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The locations of triangular equilibrium points in elliptic restricted three-body problem under the oblateness and radiation Effects

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Abstract. The Restricted Three-Body Problem (R3BP) considers motion of a third infinitesimal object under the gravitational influences of the primaries (bigger and smaller massive bodies) whose orbits are around the center of mass. If the orbits are elliptical, this belongs to Elliptic R3BP (ER3BP). In planar case it possesses five equilibrium points consisting of three collinear (L_1, L_2, and L_3) and two triangular (L_4 and L_5). To mimic a better astrophysical R3BP, such as motion of a satellite in star-planet system, the classical problem can be generalized by considering the effects of radiation pressure and oblate spheroid shape on the primaries. We study analytically the locations and the stability of L_4 and L_5 equilibrium points in the frame of ER3BP with incorporating the effects of radiation for bigger primary and oblateness for smaller primary. Our study suggests that the oblateness factor (A_2) and the radiation factor (q_1) shift the positions of L_4 and L_5 points compared with the classical ones. We also find that there is a stability limit for motion of the third body around these points.

1. Introduction

The Circular Restricted Three-Body Problem (CR3BP) considers motion of a third infinitesimal object under the gravitational influences of the primaries (bigger and smaller massive bodies) whose orbits are circular around the centre of mass. In planar case it possesses five equilibrium points consisting of three collinear (L_1, L_2, and L_3) and two triangular points (L_4 and L_5). The classical problem of CR3BP was modified by including additional forces. The effect of radiation pressure in CR3BP was first studied by Radzievskii in 1950 for the solar problem [3]. In both papers, however, Radzievskii did not consider the linear stability. The linear stability of equilibrium point was discussed by Chernikov in 1970 [3] who considered the relativistic Poynting Robertson effect which may be treated as perturbation, and Simmons in 1985 [4] with the effect of radiation pressure for all range of value.

This treatment may also be applicable to Elliptical Restricted Three-Body Problem (ER3BP). Singh and Umar in 2012 [1] considered the motion under the influences of an oblate spheroid in bigger primary and a radiation pressure in smaller primary. Kumar and Narayan in 2012 [2] studied the existence and stability of collinear equilibrium points in ER3BP under the effects of oblate spheroid primaries with the bigger primary has radiation pressure. Here we study the locations of L_4 and L_5 in ER3BP with bigger primary contain the radiation and smaller primary is an oblate spheroid.

2. Equations of motion

We assume that m_1 and m_2 are masses of the primaries, f is the true anomaly of system, r is the distance between the primaries, e is the eccentricity, and a is the semi-mayor axis of the primaries. In the case of three-body problem, it is more convenience to introduce the third body in the rotational (ξ, η, ζ) rather than in the inertial (x, y, z) coordinate system. The unit of time is chosen to make the gravitational constant $G = 1$. Introducing a transformation to a dimensionless coordinate system $\bar{\xi} = a\xi / r$, $\bar{\eta} = a\eta / r$, equations of motion of the third object in planar case are

$$\frac{d^2\bar{\xi}}{df^2} - 2\frac{d\bar{\eta}}{df} = \frac{dV}{d\bar{\xi}}, \quad \frac{d^2\bar{\eta}}{df^2} + 2\frac{d\bar{\xi}}{df} = \frac{dV}{d\bar{\eta}}.$$

The potential function V is defined by

$$V = (1 + e\cos f)^{-1}U \qquad U = \frac{1}{2}(\bar{\xi}^2 + \bar{\eta}^2) + \frac{1}{n^2}\left(\frac{(1-\mu)}{r_1} + \frac{\mu}{r_2}\right),$$

where n is mean motion of elliptical orbit, $\mu = m_2 / (m_1 + m_2)$ with $\mu \le 1/2$ and

$$r_i^2 = (\bar{\xi} - \bar{\xi}_i)^2 + (\bar{\eta} - \bar{\eta}_i)^2 \quad (i = 1, 2).$$

with (ξ_1, η_1) and (ξ_2, η_2) are the primaries coordinate.

