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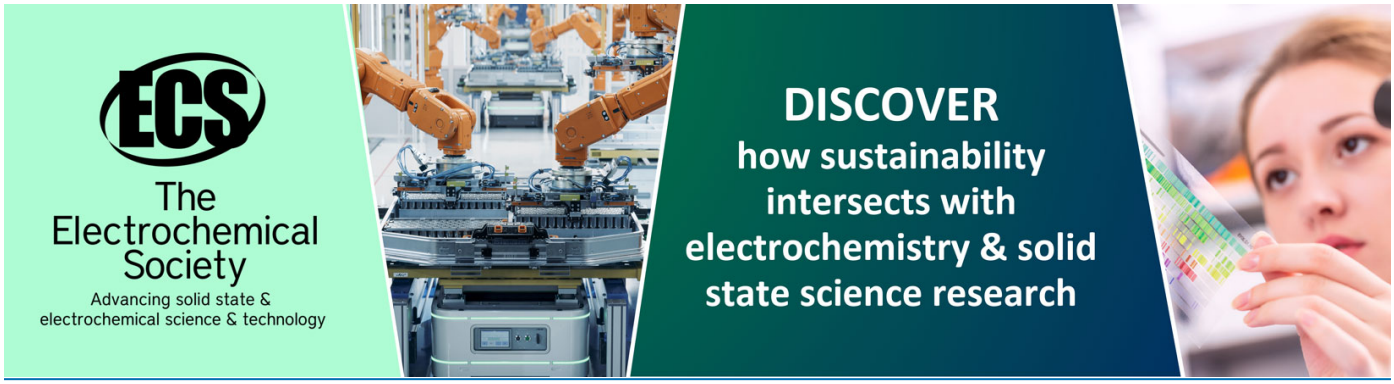

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Comparison between a typical and a simplified model for blast load-induced structural response

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Abstract. As explosive blasts continue to cause severe damage as well as victims in both civil and military environments. There is a bad need for understanding the behavior of structural elements to such extremely short duration dynamic loads where it is of great concern nowadays. Due to the complexity of the typical blast pressure profile model and in order to reduce the modelling and computational efforts, the simplified triangle model for blast loads profile is used to analyze structural response. This simplified model considers only the positive phase and ignores the suction phase which characterizes the typical one in simulating blast loads. The closed from solution for the equation of motion under blast load as a forcing term modelled either typical or simplified models has been derived. The considered herein two approaches have been compared using the obtained results from simulation response analysis of a building structure under an applied blast load. The computed error in simulating response using the simplified model with respect to the typical one has been computed. In general, both simplified and typical models can perform the dynamic blast-load induced response of building structures. However, the simplified one shows a remarkably different response behavior as compared to the typical one despite its simplicity and the use of only positive phase for simulating the explosive loads. The prediction of the dynamic system responses using the simplified model is not satisfactory due to the obtained larger errors as compared to the system responses obtained using the typical one.

1. Introduction

Accidental explosions and deliberate bombings by terrorists cause catastrophic structural damage which results in downtime such structures and create a negative impact on society. This phenomenon may lead to severe damage or even damage to numerous surrounding structures. Moreover, loss of life can result from the collapse of structures [1-4]. Several research works have investigated the different models of air blast and the effect of air blast loads on reinforced concrete structures. The evolution of blast pressure with time can be simulated either by exponential distribution, which depends on explosive charge size, type, and distance to a target, or by triangular distribution through neglecting the suction stage which leads to the simplified model [5-7].

The methodologies available nowadays for prediction of blast effects on building structures can be divided into experimental, analytical and numerical procedures. Although the experimental tests are unsafe, expensive and dangerous, they can provide a wealth of useful data.

The theoretical approach usually utilizes idealized models and building structures modelled as single-degree-of freedom systems (SDOF). Although this procedure is based on several assumptions and approximations, it is considered as the most applicable one and rather applicable in routine design where it provides researchers and structural designers with useful data and results [8]. This procedure substitutes the structural element by an equivalent stiffness, SDOF structural system and an elastic-plastic response spectra to capture the peak responses of modelled system. With the development of computer technology, a lot of researchers paid attention for conducting research works concerning the blast induced structural response numerically. The equations of motion are widely used to describe complex models in various field of engineering, particularly in structural dynamic. The wide applications of these equations are the main reason behind attracting mathematicians for performing blast analysis in the last decades. In order to understand the physical mechanism of phenomena in nature described by second order non homogeneous ordinary differential equation, exact solutions have to be explored. The study of these equations becomes one of the most important topics in mathematical physics.
This research aims to drive the exact solution of the dynamic response of buildings under blast loads for two types of pressure configurations, more accurate exponential distribution of blast pressure and simplified triangular blast pressure.

2. Modelling and idealization

The structure under study is one which can be represented by idealized SDOF mathematical model for predicting dynamic response of concrete structures subjected to blast loading as shown in figure 1. A rigid deck is connected to a base through massless columns of stiffness *k*, representing the resistance of structure against deformation, and damping coefficient *c*. The values of structural stiffness and damping coefficients can be calculated from the formulas [9].

$$k = \frac{4\pi^2 m}{T^2}; \quad c = 2\xi\sqrt{km} \tag{1}$$

where *T*, and ζ denote the natural structural vibration period and structural damping, respectively.

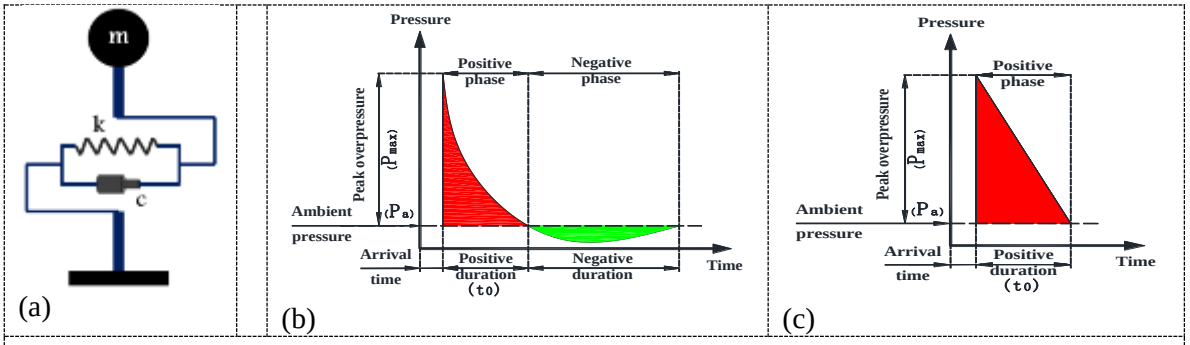

Figure 1. (a) Idealized SDOF building model (b) typical blast load profile (c) simplified blast load profile.

The equation of motion of the idealized building model presented in figure 1(a) and subjected to a blast pressure *P(t)* can take the form:

$$m\ddot{u} + c\dot{u} + ku = A_{eq}\, p(t) \tag{2}$$

where u, $\dot{u}, and$ $\ddot{u}$ represent the displacement, velocity, and acceleration of the superstructure respectively, while, A_{eq} is the exposed surface area of the building model. The blast pressure distribution $p(t)$ as function of time *t*, the positive phase duration t_0, and the peak overpressure P_{max} can take either the typical form (figure 1(b)):

$$p(t) = p_{\max}(1-\frac{t}{t_0})\exp\left(\frac{-bt}{t_0}\right)A_{eq.} \tag{3}$$

Or the simplified form (figure 1(c)):

$$p(t) = p_{\max}(1-\frac{t}{t_0}) \tag{4}$$

where *b* is a shape parameter depending on the dimensionless characteristic scaled distance *Z* [5] as:

$$Z = \frac{R}{W^{1/3}} \tag{5}$$

where *W* is an explosive charge, expressed in Kilograms of TNT, and *R* is the standoff distance, or distance between a point of interest and the blast epicenter.

