of two oscillons at $v_{\rm in} = 0.44195$

Figure 2. Formation of two oscillons in the kink-antikink collision.



Figure 3. The period of oscillations of the oscillons as a function of the initial velocity of the colliding kinks. The blue-shaded areas denote the escape of oscillons. The green-shaded area denotes the 2-bounce escape window for kinks, the pink-shaded areas denote the 4-bounce escape windows for kinks.

At the initial velocities $v_{\rm in} < v_{\rm cr}$ there is the kinks' capture and formation of their bound state, and many escape windows. The picture looks similar to the φ^4 case.

Besides that, in the range $v_{in} < v_{cr}$ we have found new interesting result, which has not been observed for the φ^4 kinks. In many cases the final configuration looked like two oscillons, which can form a bound state, figure 2(a), or escape to infinity, figure 2(b). In a bound state the oscillons' oscillate near each other with the period which depends on the initial velocity of the colliding kinks, see figure 3.

4. Conclusion

We studied the kink-antikink scattering in the sinh-deformed φ^4 model.

There is the critical initial velocity $v_{\rm cr} \approx 0.4639$, which separates two regimes of the kinks scattering. At $v_{\rm in} > v_{\rm cr}$ the kinks escape to infinities after one collision. At the initial velocities $v_{\rm in} < v_{\rm cr}$ we observed formation of kinks' bound state and resonance phenomena. This pattern is similar to that found in the φ^4 model. Moreover we found new interesting phenomenon — formation of two oscillons which can form a bound state or escape to infinities. It would be very interesting to investigate the nature of appearance of oscillons. We believe that it could become a subject of further study.

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