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Maximum power for a power plant with two Carnot-like cycles

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Abstract. A stationary power plant with two Carnot-like cycles is optimized. Each cycle has the following irreversibilities: finite rate heat transfer between the working fluid and the external heat sources, internal dissipation of the working fluid, and heat leak between reservoirs. The optimal allocation or effectiveness of the heat exchangers for this power plant is determined by applying, two alternating design rules: fixed internal thermal conductance or fixed areas. The optimal relations obtained are substituted in the power and the maximum power, according to the isentropic ratio of each one of the Carnot-like cycles of the power plant, is calculated. Additionally, the efficiency to maximum power is presented.

1. Introduction

Recently in [1], a methodology of optimization was applied to an irreversible Carnot-like stationary power plant, where the characteristic parameter was the allocation or effectiveness of the heat exchangers of this plant. This standard irreversible Carnot-like cycle has been studied at length for many objective functions, different transfer heat laws and several characteristic parameters (see [2]-[7] for further details).

[8-11] have presented a *n*-stage combined Carnot cycle optimizing the specific power and efficiency for the isentropic temperature ratios x_i ; $j = 1, \dots, n$, and effectiveness. In this last case, the law of heat conduction was used.

In this paper, we extend the results of allocation or effectiveness from one cycle to a combined cycle with two Carnot-like cycles using the Bellman's Principle [9], which has been successfully applied in [8 and 9]. We found optimal relations for two constraints (i.e. design rules): constrained internal thermal conductance or fixed total area of the heat exchangers from the hot and cold sides. Finally, the maximum power and the efficiency of maximum power for the stationary power plant with two Carnot-like cycles are calculated.

2. Power plant with two Carnot-like cycles

The stationary power plant with two Carnot-like cycles is shown in Figure 1. Each cycle satisfies the conditions expressed in [2, 4]: leak heat \hat{Q} and finite heat transfer rates \hat{Q}_i , and internal dissipations of the working fluid expressed by constants I_i (i = 1, 2), such that $I_i = \frac{\Delta S_{i+1}}{\Delta S_i} \ge 1$; i = 1, 2 [4] in which the Claussius inequality becomes equality (see [3]). Each cycle of the power plant consists of two isothermal and two adiabatic processes, denoting, for each cycle the temperatures of the working fluid during the hot and cold isothermal processes as T_i and T_{i+1} (i=1,2,3), respectively, and the end temperatures as T_H and T_L .

Following [1 and 5], the thermal efficiency of the power plant is given by:

$$\eta = \frac{P}{f(x_1, x_2)P + Q}$$

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(1)

where $x_i = \frac{T_{2i}}{T_{2i-1}}$ correspond to the isentropic temperature ratio for each cycle i (i = 1, 2), P is the power of the plant, Q is the leak heat and the function $f(x_1, x_2) = \frac{1}{1 - I_1 I_2 x_1 x_2}$. Note that this functional form for one cycle has only appeared in [1, 2, and 5]



Figure 1: A power plant stationary of two Carnot-like cycles with linear leak heat and finite heat transfer rates, and internal dissipations of the working fluid

According to the second law of Thermodynamics, for each i = 1, 2, we have [26]: $Q_2 = I_1 x_1 Q_1; Q_3 = I_2 x_2 Q_2;$ thus, $Q_3 = I_2 x_2 Q_2 = I_1 x_1 Q_1$ so that, the efficiency is given by:

$$\eta = \frac{P}{\frac{Q}{R}} = \frac{Q}{\frac{Q}{H} - Q}{\frac{Q}{L}} = \frac{Q}{\frac{Q}{1} - Q}{\frac{Q}{3}} = \left(1 - \frac{Q}{\frac{Q}{3}}\right) \frac{Q}{\frac{Q}{1}} = \frac{(1 - I_1 I_2 x_1 x_2)Q}{\frac{Q}{1}}$$
(2)

From equation (5),

$$Q_{1} = \frac{P}{1 - I_{1}I_{2}x_{1}x_{2}}$$
(3)

and equation (3), (1) is obtained.

