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## Eye tracker uncertainty analysis and modelling in real time

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## Eye tracker uncertainty analysis and modelling in real time

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**Abstract.** Techniques for tracking the eyes took place since several decades for different applications that range from military, to education, entertainment and clinics. The existing systems are in general of two categories: precise but intrusive or comfortable but less accurate. The idea of this work is to calibrate an eye tracker of the second category. In particular we have estimated the uncertainty both in nominal and in case of variable operating conditions. We took into consideration different influencing factors such as: head movement and rotation, eyes detected, target position on the screen, illumination and objects in front of the eyes. Results proved that the 2D uncertainty can be modelled as a circular confidence interval as far as there is no stable principal directions in both the systematic and the repeatability effects. This confidence region was also modelled as a function of the current working conditions. In this way we can obtain a value of the uncertainty that is a function of the operating condition estimated in real time opening the field to new applications that reconfigure the human machine interface as a function of the operating conditions. Examples can range from option buttons reshape, local zoom dynamically adjusted, speed optimization to regulate interface responsiveness, the possibility to take into account the uncertainty associated to a particular interaction. Furthermore, in the analysis of visual scanning patterns, the resulting Point of Regard maps would be associated with proper confidence levels thus allowing to draw accurate conclusions. We conducted an experimental campaign to estimate and validate the overall modelling procedure obtaining valid results in 86% of the cases.

## 1. State of the art

The eye tracker is an instrument that allows to determine the point where the user is looking at. This kind of technology has two main fields of application: analysis of visual scanning patterns and human machine interfaces (HMI).

Since it is commonly agreed that the point or region a person is looking at is the point where that person is focusing his attention, except in some cases with very simple task as stated by Rayner and Keith in [1], knowing the visual scanning pattern it's a great way to study human cognitive processes. Applications may be several starting from market to web searches. Moreover the study of visual scanning path may be used in medicine to diagnose mood, perception, learning and attention disorders. Also in the field of human machine interface the use of the eye as input have multiple advantages. The first and more evident reason to use the eye as input, is to allow users that are not able to use other input, such as joystick and keyboard, due to some diseases that partially or completely limit their movement. Anyway the eye tracker may be useful to any kind of user. Indeed eye movement is one of the fastest human movement, although are mainly conceived for exploration, less for control. A good insight can be found in [2].

There are different technologies implemented in order to track the eye [3]. Through all this technologies it is possible at first to distinguish between intrusive and non intrusive one. In the first category falls all that kind of devices that are in some way fixed with the user's head. Some examples are: contact lenses, electrodes and head mounted devices. This kind of technology have two main advantages: accuracy and robustness with respect to head movement. The main drawback of intrusive devices is that may be tedious for the user. The alternative are the non intrusive, or remote, eye tracker [4]. This kind of devices are mostly based on computer vision techniques. They consist of one or more cameras to capture the eye's image. Those kind of system are more comfortable and can be used for long periods. This make them suitable to be used as HMI. Due to the scope of this work we are more interested in remote eye trackers.

The most popular kind of remote eye trackers use features detection to track the eye. Eye's characteristic that can be tracked are, e.g., the limbus and the pupil. The problem of the limbus is that it's covered by the eyelids. On the other hand the problem of the pupil is the weaker contrast with the iris, that makes it harder to detect it in the image. This will obviously depend also on the specific user characteristics, such as shape and size of the eye and also the color. Anyway a solution commonly adopted is the usage of infrared light. Indeed IR light enhance the above mentioned contrast. Since IR is not visible it won't distract the user.

The need of a common way to estimate eye trackers' performances has been discussed by Holmqvist and Nyström in [5]. In particular they determine three principle characteristic in order to determine the performances of the eye tracking devices:

- Accuracy: is the bias (**systematic effect**), i.e. the difference between the estimated Point of Regard PoR (mean of the collected data) and the point the user is actually looking at.
- Precision: that is the **repeatability** of the estimate. It is usually estimated as the RMS or as the standard deviation of the available data.
- Robustness: capability to work with different users. Usually eye trackers do not work with everybody or anyway do not have same performances changing users. This may be due to different characteristics of the user, both cognitive (e.g. the ability to follow a task) and physics (e.g. the physiology of the eye).

A common problem underlined from many authors is that the uncertainty provided by the manufacturers seems not to correspond to real cases. This is mainly due to the fact that the values obtained by the manufacturers are obtained in ideal conditions. Knowing the real accuracy and precision allow better design of HMIs [9].

In order to calibrate a commercial eye tracker we identified the following parameters of influence:

- Illumination;
- Relative motion between the user head and the instrument (rotation and translation);
- Objects in front of the eyes;
- Different user.

Many authors have tried to identify the influencing parameters in order to find a way to decrease their effect [9, 10, 11, 12, 13, 14]. It is commonly considered that two of the main problem of the remote eye tracking systems are illumination change and head movement.

Illumination: the influence of light largely depend on the usage of the eye tracker. For example if it is used as HMI on a computer it is possible to place that computer in a spot with optimal illumination. For example far from windows (illumination change during the day). Hansen and Pece in [9] propose a system that use image statistic instead of features detection. As described by the authors “the underlying idea is that a large image gradient is likely to arise from a boundary between object and background”. Indeed they obtain a system that is robust with respect to light changes. They use an algorithm based on particle filters and EM Contour, the main drawback is that head movement have to be very limited.

Relative motion between the user and the instrument (rotation and translation): the problem of head movements will largely depend on the user characteristics: several disabled user may not be able to move the head or may not be able to keep it fixed depending on the diseases. Also a healthy user will slightly move the head in most activities. Usually to test the eye tracker it is used a chin rest or a bite bar that eliminate those kind of movement. The results obtained will be largely different from the ones achievable in real operating conditions. In literature multiple solutions to the head movement problem can be found: in [10] White, Hutchinson and Carley propose to include a second reference light source to distinguish between head and eye rotation. They proof the validity of the solution for large lateral displacement, but only with simulated data. Cerrolaza et al. in [11] instead consider the problem of head displacements in the direction perpendicular to the screen. In particular they proposed two methods. The first one is to include information about the different positions in the calibration procedure in order to improve the tolerance to this kind of displacement. A noticeable drawback for this method is the increase of the calibration time. For this kind of system a too long calibration time may increase the stress of the user and the distraction. The second method is based on the hypothesis that the PoR estimation error mainly depend on the system and not on the different user. Hence the error can be modeled for a particular configuration and used to compensate the estimation. Both method seems to improve the robustness to head movement. In [12] Guestrin and Eizenmanit stated that with more than one light source, as in the case of the system we are going to use, it is possible to tolerate certain movement of the head. That are from 2 to 4 cm depending on the direction of the displacement. That requires a multiple point calibration. In addition they show that the same results can be obtained with a single point calibration using both multiple light sources and multiple cameras. The idea of using more than one camera is exploited in more than one paper. For example in [14] multiple cameras and multiple lights are used to eliminate the usage of the user dependent parameters. The estimate is done using 3D computer vision technique and allows to completely eliminate the calibration phase. On the other hand in [13] the new camera is used in order to allow free head movement in a volume with 40 cm of diameter. This may result very interesting in the case of a user in front of a computer screen, since the natural movement won't be much bigger than that.

Anyway what we are actually interested in is not to change how the eye tracker works but to determine:

1. the accuracy estimated in the LCD plane in relation to the influencing factors;
2. the effect of the influencing factor on eye tracker accuracy and its associated covariance in real time.

The first goal is something similar to what Nystrom and al. reported in [6]. Here the authors consider calibration method and eye physiology as influencing parameters and study their effect on data quality. The test are done with a tower mounted video based eye tracker. In particular they study three different calibration procedures to verify which one works better. The difference through the three is the way to determine if the calibration has to be considered good or discard. The three kinds are: operator controlled, participant controlled and automatic calibration. From the experimental data it seems that participant controlled calibration is the one that gives the better results, followed by operator controlled one. During the test both the user and the operator can see if the system is capturing the eye position in the right way. In addition they develop a series of test to verify how accuracy, precision and the number of valid fixation samples vary changing the following influences parameters: position on the screen, time, eye physiology and visual aids. In the following the results they obtain:

- Position on the screen: The off center positions correspond to an higher visual angle that makes it harder to distinguish the eye, e.g. the pupil may be covered by eyelashes.
- Time: Data quality get worst with time. This may be due to the fact that person move from his initial position, even if with the chin rest this should not be a problem. The problem of moving from calibration position will be discussed later also in this work.
- Eye physiology: Eyelashes pointing downwards and smaller pupil has a negative effect on accuracy. Precision is worst for blue eyes. This is mainly due to the fact that blue eyes have a lower contrast between iris and pupil.
- Visual aids: Worst result both with glasses and contact lenses. The problem with contact lenses is that they can slight with respect to the eye, while the issue with glasses is that they absorb some of the infrared light (depending on the material they are made of).

In this article the authors provide the first comprehensive set of data showing how the calibration method, the operator, participants' eye physiology, and visual aids affect the quality of data.

Based on the work of Holmqvist et al. [5], it has been developed a software 'Accuracy Test Tool', for precision and accuracy measure by Tobii Technology. In particular a description of the software and test specification can be found in [7]. As said by the authors the document presents a suitable methodology to test and compare the performance of different remote eye tracking systems. It outlines a series of extensive tests that identify and control for external parameters that illustrate the accuracy and precision of the system under different scenarios (e.g. subjects position in the track box, environmental light levels, and large gaze angles). Unfortunately in this paper they do not take into account the influence parameters that depend on user characteristics (eye physiology, glasses...). Indeed they consider only data acquired by "suitable eye tracking individuals". This means that people with glasses, lenses or anyhow poor performances are excluded. The different scenarios they consider are:

- Ideal conditions: best scenario, unattainable in real working conditions.
- Large gaze angles: stimuli placed far from center.
- Varying illumination: variation of illumination level.
- Head positions: the eye tracker is moved of 5 cm in one direction (X, Y or Z) for test. The head always remain on the chin rest.

It has to be noticed that in all this test the user's head lay on a chin rest. This is not what happened in real situation where the user may move the head. Another calibration procedure, based on this idea, was carried out by Clemotte et al. in [8]. The purpose of the work is to "identify the precision and accuracy of the Tobii x2-30 with non disabled people under non ideal conditions (without any chin rest)". They found out that accuracy(systematic effect) and precision (repeatability) values obtained in these conditions highly differ from the ones reported on the data sheets: the accuracy range from 0.4 to 2.46 degrees, while the precision range from 0.2 to 1.91 degrees.

Despite several works can be found regarding the device calibration, at the current state of the art there are no works proposing the estimation of the accuracy as a function of the actual conditions. We believe that this will open the scene to a more effective use of the device. In the analysis of visual scanning patterns the resulting PoR maps would be associated with proper confidence levels that for sure would represent a fundamental instrument to draw accurate conclusions. In the field of HMI examples can range from option buttons reshape, local zoom dynamically adjusted, speed to regulate interface responsiveness, the possibility to take into account the uncertainty associated to a particular interaction. This part represents the most innovative contribution of our work.

## 2. Experimental set up and test description

In the experimental campaign we adopted the EyeAssist developed by Xtensa, a low-cost eye tracking platform developed for clinical use<sup>3</sup>. Using an infrared camera, it maps the gaze of the user on the screen by detecting the location of the pupil with respect to a known pattern of light. This system requires a first phase of calibration to fit the different users. This phase consists in a point moving on the screen in a random way between 9 possible positions. In particular we target the PoR together with its covariance that we estimate as a function of the operating conditions in real time, namely the lighting, head position and rotation, target position on the screen, number of eyes detected and user characteristics.



**Figure 1:** Eye tracker system mounted on the screen

The eye tracker communicate data obtained on a UDP port. The software developed in Unity was able to read from this port and also to communicate messages to the eye tracker, always through the UDP. This to communicate the data needed to load the calibration. The script that allows the communication was obtained starting from [15].

During the test, the software counts both the valid and the wrong data. If the number of wrong data overcomes 130, the target is moved to the following position. The data obtained contain:

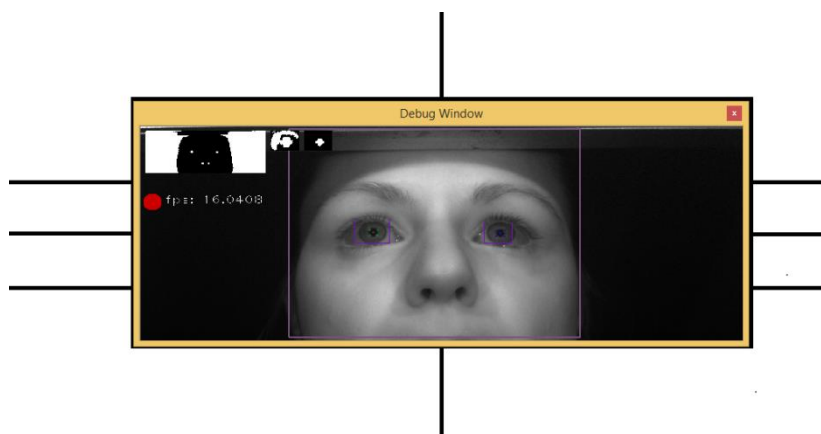
- Timestamp;
- X coordinate of the PoR obtained considering only the left eye;
- Y coordinate of the PoR obtained considering only the left eye;
- X coordinate of the PoR obtained considering only the right eye;
- Y coordinate of the PoR obtained considering only the right eye;
- X coordinate of the PoR obtained as a medium value;
- Y coordinate of the PoR obtained as a medium value;
- X position of the left pupil on the image;
- Y position of the left pupil on the image;
- X position of the right pupil on the image;
- Y position of the right pupil on the image;
- X coordinate of the target;
- Y coordinate of the target;
- Number of wrong data.

The test procedure used to collect the data is similar to the one used for calibration. The software employed to develop the test was Unity<sup>TM</sup>. Unity is an integrated multiplatform to develop animations in many different fields. In our case we used it to control a circle on the screen to appear randomly on a grid of 5x5 that covers the whole screen. The circular target remained in each location on the screen until a sufficient number of valid data has been collected. This number was set to 130.

A problem that we considered was the relative motion between the screen and the head. In order to minimize this effect we added a bar to the set up to hold the user forehead. Moreover it has been drawn a scheme that allow the user to verify if he lies in the center of the camera in order to locate the head always in the same position. Both solutions can be seen in the following images, Figure 2 and Figure 3.



