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**Improved synchronization criteria for time-delayed chaotic Lur'e systems using sampled-data control**

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**Abstract.** This paper is concerned with the synchronization for a class of time-delayed chaotic Lur'e systems using sampled-data control. Both of time-varying and time-invariant delays are considered. New criteria are proposed in terms of linear matrix inequalities (LMIs) by employing a modified LKF combined with the delay-fraction theory and some novel terms. The criteria are less conservative than some previous ones and a longer sampling period is achieved under the new results. Furthermore, the derived conditions are employed to design a sampled-data controller. The desired controller gain matrix can be obtained by means of the LMI approach. Finally, a numerical examples and simulations on Chua’s circuit is presented to show the effectiveness of the proposed approach.

1. **Introduction**

The synchronization of chaotic system has arisen a very hot topic in the nonlinearity community since the pioneering work of [1], where the chaotic system synchronization was first studied. Meanwhile, time-delays are commonly encountered in practice and are often attributed as a source of poor performance and instability. Thus, the synchronization of chaotic Lur'e systems with time delays attains considerable significance. Note that recent improvements of the synchronization criteria for chaotic Lur'e systems with time-invariant delays have been obtained in [2-8]. In order to further reduce the conservativeness of the synchronization criteria in [7,8], a novel integral inequality was developed based on the celebrated Jensen’s integral inequality [9] and Wirtinger integral inequality [10]. However, there still exists room for improving the synchronization criteria of the above literature.

**Notation:** The notation \( P > 0 \) \((< 0)\) mean that matrix \( P \) is positive (negative) definite. \( I \) denotes an identity matrix with appropriate dimensions. \(*\) denotes the symmetric terms in a block matrix and \( \text{diag}\{\cdots\} \) denotes a block-diagonal matrix. \( e_i \) \((i = 1,\cdots,m)\) are block entry matrices. For example, \( e_2^T = \text{diag}\{0, I, 0,\cdots, 0\} \). \( \text{Sym}\{B\} = B + B^T \).

2. **Problem formulation**

Consider the following general master-slave type of time-delayed chaotic Lur'e systems with sampled-data feedback controller:
which consists of the master system $M$, the slave system $S$ and the sampled-data feedback controller $C$. The master system and the slaver system are identical Lur'e chaotic systems when $u(t) = 0$ with state vectors $x(t), z(t) \in \mathbb{R}^n$, outputs of subsystems $p(t), q(t) \in \mathbb{R}^r$. $A \in \mathbb{R}^{nxn}, B \in \mathbb{R}^{nxm}, C \in \mathbb{R}^{nxn}, D \in \mathbb{R}^{nxm}$ and $H \in \mathbb{R}^{nxn}$. $u(t) \in \mathbb{R}^r$ is the slaver system control input and $K \in \mathbb{R}^{nol}$ is the sampled-data feedback control gain matrix to be designed. $d(t)$ is the time-varying delay satisfying the following condition with some scalars $d_1, d_2, d_\mu$: $d_1 \leq d(t) \leq d_2, \dot{d}(t) \leq d_\mu < 1$. $\sigma(\cdot): \mathbb{R}^n \mapsto \mathbb{R}^n$ is a diagonal nonlinearity with $\sigma_i(\cdot)$ belong to $[\omega_i, \omega_i^+]$, i.e., $[\sigma_i(\xi) - \omega_i^+ \xi][\sigma_i(\xi) - \omega_i^- \xi] \leq 0$. Suppose that the updating signal (successfully transmitted signal from the sampler to the controller and to the ZOH) at the instant $t_k$ has experienced a constant signal transmission delay $\eta$. Therefore, we have $t_{k+1} - t_k + n \leq h + \eta = \tau_M$.

Now, we define the synchronization error as $e(t) = x - z$, $\tau(t) = t - t_k + \eta$, $h_i(t) = h + \eta - \tau(t)$, then $\dot{e} = Ae + Be(t - d(t)) + Hf(D^T e) - KCe(t - \tau(t))$, where

\[
[f_i(d_i^T e, z) - \omega_i^+ d_i^T e] f_i(d_i^T e, z) - \omega_i^- d_i^T e] \leq 0, \quad (2)
\]

$d_i^T$ denotes the i-th row of $D^T$. $K_1 = \text{diag}(\omega_1^+, \cdots, \omega_n^+)$ and $K_2 = \text{diag}(\omega_1^-, \cdots, \omega_n^-)$.

