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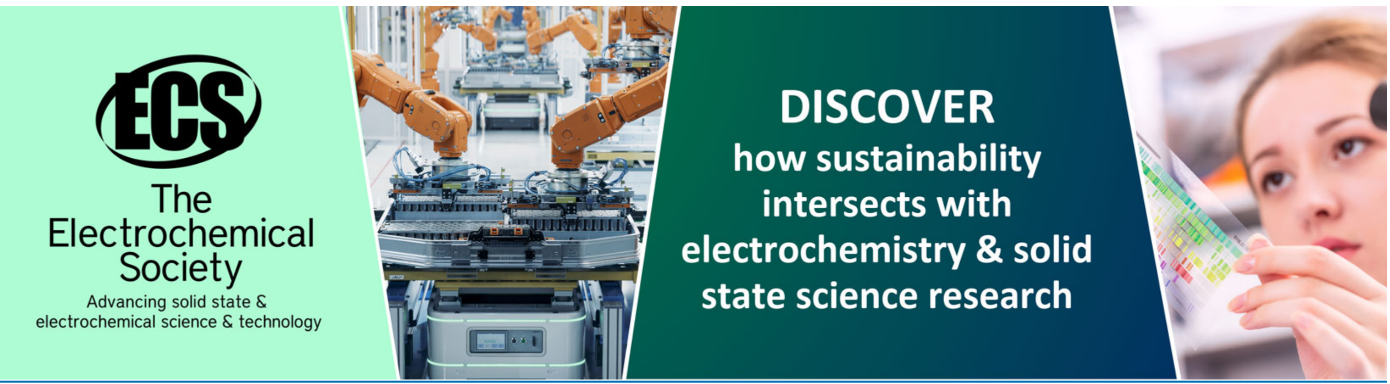

Low Reynolds number flow's heat transfer influenced by strong magnetic field

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Abstract. For the last 20 years research concerning the strong magnetic field influence on the weakly magnetic substances has been dynamically developing. The published papers refer mainly to natural convection problems connected with the impact of strong magnetic field. This paper follows previous Authors' approach to forced convection modification by the additional magnetic force. Presently, attention was paid to the heat transfer processes and their quality assessment done in the basis of Nusselt number for low Reynolds number flow. The analysis was done for the geometry from Graetz-Brinkman problem with the magnetic coil located at the position of adiabatic-thermal boundary condition change. The numerical analysis was performed with Ansys software and application of the user-defined functions. Presented results revealed the influence of magnetic field on the flow structure and heat transfer.

1. Introduction

Since XIX century it is widely known that the substances may be divided into ferromagnetics, diamagnetics and paramagnetics according to their magnetic properties. In the common knowledge the first type refers to the substances strongly attracted by the magnetic field, the second one to weakly attracted and the third one being weakly repelled. These differences are connected with substance magnetic susceptibility [1]. Magnetic susceptibility of ferromagnetics is a non-differentiable function below the Curie's temperature. Magnetic susceptibility of paramagnetics is described by the Curie's Law. The change of diamagnetics magnetic susceptibility is connected with change of density, being the result of temperature variation.

Weak magnetism was a phenomenon unable to be utilized for a long time. It required the discovery of high-temperature superconductivity, which pushed the things forward. It was accomplished by Bednorz and Muller in 1986 [2] and was granted with the Nobel prize. This discovery, connected with many others, led quickly to the construction of superconducting magnets. Nowadays, they can generate the magnetic field of induction up to 26 T and higher values are aimed for.

However, it was not necessary to reach the superconducting magnets state to understand the presence of magnetic force acting on various substances. At first it was reported by Faraday in 1847. Unfortunately, due to lack of appropriately strong magnetic field sources the progress in research stopped for almost one hundred years. Faraday's work was undertaken by Pauling in 1946, who used this knowledge to construct oxygen analyzer [3].

Nevertheless, the superconducting magnets and their ability to generate strong magnetic fields contributed to advancement in the field of fluid mechanics and heat transfer. In 1991 Braithwaite et al. proved experimentally that magnetic field was able to enhance heat transfer during natural convection

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of paramagnetic fluid and proposed the mathematical description of forces acting on this kind of fluid in the magnetic field [4]. Some other phenomena referring to strong magnetic field influence on paramagnetic fluids such as: combustion promotion [5], breath support [6] or Wakayama jet [7] were also reported. The non-mechanical air flow in the strong magnetic field [8] and the most famous magnetic levitation of droplet of water [9] were discovered afterwards.

While the above mentioned papers concerned experimental investigations, the numerical approach to these issues was also done. