Some Standard model problems and possible solutions

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Some Standard model problems and possible solutions

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Abstract. Three problems of the standard model of elementary particles are studied from a phenomenological approach. (i) It is shown that the Dirac or the Majorana nature of the neutrino can be studied by looking for differences in the $\nu$-electron scattering if the polarization of the neutrino is considered. (ii) The absolute scale of the neutrino mass can be set if a four zero mass matrix texture is considered for the leptons. It is found that $m_{\nu_3} \sim 0.05$ eV. (iii) It is shown that it is possible -within a certain class of two Higgs model extensions of the standard model- to have a cancelation of the quadratic divergences to the mass of physical Higgs boson.

1. A beautiful model
Louis de Broglie once said that if someone try to summarize the essential characteristics of physics in three words, those words would be: corpuscular, relativistic and quantum [1]. It was later that century that the concept of corpuscular was replaced with the idea of fields. These two concepts, fields and quantum were united in a single theory that now we call the quantum field theory (QFT). Thus we may say, paraphrasing de Broglie, that the essential of physics may be encompassed by QFT. In this spirit, it is natural that fundamental questions such as What is the universe made of? or what are the fundamental interactions of nature? should be tackled within the QFT formalism. Indeed, the Standard model of elementary particles (SM), the current theory that contains the fundamental building blocks of the ordinary matter, is a gauge quantum field theory. The internal symmetry that defines the SM is the local $SU(3) \times SU(2) \times U(1)$ gauge symmetry and the most general Lagrangian that describes the dynamics of the fields involves only 19 parameters. The last one of those parameters was finally measured by the LHC and it was announced on July 4th, of 2012: The mass of the Higgs boson which is $125.3$ GeV [2, 3]. The other 18 parameters are: 6 quark masses, 3 masses of the charged leptons, 3 mixing angles in the quark sector and one CP violating phase (that give rise to the Cabibbo-Kobayashi-Maskawa, CKM, matrix), 3 coupling constants, the vacuum expectation value and the $\Theta_{QCD}$.

It is almost incredible that with only those 19 parameters and within the formalism of QFT is possible to compute a huge number of processes involving elementary particles. Those processes cover energies from few MeV to the highest energies reached in both particle accelerators: man made or astrophysical. All those reactions can be computed and once the numerical values of those parameters are introduced, all the theoretical predictions match the experimental values within the one sigma error, except for few anomalies that are currently under study.
Table 1. Ratio of the solar neutrino events measured in different experiments over the expected number of events predicted by the Solar Standard Model.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>data/SSM</th>
<th>Experiment</th>
<th>data/SSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{37}$Cl</td>
<td>0.310 ± 0.030</td>
<td>$^{71}$Ga</td>
<td>0.540 ± 0.039</td>
</tr>
<tr>
<td>Super-K</td>
<td>0.413 ± 0.015</td>
<td>SNO$^{\text{Scatt}}$</td>
<td>0.413 ± 0.05</td>
</tr>
</tbody>
</table>

Nevertheless such success, which in some cases corresponds to a precision in a billion of some computed quantities, such as the gyromagnetic moment of the muon, the task is not complete. Although there is a machinery that allow us to compute such variety of phenomena, there are some new insights that tell us that the SM is not the final answer in the description of the fundamental blocks of the ordinary matter.

2. The $\nu$-SM

The first indication that the SM need an extension came from the Sun. It was called the Solar Neutrino Problem and it consisted in a deficit between the expected number of neutrinos and the measured number of events at terrestrial detectors. Indeed, the ratio of the measured neutrinos over the expected number of neutrinos, predicted by the Solar Standard Model (SSM) was always different of one in all the experiments designed to measure the neutrinos coming from the Sun as can be seen in table 1.