3. Closed form solution

Case 1: equation of motion with forcing term modeled as typical model

Referring to equations (2) and (3), the equation of motion of the building model under typical blast pressure is

$$m\ddot{u} + c\dot{u} + ku = A_{eq}\,p_{\max}(1-\frac{t}{t_0})\exp\left(\frac{-bt}{t_0}\right) \tag{6}$$

With initial conditions:

$$u(0) = 0,\ \ \dot{u}(0) = 0 \tag{7}$$

Equation (6) together with initial conditions in equation (7) can be solved analytically in terms of the particular and homogenous solutions to determine the time-dependent displacement, velocity, and acceleration of the system following any of the available textbooks [10] as:

$$u_p = (A + Bt)\exp\left(\frac{-bt}{t_0}\right) \tag{8}$$

where u_p is the particular solution. *A* and *B* are constants to be determined applying the method of determined coefficients as:

$$B = \frac{-p_{\max}A_{eq}}{t_0(m\frac{b^2}{t_0^2} - c\frac{b}{t_0} + k)} \tag{9}$$

$$A = -Bt_0 + \frac{B^2}{p_{\max}A_{eq}}(c - 2m\frac{b}{t_0})t_0 \tag{10}$$

The homogenous solution u_h can be written as:

$$u_h = \exp(\alpha t)\left[c_1\cos\beta t + c_2\sin\beta t\right] \tag{11}$$

where c_1, c_2, α and β are another constants to be determined employing the characteristic equation and the initial conditions.

Applying the characteristic equation α and β have been found in terms of mass, stiffness and damping coefficient of the building model as:

$$\alpha = \frac{-c}{2m}, \quad \beta = \frac{(c^2 - 4mk)^{1/2}}{2m} \tag{12}$$

Knowing that the general solution *u* is the summation of both homogeneous and particular solution:

$$u = u_h + u_p \tag{13}$$

$$u = \exp(\alpha t)\left[c_1 \cos \beta t + c_2 \sin \beta t\right] + (A + Bt)\exp\left(\frac{-bt}{t_0}\right) \tag{14}$$

Derivative of *u* with respect to time *t* can be derived as:

$$\begin{aligned} \dot{u} = {} & \alpha \exp(\alpha t)\left[c_1 \cos \beta t + c_2 \sin \beta t\right] + \exp(\alpha t)\left[-c_1 \beta \sin \beta t + c_2 \beta \cos \beta t\right] \\ & + B \exp\left(\frac{-bt}{t_0}\right) - \frac{b}{t_0}\exp\left(\frac{-bt}{t_0}\right)(A + Bt) \end{aligned} \tag{15}$$

Similarly, derivative of $\dot{u}$ with respect to time t can be derived as:

$$\begin{aligned} \ddot{u} = {} & \alpha^2 \exp(\alpha t)\left[c_1 \cos \beta t + c_2 \sin \beta t\right] + 2\alpha \exp(\alpha t)\left[-c_1 \beta \sin \beta t + c_2 \beta \cos \beta t\right] \\ & - \exp(\alpha t)\left[c_1 \beta^2 \cos \beta t + c_2 \beta^2 \sin \beta t\right] - 2B \frac{b}{t_0}\exp\left(\frac{-bt}{t_0}\right) + \frac{b^2}{t_0^2}\exp\left(\frac{-bt}{t_0}\right)(A + Bt) \end{aligned} \tag{16}$$

where *A*, *B*, α and β determined as before.
In order to find the constants c_1 and c_2 one needs to apply initial conditions to equations (14) and (15).

$$c_1 = -A \tag{17}$$

$$c_2 = \frac{A(\alpha + \frac{b}{t_0}) - B}{\beta} \tag{18}$$

Case 2: equation of motion with forcing term modeled as simplified model
Referring to equations (2) and (4), the equation of motion of the building model under simplified blast pressure is

$$m\ddot{u} + c\dot{u} + ku = p(t) \tag{19}$$

where

$$P(t) = \begin{cases} P_{\max}(1 - \frac{t}{t_0})A_{eq} & t \le t_0 \\ 0 & t \ge t_0 \end{cases} \tag{20}$$

With the same initial conditions mentioned in equation (7).