Now, let $z \neq x_i$ (i = 1, 2), the Corollary given at [1], extend to:

"The power P achieves a maximum value in z_{mp} if and only if the efficiency η achieves a maximum value in the value: $z_{mp} = z_{me}$ "

The proof is similar to that presented in [1]. It is easy to see that in the computations of the proof for the Corollary, no transfer heat law has been used. Thus, in the optimization of the power plant with

two Carnot-like cycles, with respect to z, it is enough to find the maximum power by,

$$\frac{\partial P}{\partial z}\Big|_{z_{mp}=z_{me}} = 0 \text{ and } \frac{\partial^2 P}{\partial z^2}\Big|_{z_{mp}=z_{me}} < 0 \tag{4}$$

The optimization performed, with respect to z, is a property independent from the heat transfer law. As a result, $z = \phi$ or ψ , depending on which design rule is applied.

3. Optimal relations for the allocation and effectiveness of the power plant's heat exchangers

In this section, x_1 , x_2 will be fixed and we will assume that the law of heat transfer can be any law, including heat leak. Next, we will discuss the following two design rules: *fixed internal thermal conductance* or *fixed areas for the heat exchangers*, which will be alternately applied.

The first design rule is that the internal conductance of the Carnot-like cycle is constrained to: $\sum_{i=1}^{3} \alpha_i = \Gamma$, where Γ is a constant applied to the allocation of the heat exchangers from the hot and cold sides; with the same overall heat transfer coefficient U by unit of area A in both ends of the cycle i(i = 1, 2) and $\alpha_i(i = 1, 2, 3)$ are the thermal conductance correspondent to the finite heat transfers of the hot/cold sides for this cycle, respectively. Thus, $\sum_{i=1}^{3} UA_i = \Gamma$ where A_i , A_{i+1} are heat transfer areas on hot/cold sides of the cycle i(i = 1, 2). The second design rule is that the total area is constrained by: $\sum_{i=1}^{3} A_i = A$; where $A_i(i = 1, 2, 3)$ are the heat transfer areas on the hot and cold sides for the cycle i(i = 1, 2), respectively. Now, the total area A is fixed but, when distributed, it has different overall heat transfer coefficients and hence different effectiveness on each one of the hot and cold sides. How $\alpha_i = U_i A_i$ (see [1, 12]), then $A = \sum_{i=1}^{3} A_i = \sum_{i=1}^{3} \frac{\alpha_i}{U_i}$, where $U_i(i = 1, 2, 3)$ are

the overall heat transfer coefficients on the hot and cold sides of the cycle i (i = 1, 2), respectively.

From the Corollary of the section 2, it is enough to find "the maximum power of each Carnot-like cycle i (i = 1, 2) for these two design rules applied alternatel". Indeed, from [3] the dimensionless power output for the cycle, i is given by:

$$p_{i} = \frac{P_{i}}{\alpha_{i}T_{2i-1}} = \frac{1 - \sqrt{I_{i}\mu_{i}}}{\frac{1}{\alpha_{i}} + \frac{I_{i}}{\alpha_{i+1}}} , \qquad (5)$$

where μ_i is the end temperature ratio of the cycle i = 1, 2. The numerator of (5) is obtained from applying the conduction heat law and optimizing it with respect to x_i ; i = 1, 2.

Next, if we apply the Bellman' Principle [9]: "to state that every part of an optimum path is optimal", we determine the optimal allocation or effectiveness of the heat exchangers.

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3.1 Constrained internal thermal conductance

