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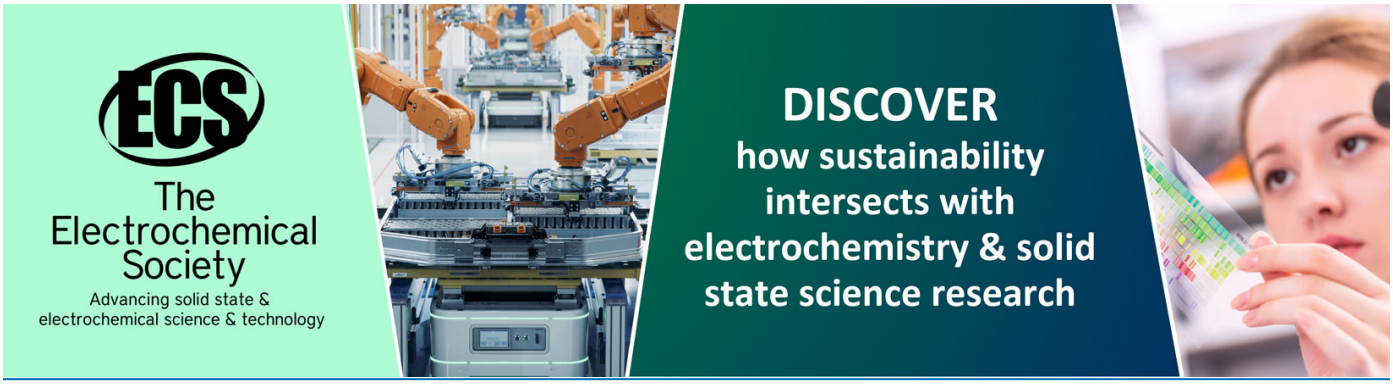

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Stability of Partial slip, Soret and Dufour effects on unsteady boundary layer flow and heat transfer in Copper-water nanofluid over a stretching/shrinking sheet

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Abstract. The stability of unsteady boundary layer flow and heat transfer over stretching/ shrinking sheet immersed in Copper-water nanofluid is studied with the presence of partial slip, Soret and Dufour effects. Tiwari and Das model is considered to solve the nanofluid boundary layer problem. The system of partial differential equations is transformed to ordinary differential equations using similarity transformation and was solved using bvp4c program in Matlab software to obtain the numerical solutions. The results were displayed graphically and the figures revealed that the dual solutions with the presence of partial slip, Soret and Dufour effects were exist for a certain range of stretching/shrinking parameter. Finally, the stability analysis is applied in order to determine the stability of the solution.

1. Introduction

Recently, the classical no-slip assumption has been replaced by velocity slip effect since that particular assumption is not consistent with all characteristics physically (Bhattacharyya et al.[1]). Mukhopadhyay [2] stated that the presence of velocity slip that proportional to local shear stress may exist when the fluid is particulate for instance emulsion, suspensions, foams and polymer solutions. Besides, the consideration of slip in the problem has important applications in various fields such as in medical Mukhopadhyay [2], polymer melts Khan et al.[3] and some other fields. There were some studies that included slip effect in boundary layer flow have been made by some researchers for example Ullah et al.[4] which investigated the slip condition on MHD flow, Pandey et al.[5] considered the stretching cylinder in Copper-water nanofluid, Aurangzaib et al.[6] studied the unsteady MHD mixed convection with stagnation point in micropolar fluid and etc. Soret effect is a term which represents the mass flux caused by temperature gradient while Dufour effect denotes the heat flux the heat and mass flux due to concentration gradient. According to Omowaye et al.[7], both effects became important in areas such as petrology, geology, hydrology and etc. when there is density gradient due to the presence of particles in the boundary layer flow. Thus, some authors have considered both effects in their work in various situations for example over different surface as investigated by Moorthy et al.[8], Alam and Samad [9] and Animasaun et al.[10] where the findings show that the flow is influenced by Soret and Dufour effects. Merkin [11] in his work has proposed

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the stability analysis when there is more than one solutions obtained. He found that the first solution was stable and reliable while the second solution was not. Following Merkin [11], some other authors such as Weidman et al.[12], Roşca and Pop [13], Najib et al.[14] and Bachok et al.[15] have performed the analysis of stability in their paper and concluded the same finding.

