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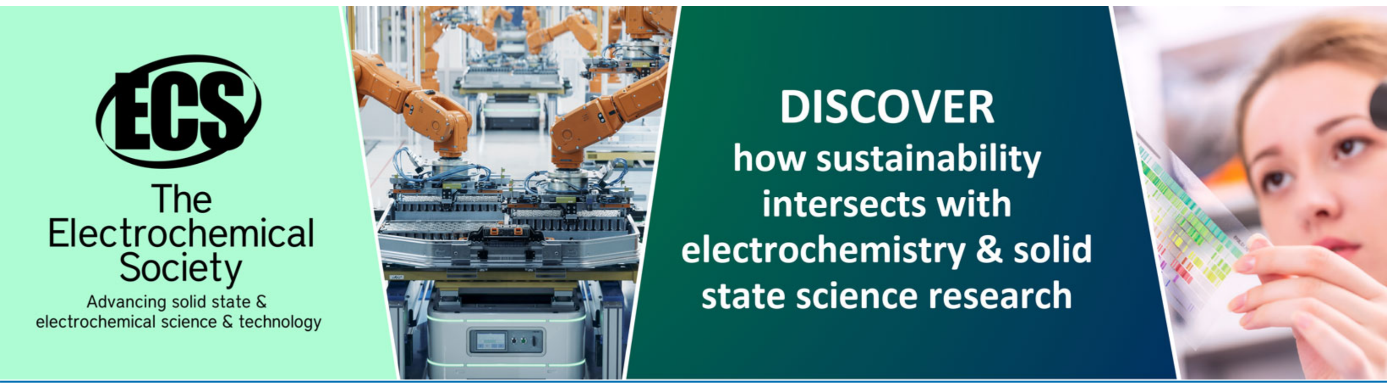

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Application of the boundary elements method for modeling of the fracture of cylindrical bodies by hydraulic fracturing

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Abstract. Experimental study of hydraulic fracturing of thick-walled cylinders with a central circular hole was carried out using the machine that creates a high oil pressure. Experiments on the compression fracture of the solid cylinders by diameter and rectangular parallelepipeds perpendicular to the ends were carried out with a multipurpose test machine Zwick / Roell Z100. Samples were made of GF-177 material based on cement. Ultimate stresses in the material under study were determined for three types of stress state: under compression, with a pure shear on the surface of the hole under frecking conditions and under a compound stress state under conditions of diametral compression of a solid cylinder. The value of the critical stress intensity factor of GF-177 material was obtained. The modeling of the fracturing process taking into account the inhomogeneity of the stress state near the hole was carried out using the boundary elements method (in the variant of the fictitious load method) and the gradient fracture criterion. Calculation results of the ultimate pressure were compared with values obtained analytically on the basis of the Lame solution and with experimental data.

1. Introduction

Lavrentyev Institute of Hydrodynamics SB RAS has an experimental machine that produces a high pulsating oil pressure, which can be used to experimentally investigate the fracturing process of samples made of brittle materials, in particular rocks and materials based on cement or gypsum. The maximum pressure can reach 40 MPa. A distinctive feature of the machine is the ability to create pulsating pressure with a frequency of up to 100 Hz. Thus, it is possible to investigate not only the formation of hydraulic fracturing cracks, but also their growth under cyclic loading.

In laboratory conditions, experiments can only be carried out with models of wells in rocks. Real wells have a significantly larger diameter than laboratory models. Therefore, it is important to investigate the scale factor, which, according to the literature, is relevant for hydraulic fracturing. Haimson [1] conducted laboratory tests on hydraulic fracturing of two rocks: limestone and granite. The diameter of the cylindrical hole imitating the borehole varied from 3 to 32 mm. With this increase in diameter, the critical pressure for limestone fell by a factor of 7, and for granite by a factor of 1.5. The results of similar experiments carried out on the same rocks are presented in [2, 3]. The diameter of the hole was changed over a wider range: in limestone – up to 50 mm, and in granite – up to

100 mm. With increasing diameter, the critical pressure decreased, asymptotically approaching a constant value.

It is known that in the presence of stress concentration, the scale factor manifests itself at a much greater degree than under a homogenous stress state. Since the borehole in the mountain massif is a stress concentrator, the inhomogeneity of the stress state near the well has a significant effect on the strength of the geomaterials. Therefore, in order to estimate the ultimate value of pressure during fracturing of rocks, nonlocal fracture criteria can be used to take into account the inhomogeneity of the stress state. It is possible to apply the criterion of average stresses, gradient criterion of fracture, etc. A distinctive feature of these criteria is the presence of a parameter having the dimension of length and characterizing the heterogeneity of the structure of the material. It is shown [4 -7] that this parameter is expressed through two standard characteristics of the material – the tensile strength σ_b at uniaxial tension and the critical stress intensity factor K_{Ic}. The values characterizing the strength, crack resistance and heterogeneity of the structure of some brittle materials and rocks can be found in [8].