**Figure 2:** Bar that limit head movement



**Figure 3:** Lines helping the user positioning his head

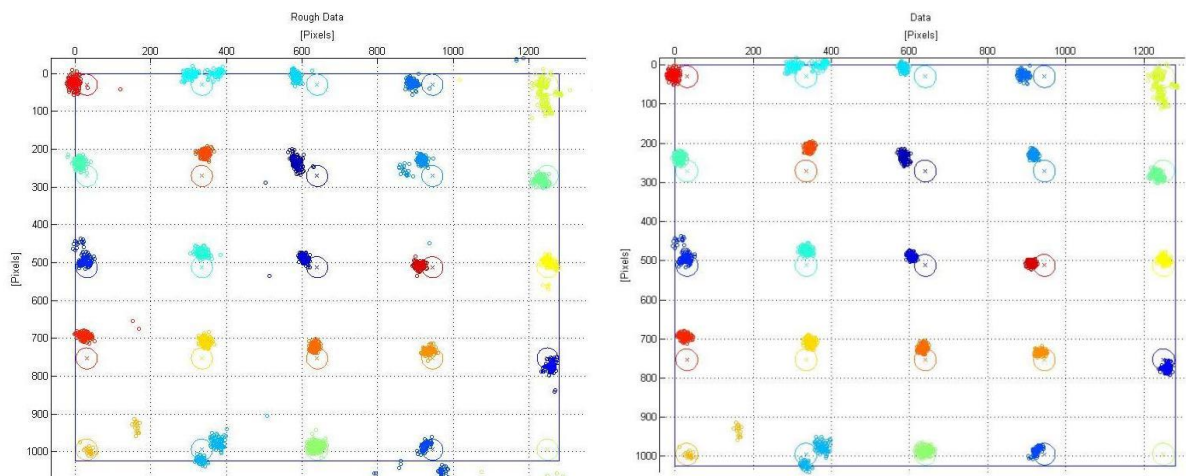
Another influencing factor is the light. Indeed ambient light may change during the day. This alteration are partially compensated by changing the camera exposure. Anyway the test



were conducted in as much limited time slots as possible, also verifying that the lighting condition didn't changed that much.

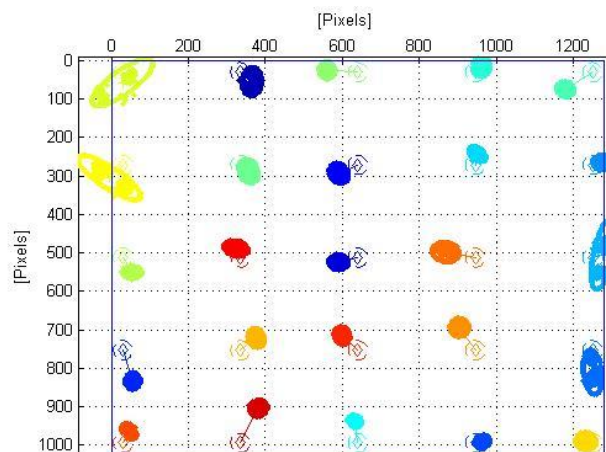
### 3. Eye tracker repeatability and accuracy characterization

The obtained data were elaborated to remove outliers by means of the Chauvenet's. We set the maximum allowable deviation to 2. The procedure was repeated until the difference between previous and current standard deviation was less than 10%. Higher reduction would be meaningless. In Figure 4 it is evident that the noise was largely reduced.



**Figure 4:** Data collected for accuracy and repeatability characterization, before (left) and after (right) the application of a filter

Having the filtered data we can calculate the systematic effect and repeatability for every target position. The repeatability regions were then calculated using a confidence threshold that allows to obtain a confidence level of 95%, as explained in [16]. Plotting the results obtained it is straightforward to see that the ellipse does not contain the true value, which is the target position.

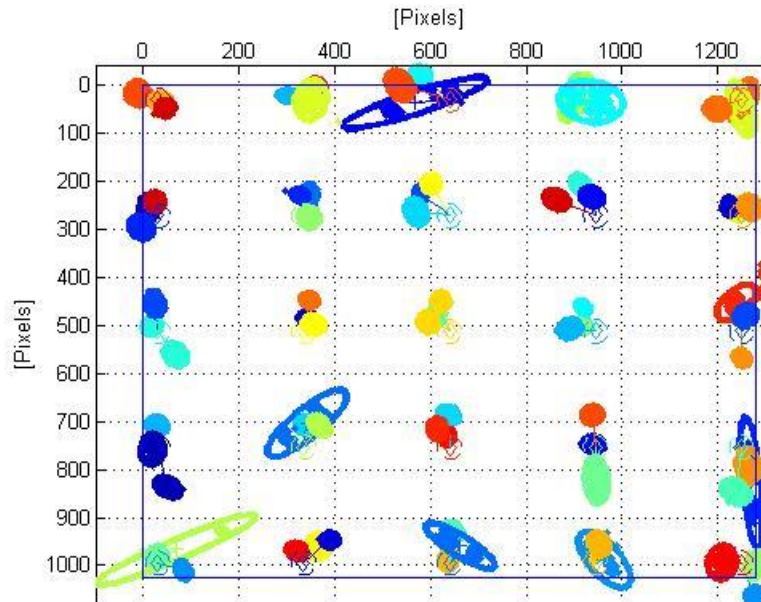


**Figure 5:** Mean and Covariance

It is evident that this problem is mainly due to the presence of a strong systematic effect that seems to have a random nature, as evident in figure 5. In traditional measurement systems, if the results are repeatable, the systematic effect can often be determined and compensated.



In order to verify the statistic nature of the systematic effect, we tried to distinguish between repeatability for the same calibration and repeatability through different calibrations. For this aim we decided to consider data coming from different calibrations. In figure 6 it is evident that the systematic effect is completely random.



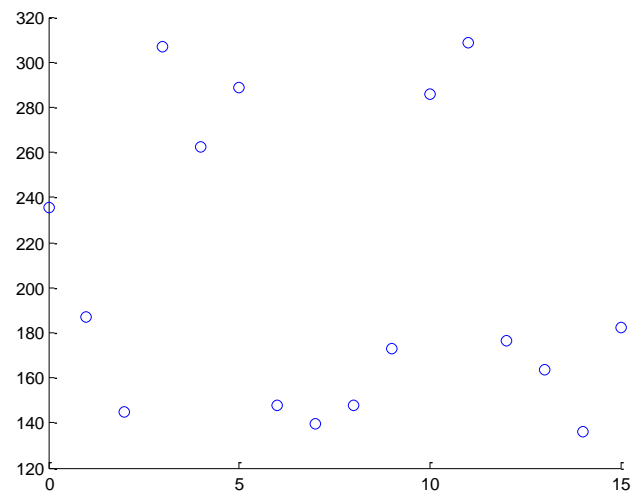
**Figure 6:** Repeatability through different calibrations

#### 4. Eye tracker accuracy characterization

The results presented previously underline the need of a different method to estimate uncertainty. It has to be noticed that we have a good estimation of the true value, that is the target position. Even though we can't say where a person is actually looking but only where he thinks to look, the target position is a good approximation since the user is asked to look at the point. Hence we can use this information to calculate an interval that includes the true values. That is we can calculate the residuals as the difference between nominal value and measured values, and use those residuals to build the covariance matrix.