**Definition 1.** The master system and slave system are said to be asymptotically synchronized if and only if the error dynamical systems is absolutely stable in the sector $[K_1, K_2]$ for the equilibrium point $e(t) \equiv 0$. That is, $e(t) \to 0$ as $t \to \infty$.


3. Main results

\[
\tau_c = \eta + \frac{h}{2}, \tau_v = \frac{h}{2}, h_i(t) = \tau_M - \tau(t), d_{12} = d_2 - d_1, e^T = [e^T e^T (t - \tau(t))], \delta^T = [\zeta^T e^T (t - \tau(t))],
\]

\[
\zeta^T = \left[ e^T \int_{t - \tau}^{t - \tau_c} e^T(s)ds \right]_{t - \tau}^{t - \tau_c}, \zeta^T = \left[ e^T (t - d_i) e^T (t - d_1) e^T (t - d_2) e^T (t - \eta) e^T (t - \tau_c) \right]
\]

\[
e^T (t - \tau(t)) e^T (t - \tau M) \frac{1}{\eta} \int_{t - \tau}^{t - \tau_c} e^T (s)ds \int_{t - \tau}^{t - \tau_c} e^T (s)ds \int_{t - \tau}^{t - \tau_c} e^T (s)ds \int_{t - \tau}^{t - \tau_c} e^T (s)ds \int_{t - \tau}^{t - \tau_c} e^T (s)ds \int_{t - \tau}^{t - \tau_c} e^T (s)ds \int_{t - \tau}^{t - \tau_c} e^T (s)ds
\]

\[
\zeta_2^T = \left[ e^T (t - d_i) e^T (t - d_1) e^T (t - d_2) e^T (t - \eta) e^T (t - \tau_c) \right]
\]

\[
\frac{1}{\eta} \int_{t - \tau}^{t - \tau_c} e^T (s)ds \int_{t - \tau}^{t - \tau_c} e^T (s)ds \int_{t - \tau}^{t - \tau_c} e^T (s)ds \int_{t - \tau}^{t - \tau_c} e^T (s)ds \int_{t - \tau}^{t - \tau_c} e^T (s)ds \int_{t - \tau}^{t - \tau_c} e^T (s)ds
\]

**Theorem 1.** For given scalars $\eta, h, d_1, d_2 > 0, d_\mu < 1, \Sigma K_1, K_2$ and a scalar $r$, the master system $M$ and slave system $S$ in (1) are synchronized if there exist positive matrices $P = [P_{ab}]_{4 \times 4}, Q_i, W_i, R_j, Z_j (i = 1, 2, 3; j = 1, 2), \bar{G} = [G_{ab}]_{2 \times 2}, \bar{V} = [V_{ab}]_{2 \times 2}$ and $\bar{N} = [N_{ab}]_{2 \times 2}$, positive definite diagonal matrices $A = \text{diag}(\lambda_1, \cdots, \lambda_n)$, $A = \text{diag}(\delta_1^-, \cdots, \delta_n^-), T_m (m = 1, 2)$, any appropriately dimensioned
matrices $U = [U_1^T U_2^T]^T$, $\overline{M} = [M_{ab}]_{2 \times 2}$, $\overline{V} = [V_{ab}]_{2 \times 2}$, $T, V, L, X_1, X_2$ and $X_3$ such that the following LMIs hold for $k = 1, 2$:

$$
\begin{bmatrix}
R_2 & T \\
\ast & R_2
\end{bmatrix} > 0, \\
\begin{bmatrix}
P_{11} + \tau_M \text{Sym}(X_1) & P_{12} & P_{13} & P_{14} & \tau_M X_3 \\
\ast & P_{22} & P_{23} & P_{24} & 0 \\
\ast & \ast & P_{33} & P_{34} & 0 \\
\ast & \ast & \ast & P_{44} & 0 \\
\ast & \ast & \ast & \ast & \tau_M \text{Sym}(X_2)
\end{bmatrix} > 0^*,
$$

where $G(i,t) = [e_i \quad e_{i+1}]^T$.