In the present study, the simulation of the fracturing process, taking into account the inhomogeneity of the stress state near the hole, was carried out using the boundary element method (in the variant of the fictitious load method) and the gradient fracture criterion.

2. Hydraulic fracturing experiments on cylindrical samples

Experimental research of hydraulic fracturing of thick-walled cylinders with a central circular hole was carried out using a machine that creates a high oil pressure. Cylinders were made of GF-177 material on the cement basis with polymer additives. The presence of polymer additives ensures the absence of shrinkage and cracking of the material upon solidification. The preparation of the solution was conducted by mixing the compound with water in a ratio recommended by the manufacturer – in the proportion of 0.29 liters of water per kilogram of the compound. The solution was poured into cylindrical molds located on the shaker, which used to remove air bubbles from the solution. Cylindrical samples had a diameter of 105 mm and a height of 100 mm. A central hole 10.5 mm in diameter was drilled along the axis of the sample. Thus, the sample was a thick-walled cylinder with an orifice, whose diameter is 10 times smaller than the diameter of the cylinder. With the help of such a sample, it is possible to simulate a circular hole in an infinite medium.

Loading of the sample by internal pressure was carried out under conditions of plane strain. The sample loading system consists of a metal tube 10 mm in diameter with radial holes. The tube was inserted into the sample hole. After installing the end rubber seals, a compressor oil was supplied to the tube, which proceeded via radial holes into the internal volume of the sample. The pressure determined using a sensor that transmits electrical signals to the oscilloscope, and a pressure gauge of the experimental machine. The experimental data recorded by the oscilloscope was transferred to the computer for further processing. The oil consumption was monitored by the oil flow sensor integrated into the unit.

Figure 1. Destroyed sample and the loading system.

After installing the loading system in the sample and connecting to the high-pressure oil line, the process of pressure growth started. The moment of fracture of the sample was observed by the pressure drop on the manometer, and also with the help of a video camera that observed the sample (Fig. 1)

The ultimate value of the pressure at the time of fracture was determined from a file recorded on a computer using a pressure and oscilloscope sensor. The readings of the pressure gauge and the pressure sensor almost coincided. Three samples were tested. The following values of the ultimate pressure were obtained: $p_1 = 3,3\,\text{MPa}$, $p_2 = 4,2\,\text{MPa}$, $p_3 = 3,6\,\text{MPa}$. The average value of the ultimate pressure for the three tests was $p_* = 3,7\,\text{MPa}$.

According to Lame's solution, under pressure p on the surface of a circular hole in an infinite medium, two stresses are acting at points on this surface: the circumferential tensile stress $\sigma_\varphi = p$ and the radial compressive stress $\sigma_r = -p$. Then the ultimate tensile stress during hydraulic fracturing is the circumferential stress at the surface of the hole $\sigma_\varphi = p_* = 3,7\,\text{MPa}$. Should be noted, that according to Lame's solution there is a stress state of pure shear on the surface of the hole and at other points of the infinite medium.

3. Fracture of solid cylinders by diametral compression

Smaller cylinders: 61 mm in diameter and 31 mm in thickness were produced from the same GF-177 material. When a solid cylinder is compressed along a diameter between rigid plates (Brazilian test), a constant tensile stress acts in the section passing through the load application points acting perpendicular to this section. With the exception of the vicinity of the load application points, the magnitude of this stress is practically constant and equals $\sigma = \frac{P}{\pi R t}$, where P is the compressive force, R and t are the radius and thickness of the cylinder. In addition, at the points of this section a compressive stress is applied along the cross-section, i.e. perpendicular to the tensile stress. Thus, in the section passing through the load application points, compound stress state is taking place. The compressive stress in the section is not constant, in the center of the cylinder it reaches a minimum in absolute value and is equal $\sigma_c = -\frac{3P}{\pi R t}$, i.e. three times higher than the tensile stress. Nevertheless, cylinders made of brittle materials, in which the tensile strength is much less than compression strength, are fractured by splitting the disk along the plane connecting the load application points. Thus, the fracture occurs mainly due to the action of tensile stresses, although the presence of a compressive stress affects the magnitude of the tensile stress upon fracture.

Three solid cylindrical samples were tested by diametral compression. Following values of the ultimate tensile stress were obtained $\sigma_1 = 1,91\,\text{MPa}$, $\sigma_2 = 1,72\,\text{MPa}$, $\sigma_3 = 1,82\,\text{MPa}$. The average value of the ultimate tensile stress for the three tests is $\sigma_* = 1,82\,\text{MPa}$. This value is 2 times smaller than the ultimate tensile stress during hydraulic fracturing.