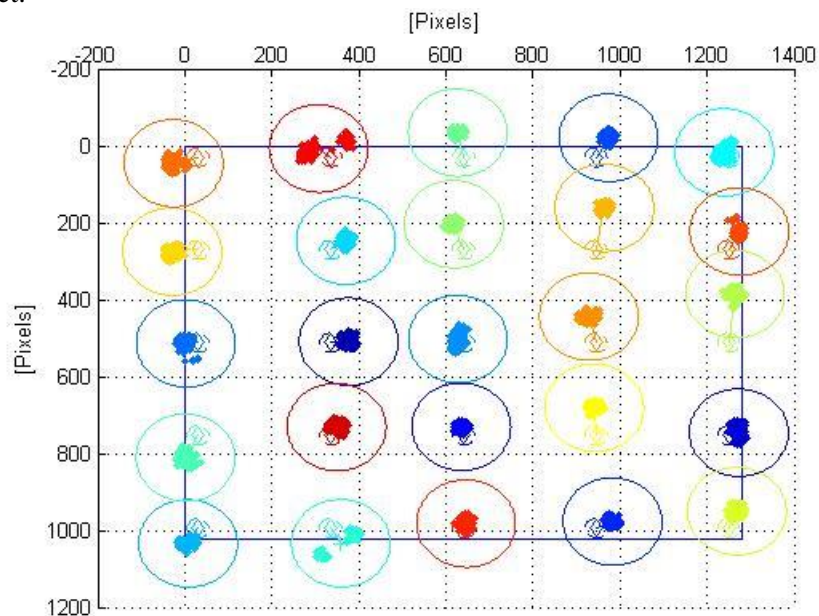
We use more than one test in order to have more data and also to have data coming from different calibrations. Indeed we have seen that data coming from different calibrations are not repeatable. Anyway all the data are still coming from the same user. Note that the covariance calculated with the distance from the target do not need a confidence threshold of two to achieve the 95% of confidence, as in the pure repeatability case, but a confidence threshold of one is more than enough because the repetitions are always concentrated several pixels away from the reference value.

The covariance (equiprobable) ellipse can be expressed by three parameters that represent the major, the minor axis and the orientation (i.e. correlation factor). Looking at the available data, see Figure 7, it is evident that the orientations are random.



**Figure 7:** Values of the covariance ellipse's angle

Because there isn't a prevalent orientation of the equiprobable ellipses it has no sense to model it as an ellipse. As a consequence we modelled the uncertainty to be equally distributed along every direction, i.e. as a circle of radius  $R$  equal to the semi major axis. We found the value of  $R$  that contains the 95% of the values to be  $R=114$  pixels. In Figure 8 can be seen this value applied to a validation set.



**Figure 8:** Covariance 'circle' with radius  $R=114$  pixels

All the targets are now inside the covariance ellipse, that is actually a circle, except for one. Using a wider series of data we verify that in the 95% of the cases the circle include the true value. Since the screen used is 1280x1024, this value is more or less the 10% of the screen dimension.

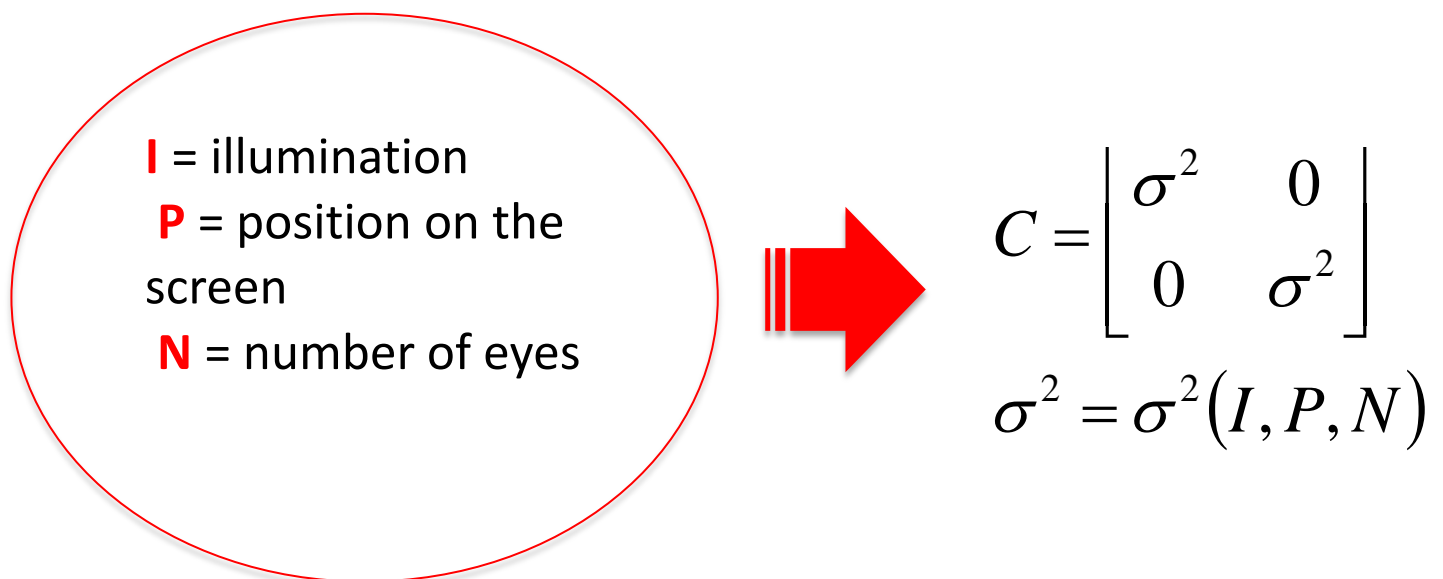
## 5. Modelling of the uncertainty as a function of the Influencing Parameters in Real Time

In this chapter we are going to model the uncertainty circle as a function of the operating conditions. The influencing parameters that have an effect on the variance are the following:

- **Target position on the screen:** as already underlined in the previous section in different positions of the screen we have different behaviors of the eye tracker. For example in the extreme points of the screen is more probable to have the eye covered by eyelashes or eyelid.
- **Illumination:** in a system that is not moving and placed in an appropriate spot, there should be limited illumination changes. That is e.g. far from window. Anyway considering a mobile application there can be wide changes in illumination.
- **Eyes detected:** the eye tracker find both the PoR of right and of left eye. The estimate is a mean weighted considering which one is the dominant eye. In some cases, usually the most external part of the screen, the eye tracker may not be able to identify both. If the detected eye is not the dominant one performances may largely change.
- **Relative movement between user and instrument:**
  - Displacement: both in the direction parallel to the screen and in the direction perpendicular to the screen.
  - Rotation: sometimes to look somewhere a person move both the eyes and the head or maybe just the head. Rotation may lead to difficulties in detecting the eyes.
- **Object in front of the eyes:** like glasses, contact lenses and hair.
- **Different user:** in this category may fall a lot of characteristic, such as eye's color, the cognitive ability, shape of the eyes and so hence so forth. Note that in this work all the tests are done by the same person, this means that those factors won't be considered. It is clear that this may be an interesting topic for future research.