$$
\Theta_e^k = \Pi^k_e + \tau_M \text{Sym}([e_i \quad e_j]X_{i+1} \quad 0^T) + \tau_M [e_i \quad e_{i+1}]\overline{N}[e_i \quad e_{i+1}]^T + G(2,i) < 0,
$$

$$
\Xi_k = \begin{bmatrix}
\Pi^k_e - \tau_M e_T N_{i+1} e_i^T + G(1,t) & \Omega_{12} \\
\ast & -\tau_M N_{i+2}^T
\end{bmatrix} < 0,
$$

Proof: Construct an LKF candidate as $V(t) = \sum_{i=1}^6 V_i(t)$ with

$$
V_i = \sum_{i=1}^n \int_{t_i}^{t_{i+1}} \left[ \lambda_i(\omega_i s - f_i) + \delta_i(\phi_i - \phi_i s) \right] ds, \quad V_2 = \varepsilon^T P \varepsilon + h_i(t) e^T \bar{X} e, \quad V_3 = \int_{t_{i+1}}^{t_{i+2}} e^T Q e ds + \sum_{i=1}^{d_i-1} e^T Q e ds
$$

Moreover, the sampled-data controller gain matrix is given by $K = V^{-1} L$.

$$
\text{Proof: Construct an LKF candidate as } V(t) = \sum_{i=1}^6 V_i(t) \text{ with }
$$

$$
Q_{11} = \begin{bmatrix}
-R_2 & R_1 - T \\
\ast & -2R_2 + T + T^T & R_2 - T \\
\ast & \ast & \ast & \ast & 12W_{i+1}
\end{bmatrix}, \quad \Sigma_{W_i} = \begin{bmatrix}
4W_j & 2W_j & -6W_j \\
\ast & 4W_j & -6W_j \\
\ast & \ast & 12W_{i+1}
\end{bmatrix}, \quad \bar{W}_2 = \text{diag}(W_2, 3W_2), \quad \bar{W}_i = \text{diag}(W_i, 3W_i), \quad \bar{U} = \begin{bmatrix}
U_1 & U_1^T \\
\ast & -U_1 & -U_1^T
\end{bmatrix},
$$

$$
\text{Proof: Construct an LKF candidate as } V(t) = \sum_{i=1}^6 V_i(t) \text{ with }
$$

$$
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$$

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$$
\[ + \tau \int_{-\tau}^{0} e^{T} W_s e^{s} d\theta + \tau \int_{-\tau}^{\tau} e^{T} W_s e^{s} d\theta, \quad V_6 = h_s(t) \int_{t-\tau(t)}^{t} \left[ e^{(t-\tau(t))}^{T} \right] ^{T} N \left[ e^{(t-\tau(t))} \right] ds. \]

\( V_2 \) can be rewritten as \( V_2 = \frac{(t-\tau(t))}{\tau_0} P \xi + \frac{h_s(t)}{\tau_0} \int_{t-\tau(t)}^{\tau} \frac{\phi(t)}{\tau_0} d\xi ds \). We can obtain \( V_2 > 0 \) if LMI (3) holds, then the LKF is positive. For any appropriately dimensioned matrix \( U = [U_1^T \quad U_2^T]^T \), we have \( 0 = 2[e^{T} + e^{T}(t-\tau(t)) + r e^{T}] V[e - \dot{e}] = 0 \). Moreover, for any diagonal matrix \( T_i > 0 (i = 1,2) \), it follows from (2) that \( \frac{\xi(t)}{\tau_0} h(t) + \frac{h_s(t)}{\tau_0} g(2,t) \geq 0 \).

**Case I.** For \( \tau(t) \in \tau_r, r \), we have \( \dot{V}_2 = \frac{\xi(t)}{\tau_0} \left[ \text{Sym} [e_i \ e_s \ 0 \ \tau e_{s1}] PP^T \xi + \frac{h_s(t)}{\tau_0} [\text{Sym} [0 \ 0 \ \tau e_{s1} \ 0]PP^T \xi] \right] \xi \gamma_2 \xi^T \left[ \text{Sym} [0 \ 0 \ \tau e_{s1} \ 0]PP^T \xi \right] \xi - \xi^T \left[ \xi^T + \frac{h_s(t)}{\tau_0} \left[ \text{Sym} [0 \ 0 \ \tau e_{s1} \ 0]PP^T \xi \right] \right] \xi + 2h_s(t) \xi^T \left[ e_{s1} \right] \Xi [ e_{s1} \ e_{s1}]^T + 2h_s(t) \xi^T \left[ e_{s1} \right] \Xi [ e_{s1} \ e_{s1}]^T \xi_1. \]

Letting \( L = VK \), it follows from above inequalities that \( \dot{V} \leq \frac{\xi(t)}{\tau_0} \Theta^T \xi_1 + \frac{1}{\tau_0} \int_{t-\tau(t)}^{t} \left[ \xi_1(t) \right] ^{T} \xi_1 \left[ \dot{e}(s) \right] ds. \)

Therefore, LMs (3)-(5) hold for \( i = 1, k = 1,2 \), which together with Schum complement equivalence imply that \( \dot{V} < 0 \).