For the case $t \le t_0$:
The particular solution can take linear form as:

$$u_p = A + Bt \tag{21}$$

Obtaining the derivatives of equation (16) and substituting into equations (14) and comparing coefficients:

$$B = -\frac{P_{\max}}{kt_0}A_{eq} \tag{22}$$

$$A = -\frac{P_{\max}A_{eq}(kt_0 + c)}{k^2 t_0} \tag{23}$$

Substituting the values of *A* and *B* into equations (16)

$$u_p = -\frac{P_{\max}A_{eq}}{kt_0}(t_0 + \frac{c}{k} - t) \tag{24}$$

Similarly to the typical model case the homogeneous solution can be found as|:

$$u_h = \exp(\alpha t)\left[c_1 \cos\beta t + c_2 \sin\beta t\right] \tag{25}$$

The values of α and β can be calculated as in equation (12).
In order to find the constants c_1 and c_2 one needs to apply initial conditions to the general solution equation.
Knowing that the general solution *u* is the summation of both homogeneous and particular solution:

$$u = u_h + u_p \tag{26}$$

$$u_h = \exp(\alpha t)\left[c_1 \cos\beta t + c_2 \sin\beta t\right] + \frac{P_{\max}A_{eq}}{kt_0}(t_0 + \frac{c}{k} - t) \tag{27}$$

Derivative of *u* with respect to time *t* can be derived as:

$$\dot{u} = \exp(\alpha t)\left[-c_1\beta \sin\beta t + c_2\beta \cos\beta t\right] + \alpha \exp(\alpha t)\left[c_1 \cos\beta t + c_2 \sin\beta t\right] - \frac{P_{\max}A_{eq}}{kt_0} \tag{28}$$

Similarly, derivative of $\dot{u}$ with respect to time *t* can be derived as:

$$\begin{aligned}\ddot{u} = {} & \alpha \exp(\alpha t)\left[-c_1\beta \sin\beta t + c_2\beta \cos\beta t\right] + \exp(\alpha t)\left[-c_1\beta^2 \cos\beta t - c_2\beta^2 \sin\beta t\right] \\ & + \alpha^2 \exp(\alpha t)\left[c_1 \cos\beta t + c_2 \sin\beta t\right] + \alpha \exp(\alpha t)\left[-c_1\beta \sin\beta t + c_2\beta \cos\beta t\right]\end{aligned} \tag{29}$$

Now we apply initial conditions to the general solution equation (27) and equation (28).

$$c_1 = -\frac{P_{\max}A_{eq}}{kt_0}(t_0 + \frac{c}{k}) \tag{30}$$

$$c_2 = \frac{P_{max}A_{eq}}{\beta kt_0}[1 + \alpha(t_0 + \frac{c}{k})] \tag{31}$$

For the case $t \geq t_0$:

The forcing term *P(t)=0*, the general solution $u_{t \geq t_0}$ can be expressed as:

$$u_{t \geq t_0} = \exp(\alpha t)\left[G_1 \cos\beta t + G_2 \sin\beta t\right] \tag{32}$$

Derivative of $u_{t \geq t_0}$ with respect to time *t*

$$\dot{u}_{t \geq t_0} = \exp(\alpha t)\left[-G_1\beta \sin\beta t + G_2\beta \cos\beta t\right] + \exp(\alpha t)\left[G_1 \cos\beta t + G_2 \sin\beta t\right] \tag{33}$$

Derivative of $\dot{u}_{t\geq t_0}$ with respect to time t

$$\begin{aligned}\ddot{u}_{t\geq t_0} &= \exp(\alpha t)\left[-G_1\beta^2 c\cos\beta t - G_2\beta^2\sin\beta t\right] + \alpha\exp(\alpha t)\left[-G_1\beta\sin\beta t + G_2\beta\cos\beta t\right] \\ &+ \alpha^2\exp(\alpha t)\left[G_1\cos\beta t + G_2\sin\beta t\right] + \alpha\exp(\alpha t)\left[-G_1\beta\sin\beta t + G_2\beta\cos\beta t\right]\end{aligned} \tag{34}$$