3. Additional forces

3.1. Radiation pressure

The radiation pressure force (F_p) changes with distance by the same law as the gravitational attraction force of m_i (F_{gi}) and acts opposite to it. It is possible that this force will lead to a reduction of the effective mass of the massive particle. We use q_i to represent radiation pressure coefficients of the primaries. We can write the total force as

$$F_{gi} - F_{pi} = q_i F_{gi} \qquad q_i = 1 - \frac{F_{pi}}{F_{gi}}$$

3.2. Oblate spheroid

We use A_i to represent oblateness coefficients of the primaries respectively, such that $0 < A_i << 1$ and

$$F_{gi} = \frac{m_i}{r_i} + \frac{3m_i}{2r_i^4}\left(\frac{AE^2 - AP^2}{5R^2}\right) \qquad A_i = \frac{AE_i^2 - AP_i^2}{5R_i^2},$$

where AE_i and AP_i are the dimensional equatorial and polar radii of m_i. R_i is radius of m_i which assume the primary as spherical object.

To mimic the extrasolar planetary system with single star and planet, we assume that only the bigger primary is the source of radiation ($q_1 \neq 1$, $q_2 = 1$) and only the smaller primary is oblate spheroid ($A_1 = 0$, $A_2 \neq 0$).

4. Location of triangular points

Conditions of triangular equilibrium points are given as follows

$$\bar{V}_{\bar{\xi}} = \bar{V}_{\bar{\eta}} = 0. \tag{1}$$

Following [5] from (1) we obtain the location of equilibrium points $(\bar{\xi}_0, \bar{\eta}_0)$

$$\bar{\xi}_o = \frac{1}{2} - \mu + \varepsilon_1 - \varepsilon_2, \quad \bar{\eta}_o = \pm\frac{\sqrt{3}}{2}\left[1 + \frac{2}{3}(\varepsilon_1 + \varepsilon_2)\right].$$

where

$$\varepsilon_1 = -\frac{1}{2}A_2 - \frac{1}{3}(1 - q_1), \quad \varepsilon_2 = 0.$$

When the primary is oblate $r_i = 1 + \varepsilon_i$ with $\varepsilon_i << 1$. The positif value of $\bar{\eta}_0$ is the position of L_4 and the negative value of $\bar{\eta}_0$ is the position of L_5.

Figure 1 shows the displacement of triangular points under the effect of radiation and oblateness. The gray dot represents the positition of m_2. The yellow dot represents the positition of m_1. The positions of infinitesimal object are represented by the blue dots (for classical problem) and the red dots (for the presence of radiation and oblateness effects) with L_4 is located above axis and L_5 is located below axis. The triangular points shift drawn toward the bigger primary with increment of A_2 and reduction of q_1.