The three thermal conductances can be written as $\alpha_i = UA_i$; i = 1, 2, 3, where U is overall heat transfer coefficient and A_i ; i = 1, 2, 3 are the available areas for heat transfer. Thus, for the first optimization we can assume for this first design rule: $\alpha_1 + \alpha_2 = \Gamma_1$, where $\Gamma_1 = \Gamma - \alpha_3$ is supposed to be a constant. Equivalently, $\frac{\alpha_1 + \alpha_2}{\Gamma_1} = 1$. Fixing the temperature T_i (j = 3, 4) and applying only for the first cycle, $\frac{a_1\alpha_1 + \alpha_2}{\Gamma_1} = 1$; $a_1 = 1$. In parameterizing, $\phi = \frac{\alpha_1}{\Gamma_1}$; $1 - a_1\phi = \frac{\alpha_2}{\Gamma_1}$. According to the equation (6), in optimizing $\left(\frac{1}{\frac{a_1}{\phi} + \frac{I_1}{1 - a_1\phi}}\right)$ according to ϕ , we obtain $\phi_1 = \frac{1}{1 + \sqrt{I_1}}$. Solving, $\frac{1 - a_1\phi_1}{\phi_1} = \sqrt{I_1} = \frac{\alpha_2}{\alpha_1}$; so $\alpha_2 = \sqrt{I_1\alpha_1}$. For the second optimization, the constraint is now $\frac{a_2\alpha_1 + \alpha_3}{\Gamma_2} = 1$, with $a_2 = 1 + \sqrt{I_1}$. In parameterizing, $\phi = \frac{\alpha_1}{\Gamma_2}$; $1 - a_2\phi = \frac{\alpha_3}{\Gamma_2}$. Similarly, in optimizing $\left(\frac{1}{\frac{a_2}{\phi} + \frac{I_1}{1 - a_2\phi}}\right)$ according to ϕ , we obtain $\phi_2 = \frac{1}{\alpha_2 + \sqrt{I_2}}$. Solving, $\frac{1 - a_2\phi_2}{\phi_2} = \sqrt{I_2} = \frac{\alpha_3}{\alpha_1}$; so that $\alpha_3 = \sqrt{I_2\alpha_1}$.

In summary, the optimal conductance is: \sqrt{r}

$$\alpha_{2} = \sqrt{I_{1}}\alpha_{1}; \alpha_{3} = \sqrt{I_{2}}\alpha_{1}$$
and
$$a_{3} = \sum_{j=0}^{2} \sqrt{I_{j}}, I_{0} = 1.$$
(6)

3.2 Constrained areas of heat exchangers

For simplicity, we suppose $I_2 = I_1 = 1$. Applying the second design rule $A_1 + A_2 = A_1$ where $A_1 = A - A_3$, $A_1 = \frac{\alpha_1}{U_1}$; $A_2 = \frac{\alpha_2}{U_2}$, then $A = a_1\alpha_1 + u_1\alpha_2$, where $u_1 = \frac{u_1}{U_2}$; $a_1 = 1$. In parameterizing, $\psi = \frac{\alpha_1}{A}$; $1 - a_1\psi = \frac{\alpha_2}{A}$. From equation (5) $\frac{1}{\frac{a_1}{\psi} + \frac{1}{(1-a_1\psi)u_1}}$, the first optimization with respect to ψ , gives $\psi_1 = \frac{\sqrt{u_1}}{1 + a_1\sqrt{u_1}}$. Then, $\frac{1 - a_1\psi_1}{\psi_1} = \frac{1 + \sqrt{u_1}(a_1 - 1)}{\sqrt{u_1}}$. Solving, $\frac{\alpha_1}{\sqrt{a_1u_1}} = \alpha_2$; so that $A_2 = \frac{A_1}{\sqrt{a_1u_1}}$, and the results of [1] are recovered. For the second optimization, the constraint is now $a_2A_1 + \frac{A_3}{u_2} = A$, where $a_2 = \frac{1 + \sqrt{a_1u_1}}{\sqrt{a_1u_1}}$; $u_2 = \frac{u_1}{U_3}$. In parameterizing, $\psi = \frac{A_1}{A}$; $1 - a_2\psi = \frac{A_3}{A}$. Now, from (5) $\frac{1}{\frac{a_2}{\psi_1} + \frac{1}{(1-\alpha_1)}}$. The second optimization with respect to ψ , gives, $\psi_2 = \frac{\sqrt{u_2}}{1 + a_2\sqrt{u_2}}$ and

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$$\frac{1-a_2\psi_2}{\psi_2} = \frac{1+\sqrt{u_2}(a_2-1)}{1+a_2\sqrt{u_2}}$$
. Thus, $A_3 = \frac{1+\sqrt{u_2}(a_2-1)}{\sqrt{u_2}}A_1$. So that, the optimal areas, are:

$$A_{2} = \frac{A_{1}}{\sqrt{a_{1}u_{1}}}; A_{3} = \frac{1 + \sqrt{u_{2}}(a_{2} - 1)}{\sqrt{u_{2}}} A_{1}$$
and
$$a_{3} = 1 + \frac{\sqrt{I_{2}} + \sqrt{u_{2}}(a_{2} - 1)}{\sqrt{I_{2}} + a_{2}\sqrt{u_{2}}}.$$
(7)