Appling the initial conditions (as illustrated in figure (2)):

$$u_{t\geq t_0} = u_{t=t_0} \text{ and } \dot{u}_{t\geq t_0} = \dot{u}_{t=t_0} \tag{35}$$

Leads to:

$$G_1 = \frac{R - G_2\sin(\beta t_0)}{\cos(\beta t_0)} \tag{36}$$

$$G_2 = \frac{E - R\left(\alpha - \beta\tan(\beta t_0)\right)}{\beta\cos(\beta t_0)\left(1+\tan^2(\beta t_0)\right)} \tag{37}$$

where $$R = u(t_0)/\exp(\alpha t_0) \tag{38}$$

and $$E = \dot{u}(t_0)/\exp(\alpha t_0) \tag{39}$$

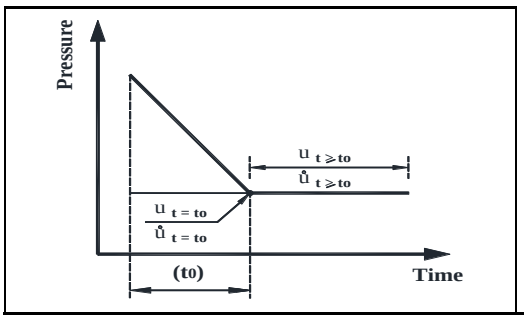

Figure 2. Initial conditions for the case $t \geq t_0$.

Figure 2 presents the simplified triangle distribution for the blast load as well as the initial condition required to get the exact solution of equation (19).

4. Results and discussion

To demonstrate the validity of the simplified blast load model, the closed form solution of the equation of motion for the building model shown in figure 1(a) using the two models of blast loads presented in figure 1(b) and (c) are provided using an explosive load of peak pressure P_{max} of 312 kpa and positive phase duration t_0 = 0.02142 second. The considered system parameters in terms of mass m, damping ratio ζ and natural period T_n, are set to be 25 $\times 10^3$ kg, 0.05 and 1.2 s respectively.

Here, for brevity, a comparison between typical and simplified blast pressure profile models is performed.

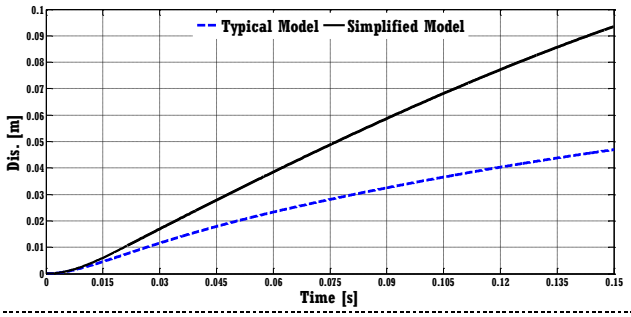

Figure 3. Storey displacement time histories with two models of blast load.

It can be seen from figure 3 that the displacement of the building model reaches its peak at the end of explosion time. Moreover, it has been noticed that, the error in predicted displacement response increases with increasing the blast duration.

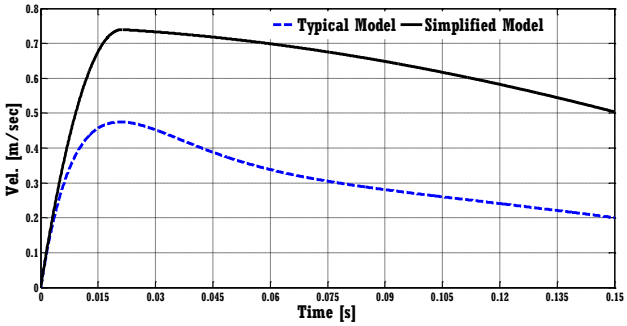

Figure 4. Storey velocity time histories with two models of blast load.