This study is an extension of Bachok et al.[16] with Soret and Dufour effects as proposed by Alam and Rahman [17].The main purpose of this present work is to investigate the characteristics of boundary layer flow, heat and mass transfers over a stretching/ shrinking sheet in nanofluid when the Soret and Dufour effects are taken into consideration for unsteady problem. The governing equations are transformed to ordinary differential equations using dimensionless similarity transformation parameter and are solved numerically by Matlab. The stability analysis is performed using bvp4c program in order to determine the stability of the numerical solutions.

2. Problem formulation

Unsteady boundary layer flow over a stretching/ shrinking surface immersed in Copper-water nanofluid is solved using Tiwari and Das model. Assume that at $t<0$, the surface is in stationary state with velocity $u_w = 0$. As $t>0$, the surface begin to stretch or shrink where the velocity with slip is $u_w = Ax/t + L\partial u/\partial y$ which $A>0$ is dimensionless acceleration parameter. v_w represents velocity of mass flux where $v_w > 0$ is for injection and $v_w < 0$ is for suction. Let the uniform temperature and concentration at the surface of the plate are T_w and C_w. T_∞ and C_∞ are the temperature and the concentration of the ambient fluid. Following the assumptions above, the governing equations of the problem are, see Bachok et al.[16] and Alam and Rahman [17];

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 v}{\partial y^2}, \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2}, \tag{4}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_m\frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m}\frac{\partial^2 T}{\partial y^2}, \tag{5}$$

subject to boundary conditions

$$\begin{aligned} &t<0,\ v=0,\ u=0,\ T=T_\infty,\ C=C_\infty \quad \text{for all } x \text{ and } y,\\ &t\ge 0,\ v=v_w,\ u=u_w(x)=\varepsilon Ax/t + L\partial u/\partial y,\ T=T_w,\ C=C_w \quad \text{at } y=0,\\ &u\to 0,\ T\to T_\infty,\ C\to C_\infty \quad \text{as } y\to\infty, \end{aligned} \tag{6}$$

where x and y are the Cartesian coordinate along and perpendicular to the plate. u and v are the velocity component in x and y directions, T is the temperature of the nanofluid, C is the concentration of the nanofluid, L is the length of the slip, p is the fluid pressure, D_m is the coefficient of mass diffusivity, c_p is the specific heat at constant pressure, T_m is the mean fluid temperature, k_T is the thermal diffusion ratio, c_s is the concentration susceptibility, respectively. While α_{nf} is the thermal diffusivity of the nanofluid, μ_{nf} is the viscosity of the nanofluid, ρ_{nf} is the

density of the nanofluid which can be referred in Oztop and Abu Nada [18]. The similarity solution of equations (1) – (5) subjected to boundary condition (6) in the following form;

$$\psi = Ax(\nu / t)^{1/2} f(\eta),\ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty},\quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty},\quad \eta = \frac{y}{(\nu t)^{1/2}}, \tag{7}$$

where η is the dimensionless similarity variable, primes denote differentiation with respect to η, ψ is the stream function which defines $u = \partial\psi / \partial y = \left(Ax / t\right) f'(\eta)$ and $v = -\partial\psi / \partial x = -A(\nu / t)^{1/2} f(\eta)$. $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ are dimensionless stream, temperature and concentration functions of the fluid in the boundary layer, respectively. v_w is represented as $v_w = -A(\nu / t)^{1/2} s$, where is s the constant mass flux which $s > 0$ for suction and $s < 0$ for injection. In order to make the equations look simpler, we let $B = 1/\left(1-\varphi\right)^{2.5}\left(1-\varphi+\varphi\left(\rho_s / \rho_f\right)\right)$ and $C = k_{nf} / k_f\left(1-\varphi+\varphi\left(\left(\rho c_p\right)_s / \left(\rho c_p\right)_f\right)\right)$, thus