4. Compression fracture and a limit state diagram scheme

Samples from GF-177 were made in the form of rectangular parallelepipeds for compression fracture tests. The average value of the compressive strength for the three tests is $\sigma_c = 10,2\,\text{MPa}$.
Based on the test results for two types of stress state, a limit state diagram was constructed using the Mohr- Coulomb strength criterion (Figure 2) and an estimation of the tensile strength of the material for uniaxial tension was obtained $\sigma_b = 3,92\,\text{MPa}$. A light square in the limit state diagram shows the result obtained during hydraulic fracturing. The experimental value of the ultimate tensile stress during hydraulic fracturing exceeds the estimation that can be obtained by the Mohr-Coulomb test, which can be explained by the influence of the inhomogeneity of the stress state during hydraulic fracturing.

5. Determination of the critical stress intensity factor

Two cylindrical samples with a central cut were made to determine the critical stress intensity factor of

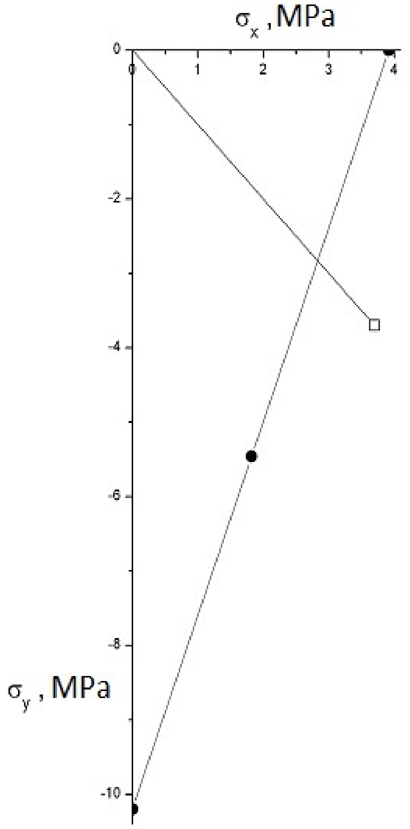

Figure 2. The limit state diagram.

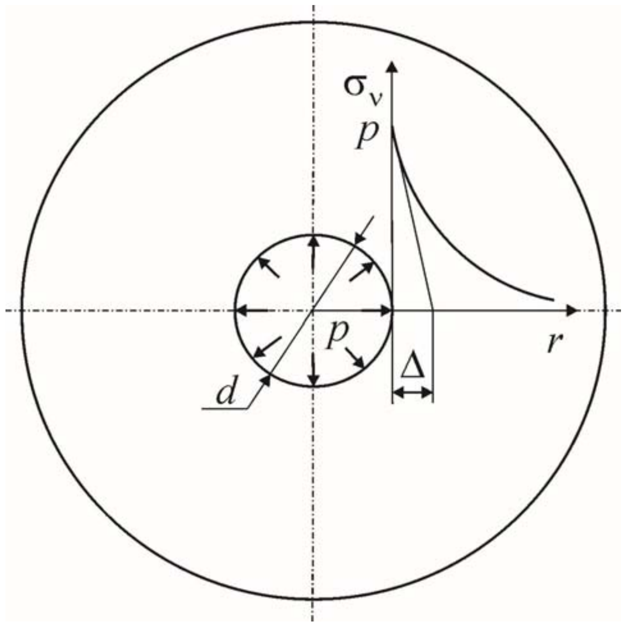

Figure 3. The sample loading scheme and the plot of the tensile stress.

GF-177 material. Samples were tested on a Zwick / Roell Z100 test machine by compressing between plates along the cutting line. According to two experiments, the average value of the critical stress intensity factor was obtained $K_{Ic} = 0{,}142\,\mathrm{MPa}\cdot\mathrm{m}^{1/2}$. Using this value of K_{Ic} and an estimation of the ultimate strength of the material $\sigma_b = 3{,}92\,\mathrm{MPa}$, the parameter that enters into the average stress criterion and the gradient criterion was calculated $L_1 = (2/\pi)K_{Ic}^2/\sigma_b^2 = 0{,}835\,\mathrm{mm}$.