4. Maximum power and efficiency to maximum power

Let P_1 , P_2 be the power of cycles 1 and 2, respectively, of Figure 1. For the first design rule, the maximum power of the power plant will be given by:

$$(P_1 + P_2)_{\max} = \frac{\sqrt{I_2} U A \left(1 - \sqrt{I \mu}\right)^2}{a_3 \left(\sqrt{I_2} + I_1 a_2\right)} \qquad ; \qquad (8)$$

and for the second design rule, the maximum power will be given by:

$$(P_1 + P_2)_{\max} = \frac{T_H a_3 a_2 (a_2 - 1) A U_1 (1 - \sqrt{I \mu})^2}{(I_1 u_1 a_2 (a_2 - 1) + a_3) (a_2 (a_2 - 1) + a_3)} , \qquad (9)$$

where $I = I_1 I_2$; $\mu = \frac{T_L}{T_H}$ using the notation of the subsections 3.1 and 3.2, respectively.

We can show (8) as follow:

$$\frac{P_1 + P_2}{\alpha_1 T_H} = \left(1 - \sqrt{I_1 \mu_1}\right)^2 + \frac{\alpha_3}{\alpha_1 a_2} \frac{T_2}{T_H} \left(1 - \sqrt{I_2 \mu_2}\right)^2$$
$$= \left(1 - \sqrt{I_1 \mu_1}\right)^2 + \frac{\sqrt{I_2}}{I_1 a_2} I_1 \frac{T_2}{T_H} \left(1 - \sqrt{I_2 \mu_2}\right)^2$$
$$= \left(1 - \sqrt{I_1 \mu_1}\right)^2 + \frac{\alpha_3}{I_1 a_2 \alpha_1} \left(\sqrt{I_1 \mu_1} - \sqrt{I_1 I_2 \mu_2}\right)^2$$

where we apply equation (6). In optimizing according to $\sqrt{I_1\mu_1}$:

$$(P_1 + P_2)_{\max} = \alpha_3 \alpha_1 T_H \frac{\left(1 - \sqrt{I_1 I_2 \mu}\right)^2}{I_1 a_2 \alpha_1 + \alpha_3}$$

= $T_H \frac{\left(1 - \sqrt{I_1 I_2 \mu}\right)^2}{\frac{1}{\alpha_1} + \frac{I_1 a_2}{\alpha_3}} = \frac{I_2 U A \left(1 - \sqrt{I_1 I_2 \mu_1 \mu_2}\right)^2}{a_3 \left(I_1 a_2 + \sqrt{I_2}\right)}$

because of $UA = \alpha_1 + \alpha_2 + \alpha_3 = \alpha_1 + \sqrt{I_1}\alpha_1 + \sqrt{I_2}\alpha_1 = \alpha_1(1 + \sqrt{I_1} + \sqrt{I_2})$, so, $\alpha_1 = \frac{UA}{a_3}$.

Now from [10], we can obtain the efficiency to maximum power:

$$\eta_{mp} = \frac{\left(1 - \sqrt{I\mu}\right)}{\frac{1}{1 + \frac{Q_L}{\Lambda T_H}}},$$
(10)

where $\Lambda = \sum_{i=1}^{3} \frac{1}{\alpha_i}$. Equation (10) is obtained similarly.

5. Conclusions

We have found and determined the optimal allocation and effectiveness of heat exchangers of a combined cycle with two Carnot-like cycles. Moreover, these relations can be satisfied for other operation regimes, e.g. algebraic combination of power and/or efficiency that have thermodynamic meaning and satisfy imposed power conditions (equation (5)). Nevertheless, the optimal isentropic temperatures ratios depend on the heat transfer law and the operation regime of the engine as discussed in [1]. Additionally, the maximum power and its corresponding efficiency to maximum were calculated. The equations (6-10) can be extended to a power plant with *n*-Carnot-like cycles of the model presented here. However, the latter requires a comprehensive study of the implications for the power plant considered. We will study such implications in our future work.

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