$$Bf''' + \left(\eta / 2\right) f'' + f' - A\left(f'^2 - ff''\right) = 0 \tag{8}$$

$$\left(1/\Pr\right) C\theta'' + \left[\left(\eta / 2\right) + Af\right]\theta' + Du\phi'' = 0 \tag{9}$$

$$\phi'' + Sc\left[\left(\eta / 2\right) + Af\right]\phi' + ScSr\theta'' = 0 \tag{10}$$

subject to boundary conditions

$$f(0) = s,\ f'(0) = \varepsilon + \sigma f''(0),\ \theta(0) = 1,\ \phi(0) = 1,\ f'(\infty) \to 0,\ \theta(\infty) \to 0,\ \phi(\infty) \to 0. \tag{11}$$

Primes denote differentiation with respect to η, $\sigma = L / \sqrt{\nu t}$ is velocity slip parameter, Sc is Schmidt number, Df is Dufour number and Sr is the Soret number which can defined as

$$\Pr = \frac{\nu}{\alpha},\ Sc = \frac{\nu}{D_m},\ Df = \frac{D_m k_T\left(C_w - C_\infty\right)}{\nu c_s c_p\left(T_w - T_\infty\right)},\ Sr = \frac{D_m k_T\left(T_w - T_\infty\right)}{\nu T_m\left(C_w - C_\infty\right)}. \tag{12}$$

The skin friction coefficient, local Nusselt number and the local Sherwood number are the quantities of physical interest in this problem and defined as

$$C_f = \frac{\tau_w}{\rho_f u_w^{\ 2}},\ Nu_x = \frac{xq_w}{k(T_w - T_\infty)},\ Sh_x = \frac{xq_m}{D_m(C_w - C_\infty)}, \tag{13}$$

where τ_w and q_w are the shear stress, heat flux and mass flux, respectively as given

$$\tau_w = \mu_{nf}\left(\partial u / \partial y\right)_{y=0},\ q_w = -k\left(\partial T / \partial y\right)_{y=0},\ q_m = -D_m\left(\partial C / \partial y\right)_{y=0}, \tag{14}$$

where μ is the dynamic viscosity of the fluid and k is the thermal conductivity of the nanofluid. Using equations (7), (13) and (14), we obtained

$$C_f \operatorname{Re}_x^{1/2} = \frac{f''(0)}{A^{1/2}\left(1-\phi\right)^{2.5}},\ Nu_x \operatorname{Re}_x^{-1/2} = -\frac{k_{nf}}{k_f}\frac{\theta'(0)}{A^{1/2}},\ Sh_x \operatorname{Re}_x^{-1/2} = -\frac{\phi'(0)}{A^{1/2}} \tag{15}$$

where $\operatorname{Re}_x = u_w x / \nu$ represents local Reynold number.

3. Stability analysis

The stability analysis is performed to investigate the stability of solutions since dual solutions are obtained. Weidman et al.[12] and Roşca and Pop [13] have shown that the first solution is stable while the second solution is not. This analysis is tested by considering equations (1) - (5) and new dimensionless time variable $\tau = \ln(t / t_0)$ is introduced where t_0 is a characteristic time (take $t_0 = 1$) and τ is associated with an initial value problem and is consistent with the question of which solution will be obtained in practice (physically realizable).

$$u=\frac{Ax}{t}\frac{\partial f}{\partial\eta}(\eta,\tau), v=-A\left(\frac{\nu}{t}\right)^{1/2} f(\eta,\tau), \theta(\eta,\tau)=\frac{T-T_\infty}{T_w-T_\infty},$$
$$\phi(\eta,\tau)=\frac{C-C_\infty}{C_w-C_\infty}, \eta=\frac{y}{(\nu t)^{1/2}}, \tau=\ln(t). \tag{16}$$