6. Hydraulic fracture modeling using a gradient fracture criterion

A gradient criterion can be used to model fracture during frecking [5-7]. According to this criterion, with an inhomogenous stress distribution to determine the onset of fracture, the ultimate tensile stress of the material should be compared with the maximum value of the effective stress σ_e instead of maximum value of the first principal stress σ_1 (assumed as equivalent), that is higher than the equivalent stress. The effective stress is proportional to σ_1 and, in addition, depends on the local inhomogeneity of the stress field in the vicinity of the point under consideration and the representative size of the material's inhomogeneity. Local inhomogeneity of stress distribution is characterized by a relative gradient of positive normal stress acting on the plane including the normal plane of the first principal stress at the considered point of the body, where the plane and the platform have a common normal. Calculating the value $|\mathrm{grad}\,\sigma_\nu|$, in some tasks is easier than the previously used value

$|\mathrm{grad}\,\sigma_1|$ [5, 6]. The relative gradient g_ν is inversely proportional to the conditional size of the stress concentration zone Δ, which is shown in Fig. 3, i.e. $g_\nu = 1/\Delta$. The relative gradient is found using the solution of the corresponding problem of elasticity theory. The expression for the effective stress is written in the form

$$\sigma_e = \sigma_1 \Big/ \left(1 - \beta + \sqrt{\beta^2 + L_1 g_\nu}\right)$$

Here, L_1 is a parameter having the dimension of length and characterizing the heterogeneity (defectiveness) of the material; β is a non-negative dimensionless parameter, which can be considered as an approximation parameter.

The parameter L_1 is found in [5] from the condition for matching the gradient criterion with linear fracture mechanics and is expressed in terms of known material characteristics – the ultimate strength σ_b and the critical stress intensity factor K_{Ic} by the formula $L_1 = (2/\pi) K_{Ic}^2 / \sigma_b^2$.

We will assume that the fracture in the vicinity of the point under consideration begins when the effective tensile stress reaches the ultimate tensile stress and initially spreads through the action area of the maximum tensile stress. In the case of hydraulic fracturing, this is the area of action of the circumferential stress, which was confirmed experimentally (Figure 1).

According to Lame's solution, when the pressure p is acting on the surface of a circular hole in an infinite medium, then the circumferential and radial stresses are determined by the formulas $\sigma_\varphi = p\dfrac{a^2}{r^2}$ and $\sigma_r = -p\dfrac{a^2}{r^2}$. In this case, the tensile stress $\sigma_\nu = \sigma_\varphi$. The distribution of this stress along the radius is shown in Fig. 3. Note that in the absence of axial forces and stresses $\sigma_z = 0$, a plane strain occurs with $\varepsilon_z = 0$, since $\varepsilon_z = \dfrac{1}{E}\left[\sigma_z - \nu(\sigma_r + \sigma_\varphi)\right]$.

Calculation of the relative gradient yields $g_\nu = 4/d$, i.e. $\Delta = d/4$. Thus, the diameter of the hole enters the gradient fracture condition, which allows one to describe the scale effect upon fracture under conditions of a non-uniform stress state. Note that in the classical conditions of fracture, the size of the hole is not taken into account.

7. Combined use of the boundary element method and gradient fracture criterion

By using the modification of the program [9] in [10-12] the algorithm for joint application of the method of boundary elements (in the variant of the method of fictitious loads) and the gradient fracture criterion was implemented. For the problem of hydraulic fracturing of a thick cylinder, this algorithm was verified by comparing the numerical results with the results obtained by solving the Lame problem for a thick-walled tube. The outer and inner contours of the cylinder were divided into 360 elements. Thus the value $\sigma_* = 2{,}83\,\text{MPa}$ was used as the limiting tensile stress in pure shear. This value was obtained by intersecting a ray passing from the origin in Fig. 2 at an angle of 45 degrees and corresponding to the loading during hydraulic fracturing, with a straight line drawn through two experimental points (compressive strength and Brazilian test ultimate strength). To determine the pressure at which fracture starts during a frecking, the effective stress in the gradient criterion was compared with σ_*. The inhomogeneity of the stress state in the vicinity of the hole leads to a decrease of σ_e in comparison with the circumferential stress σ_φ and, consequently, an increase in the limiting pressure p_* in comparison with σ_*. Thus, the joint application of the boundary element method and the gradient fracture criterion gives the ultimate pressure $p_* = 3{,}55\,\mathrm{MPa}$, which is close to the mean experimental value $p_* = 3{,}7\,\mathrm{MPa}$.

8. Conclusion

Experimental analysis of hydraulic fracturing of thick-walled cylinders with a central circular hole was carried out using a machine that creates a high oil pressure. Cylinders were made of GF-177 material on the cement basis with polymer additives. The value of the limiting tensile circumferential stress of 3.7 MPa is obtained. Also, limiting stresses in the material under study were determined under two more types of stress state: under compression and under a complex stress state under conditions of compression of a solid cylinder by diameter. The value of the critical stress intensity factor of material GF-177 was obtained. Using the method of boundary elements (in the variant of the fictitious load method) and the gradient fracture criterion, the fracturing process is modeled taking into account the heterogeneity of the stress state near the hole. A comparison of the results of calculations with the values of limit pressure obtained analytically based on Lame’s solutions and experimental data was made.

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