Through all this influencing parameters we decide to model only: target position on the screen, number of eyes and head movement with respect to the instrument.

In the test about the influencing parameters the statistical control for each of them is obtained varying only one at once. That is, e.g., changing only the illumination keeping all the other parameters as fixed as possible. The test used is the same explained previously. Those are the values that we can quantify in real time given the information we have access to. The effect of every parameter will be modelled independently thus neglecting correlations between them.

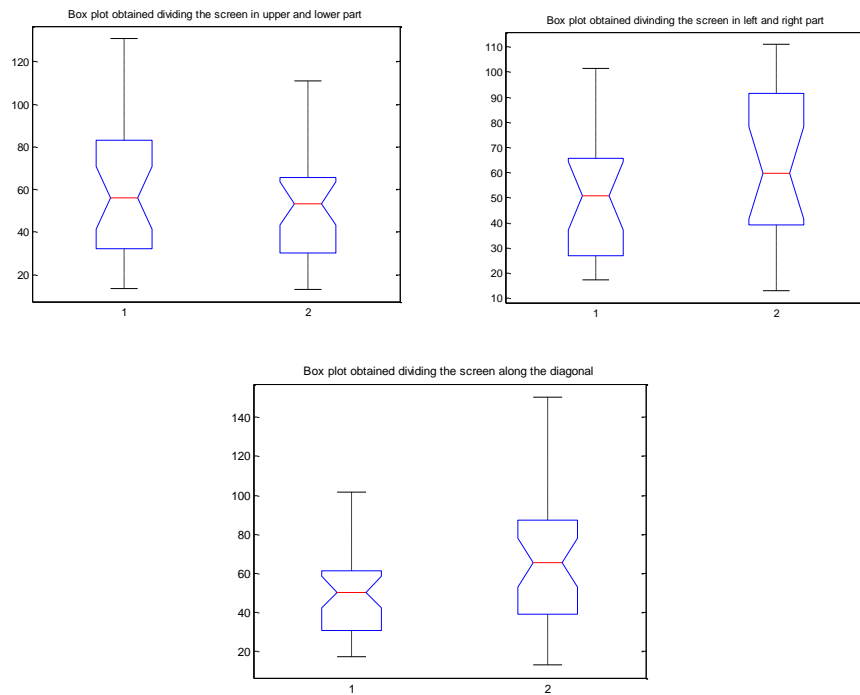


**Figure 9:** Influencing factors correlation/modelling wrt uncertainty

Note that the dependency from each parameter considered does not have to be linear. In the following we will find the relation between the parameters considered and the covariance, namely its radius  $R$ .

### 5.1. Target position on the screen.

The first influencing parameter we are going to consider is the target position on the screen. From the ANOVA test it comes out that it makes sense to divide the values along the diagonal, as can be seen from the following box plots, we will use the distance from the diagonal as a representation of the position.

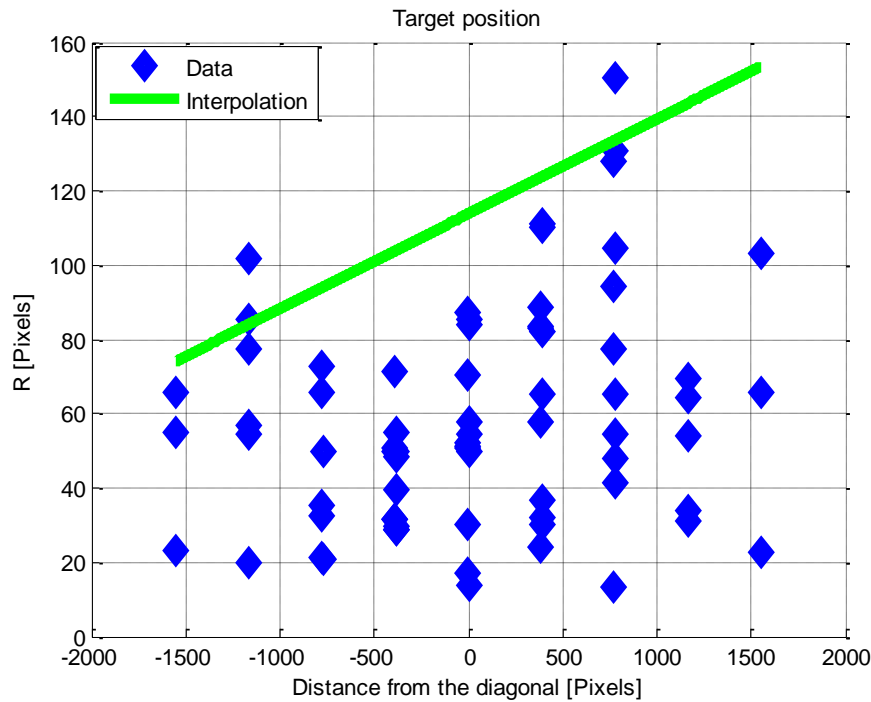


**Figure 10:** ANOVA test results

That is the influencing factor will be how far the point is from the diagonal, that is the line going from the upper left to the lower right part of the screen. This is probably due to the asymmetries of the system.

Taking the already considered three tests we model the value of the covariance radius  $R$  for every position and the corresponding distance from the diagonal. The linear model was computed as the line that covers the 95% of the radius. Hence the function will be:

$$\frac{dR}{dP} = 0.009$$

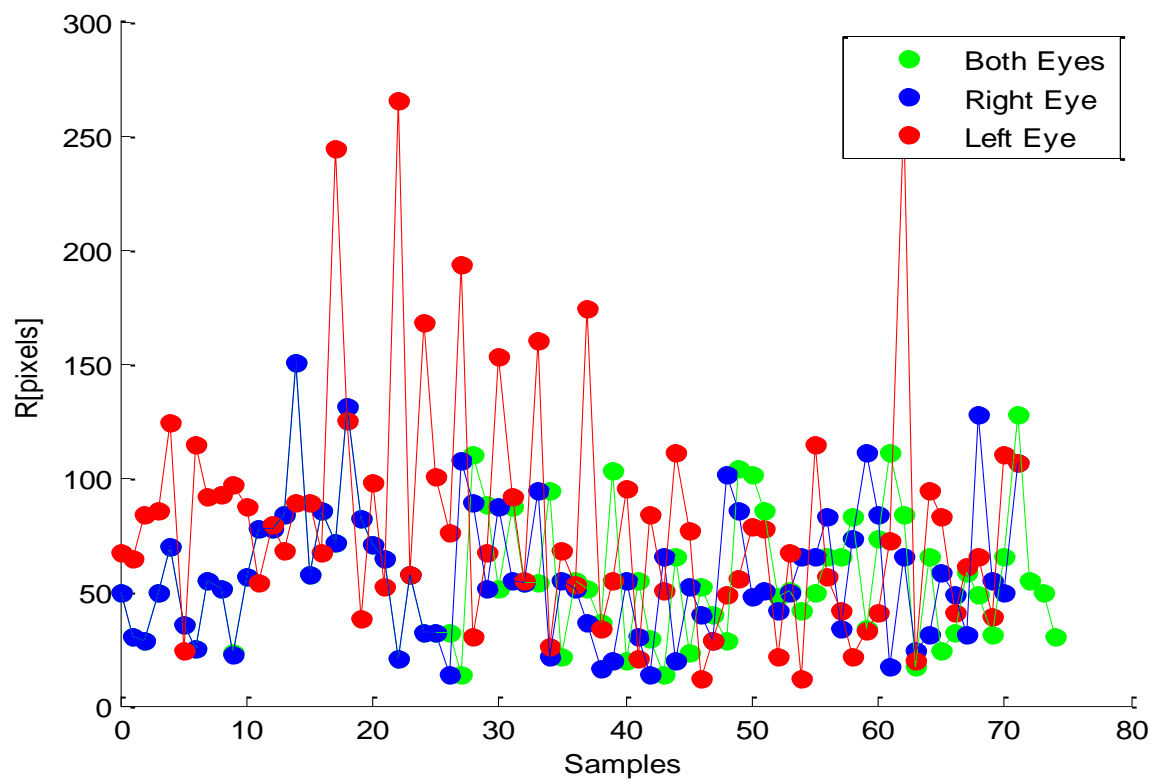


**Figure 11:** Target position on the screen.

### 5.2. Eyes detected

Unlike all the other parameters this is not a continuous value. We took three tests as before and consider the data coming from the right eye, the ones coming from the left eye and the ones coming from the mean. In Figure 12 we can see that the behaviour obtained with the information coming from only left eye strongly differs. In this case the user's dominant eye is the right one. Indeed  $R$  gets bigger when considering the data coming from only the left one. Hence we can find two values of  $R$ , one to be applied in the case we have the information from the right eye, or more in general the dominant eye, and one for the other cases. To calculate those values we consider a number that include the 95 % of the available data. The two values obtained, as can be seen in the figure, are 114 pixels for the right eye (blue line) and 188 pixels for the left eye (red line). So the model we finally obtain will be:

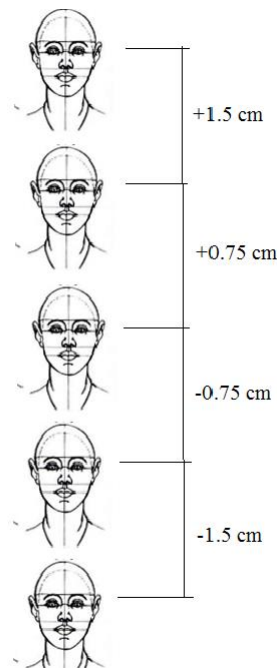
$$\frac{dR}{dN} = \begin{cases} 74 & \text{if left} \\ 0 & \text{otherwise} \end{cases}$$



**Figure 12:** Eyes detected

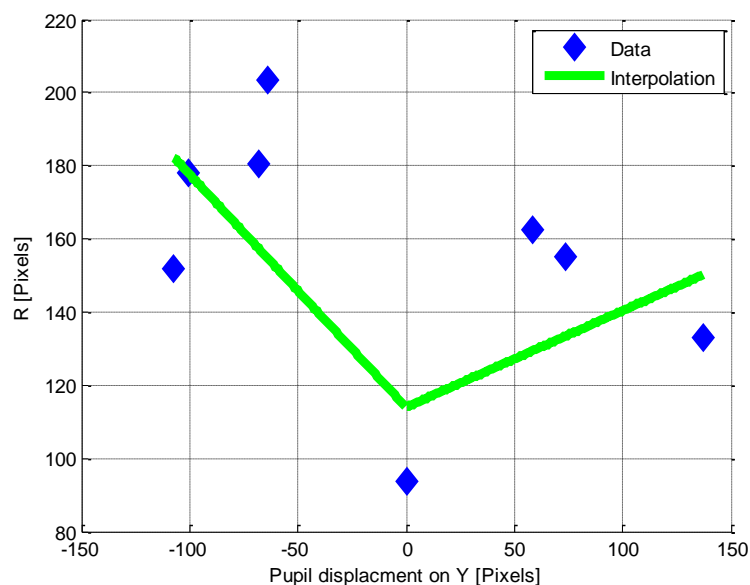
### 5.3. Displacement

In this case we have no direct access to this quantity but we can determine it from the pupil position of both eyes. This is an available data, also in real time, and help us to determine whether there was a displacement. We are going to divide the possible displacements in three categories: vertical, horizontal and depth. The vertical displacement is the motion of the head up or down with respect to the reference position.



**Figure 13:** Head positions considered for the vertical displacement

This can be calculated starting from the y position of the pupils, calculated as the mean of the left and the right one. The tests to determine this parameter are done moving the head upward of 0.75 cm and then of 1.5 cm and the same moving it downward. Over this position the eyes are out of the camera field of view and hence it has no sense to do any test. In Figure 13 can be seen the different positions, from the eye tracker point of view, at which the tests were made. In this case we take the data coming from the different tests and interpolate them with two linear functions since the behaviour does not seem symmetric with respect to the center. This can be explained by the fact that the camera used by the eye tracker is in the bottom part of the screen. The data and the interpolation can be seen in Figure 14.



**Figure 14:** Vertical displacement

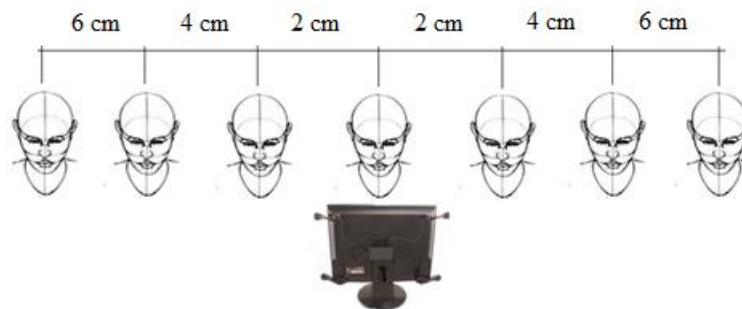


On the y axis there is the radius, while on the x axis there is the difference between the reference y pupil position and the current one. Both the quantities are in pixels. Note that the radius is always calculated considering the value that contains the 95% of the available values. The relation we finally obtain is the following:

$$\frac{dR}{dD_a} = \begin{cases} -0.064 & \text{if } D_a > D_{a0} \\ 0.264 & \text{otherwise} \end{cases}$$

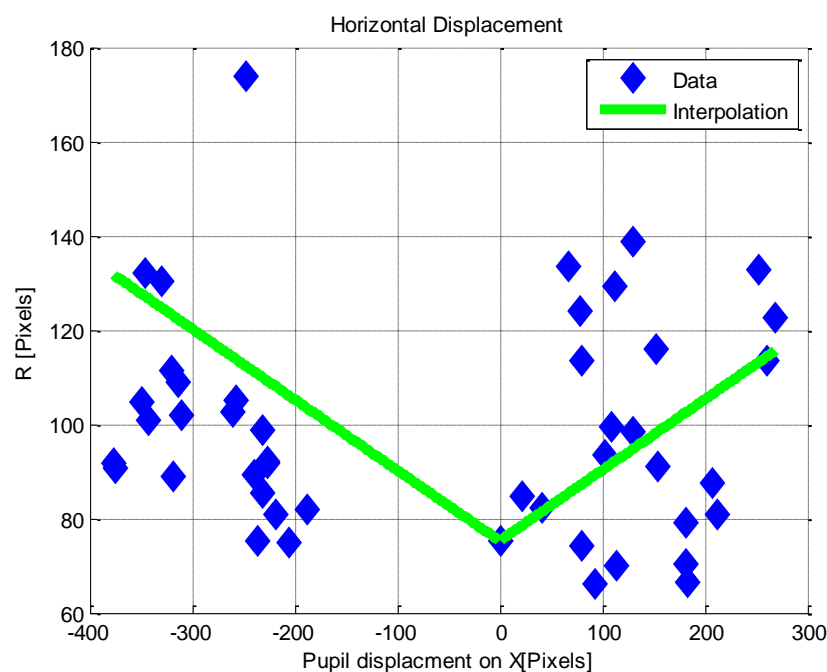
where  $D_{a0}$  is the reference value of the y pupil position, and  $D_a$  is the current one.