Similar phenomena were also observed in atmospheric neutrinos and in neutrinos from reactor experiments. It took over 30 years to understand the origin of such deficit and the current solution to the problem is called the neutrino oscillation mechanism and it means that neutrinos change their flavor due to the fact that they are massive and their flavor states are a mixture of their mass states i.e. $|\nu_\alpha\rangle = \sum_a U_{aa}^* |\nu_a\rangle$, $a = 1, 2, 3, \alpha = e, \mu, \tau$. Here the matrix $U_{PMNS}$ is the mixing matrix between flavor states and PMNS is an abbreviation for Pontecorvo-Maki-Nagakawa-Sakata since this matrix was introduced by Ziro Maki, Masami Nakagawa and Shoichi Sakata [4] to explain the neutrino oscillations and because neutrino oscillation was first predicted by Bruno Pontecorvo in 1957 [5]. These two hypothesis on the neutrino sector imply that SM needs to incorporate:

(i) Three neutrinos with (at least two non-vanishing) masses $m_1$, $m_2$, $m_3$ and
(ii) three mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$ and at least one CP violating phase $\delta$.

That is, we have to change from 19 parameters to at least 26 parameters. Many of them still unknown. The probability that neutrinos change their identity is a function of those new parameters: $P(\theta_{12}, \theta_{13}, \theta_{23}, \delta, \Delta m_{12}^2, \Delta m_{13}^2)$. As it can be seen, $P$ is not a function of the absolute masses of the neutrinos $m_i$ but a function of the differences $\Delta m_{ij}^2 = m_j^2 - m_i^2$.

There exists an experimental program trying to determine with the highest precision the values of those parameters. It involves the detection of neutrinos produced in reactor, accelerators or astrophysical sources as the Sun and active galactic nuclei or supernova remnants that produce ultra-high energetic neutrinos. All this effort has cornered the allowed region of parameters as follows: $7.02 \times 10^{-5} \text{(eV)}^2 \leq \Delta m_{12}^2 \leq 8.09 \times 10^{-5} \text{(eV)}^2$, $0.270 \leq \sin^2 \theta_{12} \leq 0.344$, $2.317 \times 10^{-3} \text{(eV)}^2 \leq \Delta m_{13}^2 \leq 2.607 \times 10^{-3} \text{(eV)}^2$, $0.382 \leq \sin^2 \theta_{23} \leq 0.643$ at the 3$\sigma$ C.L. [6].

Thus, the leptonic sector, after the discovery of neutrino oscillations, imply some questions that remain to be answered.

- What is the absolute scale of the neutrino masses?: Since neutrino oscillation data only allows the knowledge of the mass difference $\Delta m_{ij}^2$, the absolute values of $m_{\nu_1}, m_{\nu_2}, m_{\nu_3}$ remains a mystery. The strongest constraints on $m_i$ come from Cosmology: $\sum_i m_i < 0.136 \text{eV}$ [7].
• What is the hierarchy of masses in the neutrino sector?: From the neutrino data, it is not possible to determine the sign in $\Delta m_{ij}^2$, that is, either possibilities $m_1$ bigger or lower that $m_2$ are still allowed.

• Is there CP violation in the neutrino sector?: So far, the value of $\delta$ is feeble bounded, $\delta = 306^{+49}_{-20}$ degrees at the $1 \sigma$ C.L., while at $3 \sigma$ it can be from $0^\circ - 360^\circ$ [6].

• Why is the hierarchy of the CKM matrix so different to the one observed in the PMNS matrix?: Indeed, the phenomenon of mixing in the neutrino sector is very similar to the mixing in the quark sector but the structure of the mixing matrices are very different.

• *Is the neutrino a Dirac or a Majorana particle?*

The last question is not directly related with the oscillation of neutrinos, but it is a question about the fundamentals of the theory behind elementary particles. Grand Unification Theories (GUT) predict a Majorana nature for the neutrinos. Furthermore, if neutrinos are Majorana, the dynamical equation will not be the Dirac equation any more. Thus, changing our model (GUT) predict a Majorana nature for the neutrinos. Furthermore, if neutrinos are Majorana, the calculation of the mixing in the quark sector but the structure of the mixing matrices are very different.