Thus, equations (1) - (5) can be written as

$$B\frac{\partial^3 f}{\partial\eta^3}+A\left[f\frac{\partial^2 f}{\partial\eta^2}-\left(\frac{\partial f}{\partial\eta}\right)^2\right]+\frac{\partial f}{\partial\eta}+\frac{\eta}{2}\frac{\partial^2 f}{\partial\eta^2}-\frac{\partial^2 f}{\partial\eta\partial\tau}=0 \tag{17}$$

$$\frac{1}{\Pr}C\frac{\partial^2\theta}{\partial\eta^2}+\left(\frac{\eta}{2}+Af\right)\frac{\partial\theta}{\partial\eta}+Df\frac{\partial^2\phi}{\partial\eta^2}-\frac{\partial\theta}{\partial\tau}=0 \tag{18}$$

$$\frac{\partial^2\phi}{\partial\eta^2}+Sc\left(\frac{\eta}{2}+Af\right)\frac{\partial\phi}{\partial\eta}+ScSr\frac{\partial^2\theta}{\partial\eta^2}-Sc\frac{\partial\phi}{\partial\tau}=0 \tag{19}$$

subject to the initial and boundary conditions

$$f(0,\tau)=s,\quad \frac{\partial f}{\partial\eta}(0,\tau)=\varepsilon+\sigma\frac{\partial^2 f}{\partial\eta^2}(0,\tau),\quad \theta(0,\tau)=1,\quad \phi(0,\tau)=1, \tag{20}$$
$$\frac{\partial f}{\partial\eta}(\infty,\tau)\to 0,\quad \theta(\infty,\tau)\to 0,\quad \varphi(\infty,\tau)\to 0.$$

To determine the stability of the solution $f=f_0(\eta)$, $\theta=\theta_0(\eta)$ and $\phi=\phi_0(\eta)$, satisfying the boundary-value problem (8) -(11), we write (see Roşca and Pop [13])

$$f(\eta,\tau)=f_0(\eta)+e^{-\gamma\tau}F_0(\eta),\quad \theta(\eta,\tau)=\theta_0(\eta)+e^{-\gamma\tau}G_0(\eta),\ \phi(\eta,\tau)=\phi_0(\eta)+e^{-\gamma\tau}H_0(\eta), \tag{21}$$

where γ is an unknown eigenvalue parameter, and $F_0(\eta)$, $G_0(\eta)$ and $H_0(\eta)$ are small relative to $f_0(\eta)$, $\theta_0(\eta)$ and $\phi_0(\eta)$. Substituting (21) into equations (17) - (19), and take $\tau=0$, thus

$$BF_0'''+(Af_0+\eta/2)F_0''+(1-2Af_0'+\gamma)F_0'+Af_0''F_0=0, \tag{22}$$
$$(1/\Pr)CG_0''+Af_0G_0'+AF_0\theta_0'+(\eta/2)G_0'+Du\,H_0''+\gamma G_0=0 \tag{23}$$
$$H_0''+Sc(Af_0+\eta/2)H_0'+AScF_0\phi_0'+ScSr\,G_0''+Sc\gamma H_0=0 \tag{24}$$

subject to the boundary conditions

$$F_0(0)=0, F_0'(0)-\sigma F_0''(0)=0, G_0(0)=0, H_0(0)=0, F_0'(\infty)\to 0, G_0(\infty)\to 0, H_0(\infty)\to 0. \tag{25}$$

Solving the eigenvalue problem (22) – (24) we obtain an infinite number of eigenvalues $\gamma_1<\gamma_2<\gamma_3<\ldots$. If the smallest eigenvalue is positive the flow is stable and if the smallest eigenvalue is negative the flow is unstable. According to Harris et al.[19] , the range of possible eigenvalues can be determined by relaxing a boundary condition on $F_0(\eta)$, $G_0(\eta)$ or $H_0(\eta)$. For the present problem, the boundary condition $F_0'(\eta)\to 0$ as $\eta\to\infty$ is relaxed and for a fixed value of γ, the system of equations (22) – (24) subject to (25) along with the new boundary condition $F_0''(0)=1$ is solved.