Regarding the horizontal motion, we consider a displacement that goes from 2 cm to 6 cm.



**Figure 15:** Horizontal head displacement considered

Again an higher displacement will be meaningless since the eye will fall out of the camera field of view. Also in this case we take the data coming from the different tests and interpolate them with two linear functions. Unlike the vertical motion, the behaviour seems symmetric with respect to the center. Hence the two linear function will be the same. The data and the interpolation can be seen in Figure 16.

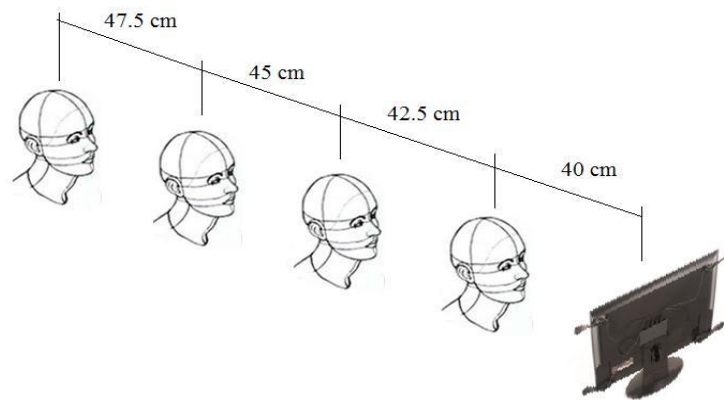


**Figure 16:** Horizontal Displacement

As before on the y axis there is the radius, while on the x axis there is the difference between the reference y pupil position and the current one. Both the quantities are in pixels. Note that the radius is always calculated considering the value that contains the 95% of the available values. The relation we finally obtain is the following:

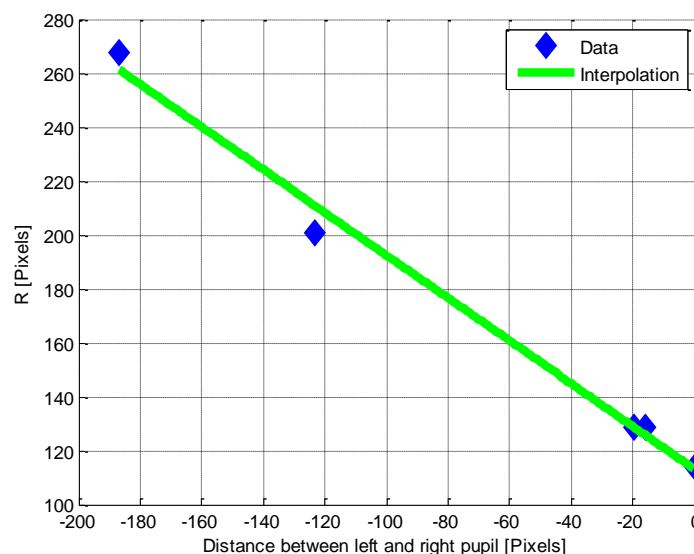
$$\frac{dR}{dD_b} = 0.15$$

Finally the last displacement we will consider is the one in the direction perpendicular to the screen. The default measurement sare done standing at about 40 cm from the screen. The tests to determine the influence of the distance are done at a distance of 42.5, 45 and 47.5 cm from the screen.



**Figure 17:** Different depth considered in the test

To understand if the user moves from the reference position we can use the distance between the two pupils. The nearest the two eyes the further the head from the screen. The data obtained and the interpolation can be found in Figure 18. We use a linear interpolation since it seems to fit well the values obtained, with some few exception that are considered as outliers.



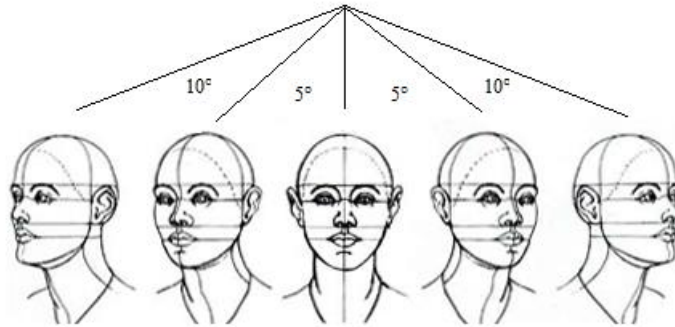
**Figure 18:** Depth displacement

In this case the model obtained is the following:

$$\frac{dR}{dD_c} = 0.767$$

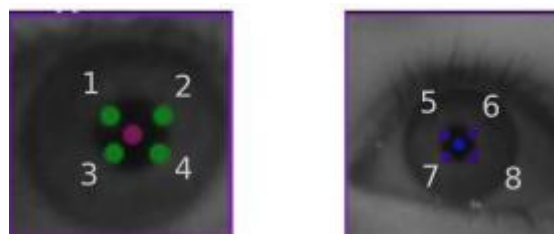
#### 5.4. Rotation

The last parameter of influence considered is head rotation. In this case we consider the rotation to the left and to the right. We make two cases for both direction verifying that the two eyes remain visible in all the situations considered. Indeed if one of the two eye is not visible any more we will fall in the previously discussed case of the number of eyes. The rotation considered are of 5 and 10 degrees both to the right and to the left.



**Figure 19:** Rotation considered

In order to determine whether a person is rotating his head or not we would like to use the information about the 4 infrared light in the eyes, see Figure 20.