3. The nature of the neutrino

It was shown some time ago that Majorana and Dirac neutrino scatter different on electrons or nucleons [9]. Nevertheless, since neutrinos have a very small mass and due to the left-handed nature of the charged weak interactions, neutrinos are born almost fully polarized. To account for such polarization, it is needed an extra state preparation factor $(1 - \gamma_5)/2$ which is usually added and the resulting amplitudes for both Dirac or Majorana neutrino scattering become identical. In any case, the calculation of the $\nu - e$ scattering for both cases, Dirac and Majorana neutrinos by including as a free variable the neutrino polarization can be performed. The result for the Dirac neutrino in the center of mass system is [9, 10]:

$$\frac{d\sigma^D}{d\Omega} = \frac{G_F^2}{8\pi^2 s} \left( m_\nu^2 (E_\nu - p^2 \cos \theta)(g_A^2 - g_V^2) + (E_\nu E_e + p^2) (g_V + g_A) \right)$$

$$+ \left( E_\nu E_e + p^2 \cos \theta \right)^2 (g_V^2 - g_A^2) - p [s^{1/2} (E_\nu E_e + p^2) s_{||}] (g_V + g_A)^2 + (E_\nu E_e + p^2 \cos \theta)$$

$$\times \left( (E_e + E_\nu \cos \theta) s_{||} + m_\nu s_{\perp} \sin \theta \cos \phi \right) g_V^2 - g_A^2 \right)^2$$

$$+ m_\nu (E_\nu (1 - \cos \theta) s_{||} - m_\nu |s_{\perp}| \sin \theta \cos \phi) (g_A^2 - g_V^2) \right) \right), \quad (1)$$

while for the Majorana case it is given by:

$$\frac{d\sigma^M}{d\Omega} = \frac{G_F^2}{4\pi^2 s} \left( (E_\nu E_e + p^2)^2 + (E_\nu E_e + p^2 \cos \theta)^2 + m_\nu (E_\nu E_e + p^2 \cos \theta) \right) (g_V^2 + g_A^2)$$

$$+ \left( E_\nu E_e (1 - \cos \theta) - m_\nu |s_{\perp}| \sin \theta \cos \phi \right) . \quad (2)$$

Here $E_e, E_\nu$ are the energy of the electron and the neutrino, $\theta, \phi$ the scattering angles and $p$ the neutrino momentum and $s$ the Mandelstam variable. Furthermore, the incident neutrino polarization vector is defined as $s_\nu = (0, s_{\perp}, 0, s_{||})$ in the neutrino rest frame, and
\[ g_\nu^e = -\frac{1}{2} + 2 \sin^2 \theta_W + \delta_{\nu e}, \quad g_\nu^\ell = -\frac{1}{2} + \delta_{\nu \ell}, \quad \ell = e, \mu, \tau \] 
are the coupling constants. Both cross sections are equal in the limit of \( m_\nu \to 0 \) and \( s_\| = -1 \), that is, a fully polarized neutrino. If the neutrino polarization evolution is considered, \( \nu - e \) scattering may shed some light into the Dirac or Majorana nature. Consider a change in the neutrino polarization that may be achieved by strong magnetic fields in astrophysical environments. Any particle, even neutral particles, possessing a magnetic moment, as the neutrino may have due to the fact that it is now a massive particle, may interact with external electromagnetic fields and consequently, its spin polarization may change.

Thus, as it can be seen from eqs. (1) and (2), even in the case of a very small neutrino mass, if the change in \( S_\| \) is big enough, there will be appreciable differences between the scattering of a Dirac or a Majorana neutrino regardless the energy of the incoming neutrino. Thus, it will be another possible way to observe differences between Dirac or Majorana neutrinos.