Figure 4 presents the captured velocity time-histories of the considered building model using the typical and simplified blast models. It is worth noting that the velocity of the structure increases rapidly in the initial time *to* reach its peak value almost at the end of the positive phase duration $t = t_0$, and the corresponding error reaches maximum at the end of positive phase as well.

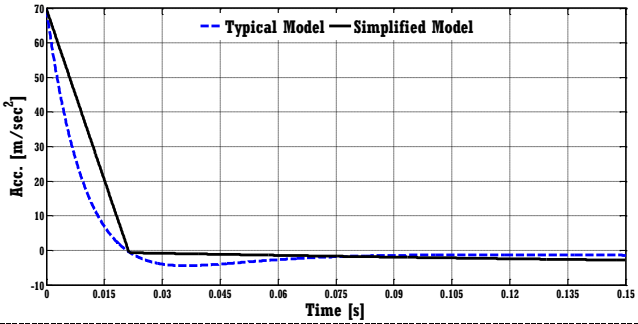

Figure 5. Storey acceleration time histories with two models of blast load.

Focusing on the acceleration curves shown figure 5, it can be seen that the accelerations start with certain maximum values immediately at the occurrence of the blast detonation followed by a sudden decrease to zero for the remaining time duration. Therefore, the acceleration response appears to be

only significant at the start of the explosion causing sudden increase in the storey accelerations. Moreover, it has been noticed that, the obtained peak acceleration response with simplified model is nearly identical to the one obtained with typical model.

The captured peak displacement and velocity for the considered herein building model subjected to the aforementioned blast load in the form of typical model are 0.0469 m, and 0.4748 m/s respectively. Considering the blast load in the form of the simplified model one increases these captured values to be 0.0935 m, and 0.7396 m/s respectively. Consequently, the simplified model overestimates the peak responses by 100% and 60% respectively. But, the peak acceleration remains constant and equal 69.7986 m/sec^2 in both of the two models.

As it can be seen from the presented figures and the obtained results, the proposed simplified model excessively increases the obtained response for the building model during the whole period of explosion and does not yield good predictions except for the peak acceleration response.

5. Conclusions

This paper presents a comprehensive investigation on the induced structural response due to applied blast loads. Both typical and simplified models have been used to represent the explosive load acting on the building model. It has been demonstrated that the simplified model for blast load yields unreasonable predictions of structural responses as compared to the typical model where a significant difference between the induced structural responses of the buildings has been found.

References

[1] Osteraas JD 2006 Murrah building bombing revisited: A qualitative assessment of blast damage and collapse patterns *J. Perform. Constr. Facil.* **20** pp 330–335.
[2] Elliot CL, Mays GC and Smith PD 1992 Proceedings of Institution of Civil Engineers, Structures & Buildings, **94** pp 287–297
[3] Corley W G 2004 J. Perform. Constr. Fac., **18** pp 68-78
[4] Remennikov A M 2003 J. of Battlefield Tech. Aragon Press Pty Ltd., **6** pp 5-10
[5] Baker W.D., Cox, P.A., Westine, P.S., Kulesz, J.J. and Strehlow, R.A., 1983 Explosion Hazards and Evaluation, Elsevier, Amsterdam, the Netherlands
[6] Kinney GF, Graham KJ 1985 Explosive Shocks in Air, Springer-Verlag, New York
[7] Bangash MYH 1993 Impact and Explosion, Analysis and Design, Blackwell Scientific
[8] Mays GC and Smith PD 1995 Blast Effects on Buildings, Thomas Telford Publications, Heron Quay, London
[9] Harris CM and Piersol AG 2002 in: *Harris' Shock and Vibration Handbook*, edited by Harris CM, and Piersol AG, McGraw-Hill publishing, New York
[10] *Zill* DG 2001 *Differential Equations with Modeling Applications*, 7th ed., Brooks/Cole Thompson Learning, Pacific Grove, CA