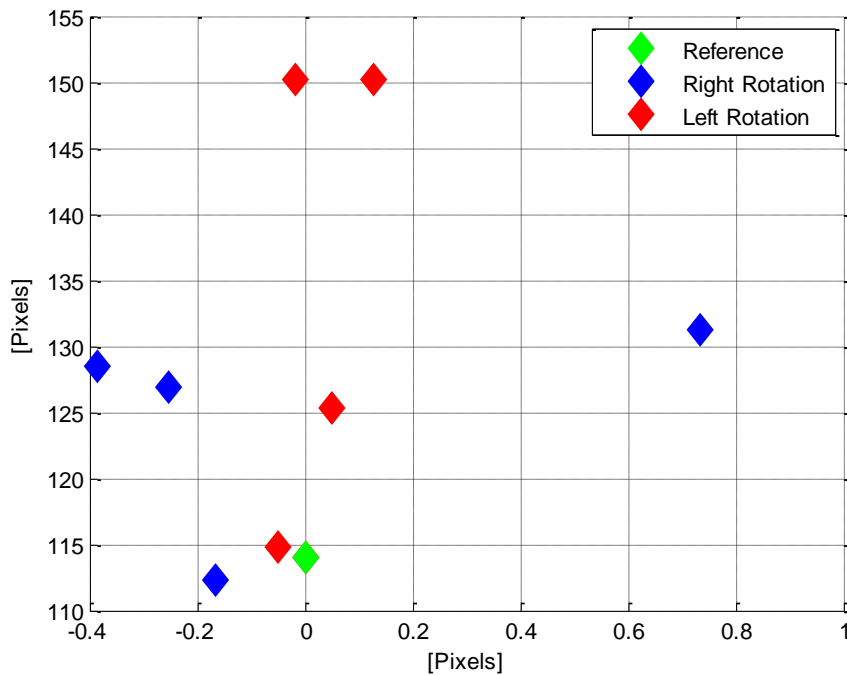
**Figure 20:** Numeration of the infrared light on the pupils

In order to obtain a value that is proportional to the rotation we take the x position of the point on the right (1 and 3 for the right eye, 5 and 7 for the left eye) minus the x position of the point on the left (2 and 4 for the right eye, 6 and 8 for the left eye). For example for the right eye we get:

$$\Delta x = \frac{(x_1 + x_3)}{2} - \frac{(x_2 + x_4)}{2}$$

To better understand the numeration see Figure 20. This quantity should diminish when rotating. The results obtained are plotted in Figure 21. As explained in the legend the green point is the

reference, the blue points are the data coming from the right rotation and eventually the red ones are the data coming from left rotation. On the x axis we have the difference between the distance of the points in the eye, calculated as explained before, and the reference distance, that is the one obtained with the reference test. In the reference test the head is obviously not rotated. On the y axis, as in the previous cases, we have the radius of the covariance circle. From Figure 21 we can see that we have two main issues. The first is that the difference between the pupils positions does not change that much, the maximum is less than 1 pixel. Those kind of displacement are so little that we can't say if they are noise or truly there is a rotation. Indeed some data result even bigger than the reference. The second issue is that the data obtained in this way are very noisy and does not seem to follow any particular trend. Indeed in some cases the performances seem to improve with the rotation. This two issues make it not reasonable to model the rotation in our case.



**Figure 21:** Rotation data

### 5.5. Validation

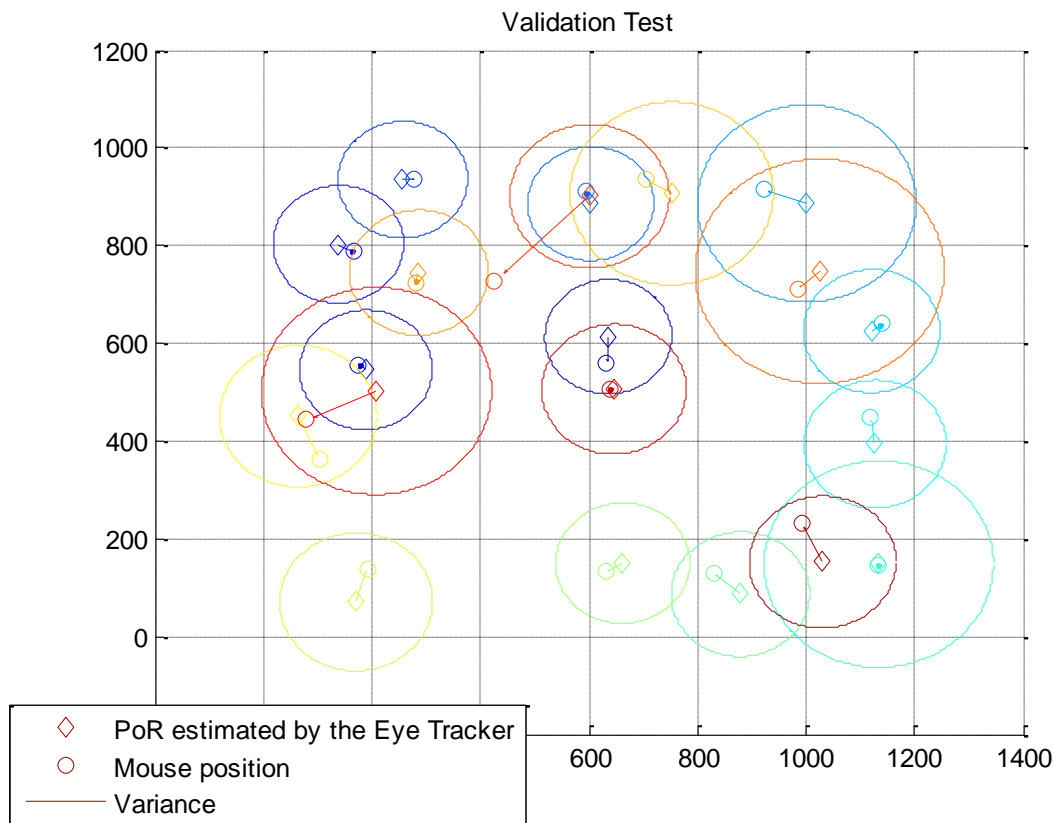
Combining the above modelled effects, we achieve the final expression that models the influence of each parameters on the uncertainty radius as follows:

$$R = R(P_0, N_0, D_{a0}, D_{b0}, D_{c0}) + 0.009\Delta P + 71n + d_a\Delta D_a + 0.15\Delta D_b + 0.767\Delta D_c \quad [\text{pixels}]$$

Where:

- $n$  it's a variable that determines if we have data coming only from the left eye. Equal to one in this case and zero in the other.
- $d_a$  it's a value that depend on the direction of the vertical displacement. It is equal to -0.064 if the head is moving upward with respect to the reference position and 0.264 in the other case.

It has been implemented a validation test. That based on the incoming data determines a value for the radius  $R$  according to the above model. The target in this case is the mouse that was moved along random positions on the screen. The user was asked to move his head during the test. The data can be seen in Figure 22.



**Figure 22:** Validation

In the above figure we have 20 data, that are a subset of the data considered for the validation, and only one is outside the estimated uncertainty. Moreover we determine the percentage of target that are inside the equiprobable circle. Doing four tests, each containing 50 data after different calibrations, we obtained 86% of valid data. That is in the 86% of the cases the target fall in the circle.

## 6. Conclusion

The eye tracker performances have been estimated focusing on its uncertainty as a function of different influencing factors. We have found that the uncertainty can be simplified to a circle, hence depending on only one parameter  $R$ . we computed  $R$  in nominal conditions taking into account also the systematic error that is indeed random. The value obtained for  $R$  in nominal conditions was 114 pixels. Since the screen was 1280x1024 this correspond to the 9% of the screen height and the 11% of the screen width. In addition to the value obtained for nominal operating conditions we estimated the dependency of the uncertainty from a set of influencing parameters. In particular we determined the relationship between the uncertainty and those parameters that can be estimated in real time: target position on the screen, eyes detected, head position and rotation with

respect to the instrument. The value obtained for uncertainty was applied to a set of validation data. For the nominal case we get that in the 95% of the cases the circle contains the true value, in the cases affected by the influencing parameters the percentage lowered to 86%. One explanation of the discrepancy could be found in that the influencing parameters were considered as not correlated.

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