4. The absolute mass of the neutrino

As we have mentioned, the oscillation of neutrinos gives only information on \( \delta m^2 \). The absolute values of \( m_i \) are still unknown. So far, cosmology gives the strongest bound on the sum of all neutrinos. Currently, PLANCK data constraint on the mass of neutrinos is \( \sum m_i < 0.136 \) eV [7]. Other limit on the absolute neutrino mass come from the analysis of the Kurie plot in \( \beta \)-decays. In particular, the Tritium \( \beta \)-decay analysis done by the Troitsk Collaboration sets an upper limit on the electron neutrino mass of \( m_\nu < 2.2 \) eV. [11]. It looks like a matter of time when high precision experiments will eventually give the value of the absolute neutrino mass. Nevertheless, from a phenomenological approach it is possible to set the scale of the neutrino mass. For instance, unification models may shed light on the absolute value of the neutrino mass.

Let us make an example of this possibility. The ideas of supersymmetry or grand unifications may set relations between the fermion masses and the mixing angles that arise in the context of left-right symmetric models which predict a Yukawa matrix texture.

Let’s assume that the Yukawa matrices texture is a hermitian four-zero texture in the flavor basis, i.e.

\[
M^f = \begin{pmatrix}
0 & C_f & 0 \\
C_f^\dagger & D_f & B_f \\
0 & B_f^\dagger & A_f
\end{pmatrix}.
\] (3)

To test this texture we have to do the following steps:

(i) Define a real and symmetric matrix \( \tilde{H} = M^f M_{f\dagger} \), with \( M^f \) the mass matrix with a texture. In our case, a four-zero texture eq. (3). By writing \( B(B^\dagger) = b e^{i\phi_b} \), \( C(C^\dagger) = c e^{i\phi_c} \), \( D = de^{i\phi_D} \) and \( A = ae^{i\phi_A} \), separate the phases of the non-diagonal elements with the help of a matrix \( P = e^{-i \frac{\pi}{2}} \text{diag} \left( e^{i \frac{\pi}{2}}, e^{i(\phi_c-\phi_D)}, e^{i(\phi_c+\phi_D)} \right) \).

Thus, the new matrix \( H \) with no phases will be:

\[
H = P^\dagger \tilde{H} P.
\] (4)

(ii) The elements of \( H \) are real and can be inverted with the help of the invariants:

\[
\begin{align*}
\text{Tr}(\tilde{H}) &= m_1^2 + m_2^2 + m_3^2, \\
\text{Tr}^2(\tilde{H}) - \text{Tr}(\tilde{H}^2) &= 2m_1^2m_2^2 + 2m_1^2m_3^2 + 2m_2^2m_3^2, \\
\text{Det}(\tilde{H}) &= m_1^2m_2^2m_3^2,
\end{align*}
\] (5)

into functions of the masses \( m_i \). This leaves only one free parameter and one phase for each mass matrix. Let’s denote as \( a_a, a_d, a_\ell, a_\nu \) and \( \phi_a, \phi_d, \phi_\ell, \phi_\nu \) those parameters.
(iii) Diagonalize $H$ with the help of the matrices $O_I$ constructed with the eigen-vectors of $H$. The theoretical mixing matrix CKM and PMNS are expressed in terms of such matrices. Indeed:

$$V_{\text{CKM}}^{\text{th}} = O_I^T P^{u-d} O_d, \quad U_{\text{PMNS}}^{\text{th}} = O_I^T P^{d-\nu} O_{\nu},$$

where $P^{u-d} = \text{diag}[1, e^{i(\phi_1^d - \phi_1^d)}, e^{i(\phi_2^d - \phi_2^d)}]$ and in a similar way $P^{d-\nu} = \text{diag}[1, e^{i(\phi_1^f - \phi_1^f)}, e^{i(\phi_2^f - \phi_2^f)}]$.

(iv) Compare the theoretical mixing matrices with the latest experimental values and find the allowed regions of the free parameters $a'_u, a'_d, a'_c, a'_\nu$, and the combinations $\phi \equiv \phi_1^u - \phi_1^d = \phi_2^d - \phi_2^f$ and $\Phi_1 \equiv \phi_1^f - \phi_1^f = \phi_2^d - \phi_2^f$.