**Case II.** For \( \tau(t) \in \tau_r, \), by following a similar line of arguments as that in Case I, LMs (3)-(5) hold for \( i = 2, k = 1,2 \), which together with Schum complement equivalence imply that \( \dot{V} < 0 \).

Hence, it follows from the Lyapunov stability theory that the synchronization error system is absolutely stable, i.e., master system \( M \) and slave system \( S \) in (1) are global asymptotical synchronization from definition 1. This completes the proof.

### 4. Case study

**Table 1.** \( h \) with \( n = 0 \).

<table>
<thead>
<tr>
<th>Methods</th>
<th>( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[3]</td>
<td>0.3582</td>
</tr>
<tr>
<td>[6]</td>
<td>0.4355</td>
</tr>
<tr>
<td>[5]</td>
<td>0.4357</td>
</tr>
<tr>
<td>[8]</td>
<td>0.4369</td>
</tr>
<tr>
<td>[12]</td>
<td>0.4395</td>
</tr>
<tr>
<td>Theorem 1</td>
<td>0.6708</td>
</tr>
</tbody>
</table>

**Table 2.** \( h \) with \( n = 0 \).

<table>
<thead>
<tr>
<th>Methods</th>
<th>( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[13]</td>
<td>0.3914</td>
</tr>
<tr>
<td>[3]</td>
<td>0.3981</td>
</tr>
<tr>
<td>[14]</td>
<td>0.48</td>
</tr>
<tr>
<td>[5]</td>
<td>0.5147</td>
</tr>
<tr>
<td>[8]</td>
<td>0.5218</td>
</tr>
<tr>
<td>Theorem 1</td>
<td>0.7243</td>
</tr>
</tbody>
</table>

**Table 3.** \( h \) with different \( n \).

<table>
<thead>
<tr>
<th>( \eta )</th>
<th>( h )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[15]</td>
<td>0.63</td>
</tr>
<tr>
<td>[5]</td>
<td>2</td>
</tr>
<tr>
<td>Theory 1</td>
<td>2</td>
</tr>
<tr>
<td>[15]</td>
<td>0.63</td>
</tr>
<tr>
<td>[5]</td>
<td>2</td>
</tr>
<tr>
<td>Theory 1</td>
<td>2</td>
</tr>
</tbody>
</table>

**Example [3,5,6,8,12].** Consider the following Chua’s circuit (1) with sampled-data feedback control under the following parameters:

\[
A = \begin{bmatrix}
-a m_1 & a \\
1 & -1 & 1
\end{bmatrix}, \quad B = \begin{bmatrix}
-c & 0 & 0 \\
-c & 0 & 0 \\
2 c & 0 & -c
\end{bmatrix}, \quad H = \begin{bmatrix}
-a(m_0 - m_1) & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}, \quad C = D = \begin{bmatrix}1 & 0 & 0\end{bmatrix}.
\]
In order to compare with some existing methods, let \( d_\mu = 0 \), i.e., \( d(t) \equiv d \) is a constant. The maximum value of the sampling period \( h \) can be found in tables 1-3. We obtain the controller gain \( \mathbf{K} = [2.5597 \ -0.1068 \ -2.9233]^T \) with \( h = 0.6708 \) using Matlab LMI Toolbox. The responses of the states \( x(t), z(t) \) and the error signal \( e(t) \) of system (1) with the sampling period \( h = 0.6708 \) are given in figure 1 and figure 2.

**Figure 1.** Response of \( x(t) \) and \( z(t) \) with \( h = 0.6708 \).

**Figure 2.** Response of \( e(t) \) with \( h = 0.6708 \).

5. Conclusions
In this paper, some new synchronization criteria are proposed for time-delayed chaotic Lur’e systems using sampled data via a modified LKF combining the delay-fraction theory and the piecewise analysis method.

6. References

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