4. Results and discussion

The themophysical properties of water and Copper can be referred from Oztop and Abu Nada [18]. The values of $A=1$, $\Pr=6.2$, $s=1$ and $Sc=1$ are fixed. The effects of partial slip on reduced skin-friction, Nusselt number and Sherwood number coefficients are presented in Figure 1. Dual solutions exist when $\varepsilon>\varepsilon_c$ and no solution can be obtained when $\varepsilon<\varepsilon_c$, where ε_c is the critical value of ε. It is observed that from Figure 1(a) as the partial slip increases $f''(0)$ gives rise for stretching sheet $(\varepsilon>0)$ and decreases for shrinking sheet $(\varepsilon<0)$. This indicates that the shear stress at the surface increases for $\varepsilon>0$ and decreases for $\varepsilon<0$ when the partial slip σ becomes larger. The partial slip

effect on the reduced Nusselt number is depicted in Figure 1(b) where it shows that $-\theta'(0)$ decelerates for $\varepsilon > 0$ and accelerates for $\varepsilon < 0$ as the partial slip increases. This means that when σ increases, the heat transfer rate at the surface increases for $\varepsilon < 0$ but when the sheet is stretched the heat transfer rate decreases. Figure 1(c) shows that for the same increasing effect the reduced Sherwood number $-\phi'(0)$ is found decreases when the sheet is stretched and increases when it has been shrunk. Apparently, the mass transfer rate at the surface increases for $\varepsilon < 0$ and for the $\varepsilon > 0$ the mass transfer rate depreciates.

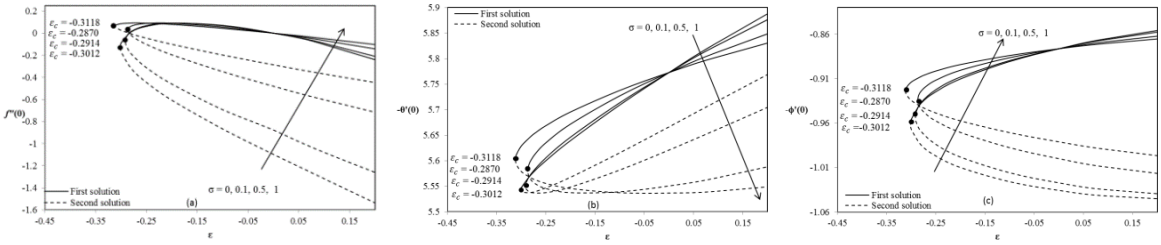

Figure 1. Variation of reduced skin friction coefficient, reduced Nusselt number and reduced Sherwood number with ε for different values of σ when $Df = 0.15$, $Sr = 0.4$, $\varphi = 0.1$, and $\sigma = 0.1$.

The effects of Soret and Dufour on local Nusselt and Sherwood numbers when the nanoparticle volume fraction φ increases from 0 to 0.2 for both $\varepsilon > 0$ and $\varepsilon < 0$ are shown in Figures 2 and 3, respectively. Figures 2(a) and 2(b) represent the increasing Soret effect on heat and mass transfer rates at the surface when the value Dufour is fixed to 0.15. Based on the figures, increasing Soret effect increases the heat transfer rate at the surface (see Figure 2(a)) but the opposite trend can be seen for mass transfer rate as shown in Figure 2(b). In addition, when the φ increases in the fluid, the heat transfer rate for first solution increases as $Sr < Df$ and decreases when $Sr \geq Df$ for both $\varepsilon > 0$ and $\varepsilon < 0$.

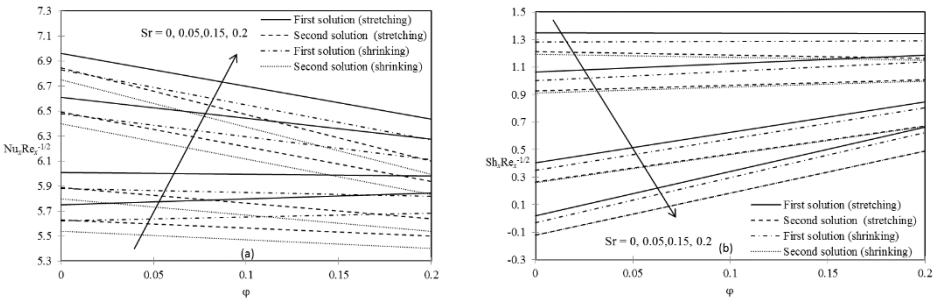

Figure 2. Variation of local Nusselt number and local Sherwood number with φ for different values of Sr when $Df = 0.15$, $\sigma = 0.1$, and ε=0.1 (stretching)/ -0.1 (shrinking).

Meanwhile the heat transfer rate for second solution for $\varepsilon > 0$ and $\varepsilon < 0$ shows the decreasing trend. Figure 3 demonstrates the effect of increasing Dufour on both heat and mass transfers with various φ. It can be seen from Figure 3(a) and 3(b), when the effect of Dufour becomes larger, the heat transfer rate at the surface decreases while the mass transfer rate increases at the surface. Besides, Figure 3(a) illustrates that when the fluid has more nanoparticle, the heat transfer are found decreases except when $Df = 0$ (without the presence of Dufour effect) the heat transfer at the surface is slightly increased for $\varepsilon > 0$ and $\varepsilon < 0$. Meanwhile, the increasing φ from 0 to 0.2 increases the mass transfer rate at the surface as illustrated in Figure 3(b).

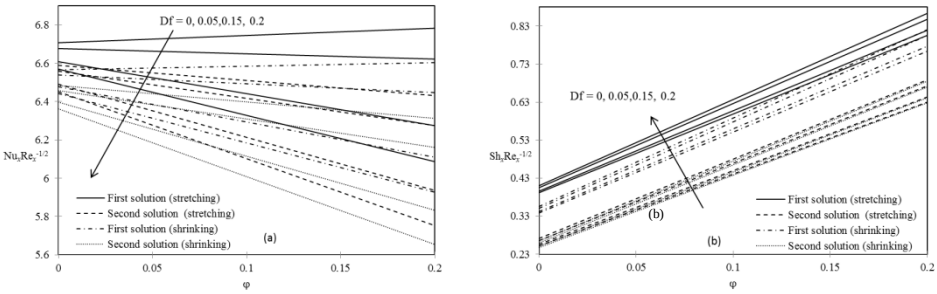

Figure 3. Variation of local Nusselt number and local Sherwood number with φ for different values of Df when $Sr = 0.15$, $\sigma = 0.1$, and ε=0.1 (stretching)/ -0.1 (shrinking) .

However, due to the space constraint profiles of velocity, temperature and concentration will not be represented here. Table 1 states the smallest eigenvalue for some values of σ and ε . According to the table, the values of γ for first solution are positive indicates that the first solution is stable while the smallest eigenvalues for second solution are negative which denotes that the second solution is unstable.

Table 1. The smallest eigenvalues γ for some values of σ and ε when $\varphi = 0.1$.

σ	ε_c	ε	γ (First solution)	γ (Second solution)
		-0.291	0.0317	-0.0314
0.1	-0.2914	-0.29	0.0589	-0.0580
		-0.2	0.4922	-0.4353
		-0.285	0.0592	-0.0583
0.5	-0.287	-0.28	0.1118	-0.1088
		-0.2	0.4034	-0.3666

5. Conclusion

The characteristic of boundary layer flow, heat and mass transfers due to partial slip, Soret and Dufour effects over stretching/ shrinking sheet in Copper-water nanofluid is studied using Tiwari and Das model. It was found that for $\varepsilon < -0.35$ the dual solutions were obtained for some values of partial-slip. Increasing partial slip parameter gave different trends for stretching and shrinking sheets for shear stress, heat transfer as well as mass transfer. The effect of increasing Soret increased the heat transfer but decreased the mass transfer at the surface. Meanwhile, increasing Dufour effect accelerated the mass transfer rate and decelerated the heat transfer rate. The stability analysis was performed and the eigenvalues for first solution are positive values indicate that the first solution was stable and reliable.

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