In the leptonic case, since the absolute value of the neutrino masses are unknown, nevertheless, from neutrino oscillation data the difference $\Delta m^2_{ij}$ are known and we can express the neutrino mass as:

$$\tilde{m}_{\nu_1} = \sqrt{1 - \frac{(\Delta m^2_{32} + \Delta m^2_{21})}{m^2_{\nu_3}}}, \quad \tilde{m}_{\nu_2} = \sqrt{1 - \frac{\Delta m^2_{21}}{m^2_{\nu_3}}}. \quad (8)$$

Thus, in addition to the parameters $a'_c, a'_\nu$ and $\Phi_1$ we have to include in the fit the neutrino mass $m_{\nu_3}$ as a free parameter. By doing a $\chi^2$ analysis, it is possible to find the allowed values of the parameters $a'_u$ and $a'_d$ and the best fit for the free parameters of quark mass matrices that reproduce the CKM matrix. Furthermore, in the leptonic case, similar regions are found $a_1, a_\nu, \Phi_1$ and, the unknown mass $m_{\nu_3}$. Results are shown in table 2. As it can be seen, the analysis shows a preferred value for $m_{\nu_3}$ to be around 0.05 eV. [12, 13]. It is important to mention the role of $\theta_{13}$ in the previous analysis. The confirmation that $\theta_{13}$ is different from zero reduce the possibilities and the allowed regions for $m_{\nu_3}$. Once the true value of $\theta_{13}$ is introduced, the fit of the theoretical $U_{\text{PMNS}}^{\text{th}}$ fixes $m_{\nu_3} \sim 0.05$ eV, as it can be seen in Fig. 1 of [14].

5. The hierarchy problem
An aesthetic criterion for a satisfactory theory is that the dimensionless ratios between the free parameters of the Lagrangian should take values of order of unity. This criterion is called as the naturalness of the theory. This is not the case of the standard model of particles. The parameters that appear on the fundamental Lagrangian vary orders of magnitude between them. Some of the naturalness problems of the standard model are:
Figure 1. Some values of $\tan \alpha$ and $\tan \beta$ that fulfill the Newton-Wu conditions for reasonable masses of the scalar particles and that sets as one of those scalar the mass of the particle found by the LHC $m_1 = 125.3$ GeV.

- Neutrino masses are too small compared with the quark top mass.
- The Higgs mass is too small compared with the Planck mass.

Related with the problem in the hierarchy of the Higgs mass compared with the Planck mass is the possibility that large quantum contributions to the square of the Higgs boson mass would inevitably lead to quadratic divergences. In 1981, Veltman realized that such quadratic divergences can be cancelled if the following relation is fulfilled [15]:

$$\frac{3}{2} M_W^2 + \frac{3}{4} M_Z^2 + \frac{3}{4} m_H^2 = \sum f N_f m_f^2.$$  

(9)

Here $f$ runs for all fermions in the Standard Model and $N_f = 3$ for quarks and $N_f = 1$ for leptons since $N_f$ is the number of color degrees of freedom. LHC discovery [2, 3] sets a mass for the Higgs boson of $\sim 125$ GeV. Thus, the relation (9) is not valid.

Furthermore, as we have seen in the previous section, the neutrino mass will be of order of few electronvolts. While the mass of the quark top is $\sim 174$ GeV. In the SM there is only one scalar, the Higgs boson that gives mass to all fermions. Thus, the Yukawa coupling of the neutrino should be around 12 orders of magnitude smaller than the Yukawa of the top quark. In order to reduce this fine-tuning problem, in 1973 T. D. Lee introduced one of the simplest and economical extensions of the SM: to introduce another doublet $\Phi_2$ [16]. Many realizations of the Two Higgs doublet model are possible. In one of them, $\Phi_2$ gives mass to the 'heavier' quarks $u,c,t$ and $\Phi_1$ to $d,s,b$ and to the leptons. Thus reducing the naturalness problem in the hierarchy of the Yukawa couplings. Is it possible to find a solution for the cancellation of the quadratic divergences of the Higgs mass within the THDM? It is possible. In 1994 Newton and Wu computed the quadratic divergences in a two Higgs extension of the SM [17].
found the quadratic divergences as functions of the gauge, the Yukawa and the quartic scalar couplings and conditions between the masses of the fermions, the bosons and the Higgses to cancel the quadratic divergences. We will call them the Newton-Wu conditions. A realization of the Newton-Wu conditions in a THDM was done in [18]. In the following, we will re-evaluate the work of [18] with the new information that we have about the mass of one of the Higgses.