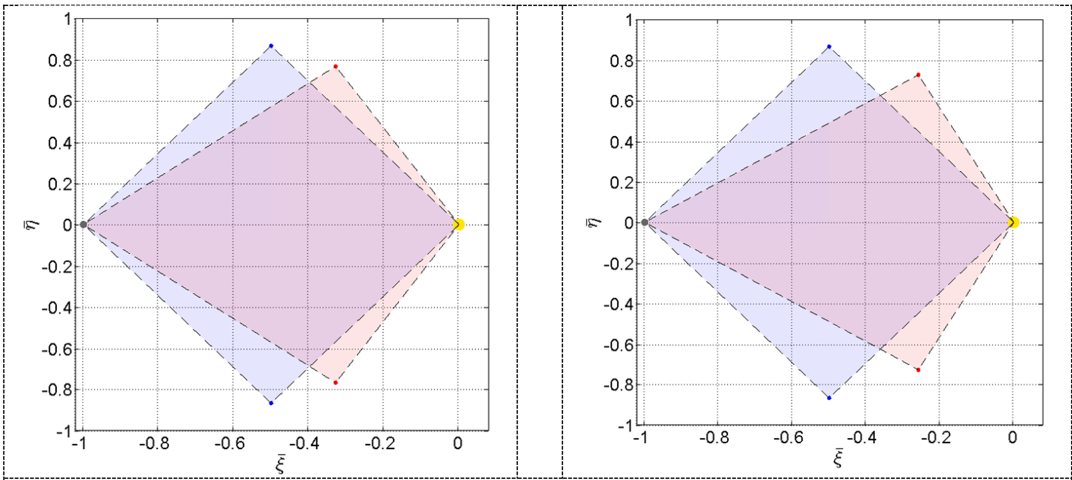

Figure 1. Variation of L_4 and L_5 locations for $\mu = 0.002$: (left) $q_1 = 0.5$; $A_2 = 0.01$, (right) $q_1 = 0.5$; $A_2 = 0.15$.

5. Linear stability

Define the displacement of triangular points by:

$$u = \bar{\xi} - \bar{\xi}_o, \quad v = \bar{\eta} - \bar{\eta}_o,$$

the equation of motion become

$$\ddot{u} - 2\dot{v} = \bar{V}_{\bar{\xi}\bar{\xi}}u + \bar{V}_{\bar{\xi}\bar{\eta}}v, \quad \ddot{v} + 2\dot{u} = \bar{V}_{\bar{\xi}\bar{\eta}}u + \bar{V}_{\bar{\eta}\bar{\eta}}v. \tag{2}$$

Substitution $u = \alpha e^{\lambda f}$ and $v = \beta e^{\lambda f}$ to (2), with α and β are constant and λ is root of characteristic equations, characteristic equation can be derived

$$\lambda^4 + (4 - \bar{V}_{\bar{\eta}\bar{\eta}} - \bar{V}_{\bar{\xi}\bar{\xi}})\lambda^2 + (\bar{V}_{\bar{\xi}\bar{\xi}}\bar{V}_{\bar{\eta}\bar{\eta}}) - \bar{V}^2_{\bar{\xi}\bar{\eta}} = 0$$

The stability of the system depends on the form of λ. If all roots are pure imaginary the system is stable. Fig. 2 shows the ER3BP's stability regions. It shows that the region is smaller when we consider q_1 and A_2.

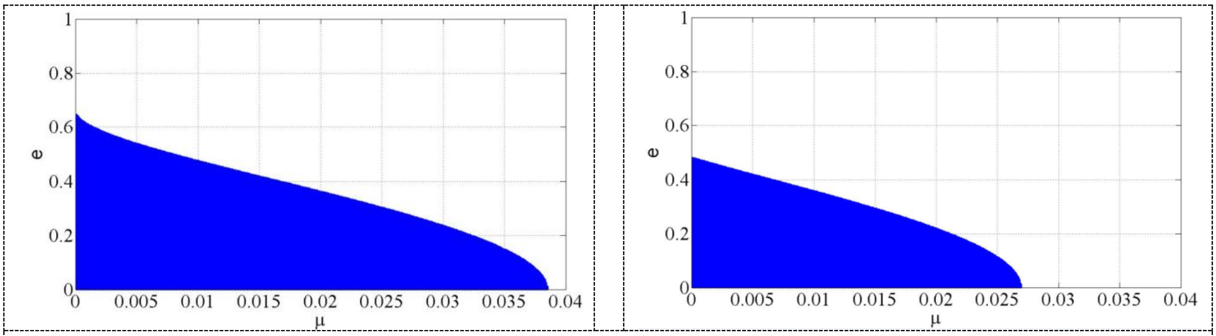

Figure 2. Stability curves for (left) q_1 = 1.0; A_2 = 0.0, (right) q_1 = 0.9; A_2 = 0.01.

6. Conclusion

The position of triangular equilibrium points in ER3BP under the effect of radiation and oblateness primaries have been studied. We conclude that the positions are shift when it recognizes the radiation factor (q_1) and oblateness coefficient (A_2). The stability regions are changed due to these effects.

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