Consider the following Higgs potential for two doublets \( \Phi_{1,2} = (\phi_1^0, \phi_2^0) \):

\[
V = \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 + \mu_{12}(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\
+ \lambda_3 (\Phi_1^\dagger \Phi_2) (\Phi_2^\dagger \Phi_1) + \lambda_4 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_5^* (\Phi_2^\dagger \Phi_2)^2.
\]

(10)

Here the vacuum expectation values \( \langle \phi_1^0, \phi_2^0 \rangle = v_{1,2} \), fulfill \( v^2 = v_1^2 + v_2^2 \), and it is defined \( \tan \beta = v_2 / v_1 \) as usual.

Thus, the Newton-Wu conditions reduce to [18]:

\[
\frac{3}{2} M_W^2 + \frac{3}{4} M_Z^2 + \frac{v^2}{2} (3\lambda_1 + 2\lambda_3 + \lambda_4) = \frac{1}{\cos^2 \beta} \sum_{f_1} N_{f_1} m_{f_1}^2,
\]

\[
\frac{3}{2} M_W^2 + \frac{3}{4} M_Z^2 + \frac{v^2}{2} (3\lambda_2 + 2\lambda_3 + \lambda_4) = \frac{1}{\sin^2 \beta} \sum_{f_2} N_{f_2} m_{f_2}^2,
\]

(11)

For simplicity, let us assume that \( \mu_{12} = 0 \) and \( \lambda_5 \) real. We have five physical scalar particles: two charged Higgses \( m_{H^\pm} \), one axial Higgs \( M_A \), and two Boson Higgs particles \( m_1, m_2 \). Furthermore, we have five quartic scalar couplings in the potential \( \lambda_{1...5} \). By rewriting the quartic scalar couplings \( \lambda_{1...5} \) in terms of the masses of the physical scalars it is possible to rewrite the Newton-Wu conditions eqs. (11) in terms of the masses of the Higgses. Direct searches of other scalar particles have been done, and lower limits on the masses of those particles have been set. In particular, the Charged Higgs boson should be bigger than 80 GeV, and there are weaker limits for the axial scalar. By fixing \( m_1 = 125.3 \) GeV as the Higgs recently discovered by the LHC it is found a region of parameters that fulfills the Newton-Wu conditions. Examples of that region are shown in Fig. 1.

6. Conclusions

Although the SM is a beautiful model with a high predictive power, few unknowns prevail as well as some problems. Most of the unknowns are in the leptonic sector and in particular the neutrino sector. One of them is to determine the absolute scale of the neutrino mass or the nature of the neutrino, either it is a Dirac or a Majorana particle. In this note we have naively tried to find alternative ways of solving those problems from a phenomenological approach.

Another problems of the SM can be considered aesthetic problems: Why is \( \Theta_{QCD} \) so small?, why is the Yukawa coupling of the neutrino so small compared with the Yukawa coupling of the quark top? Nevertheless, those questions can be related with other problems such as the cancellation of the quadratic divergences in the Higgs mass vacuum correction. Indeed, the two Higgs model extension of the SM may reduce the hierarchy problem and as shown in [18] and to cancel quadratic divergences in a class of two Higgs doublet models. In this note we have updated [18] with the constraint that one of the scalar Higgses being a scalar with the mass of \( m_1 = 125.3 \) GeV as found by LHC [2, 3].

Acknowledgments

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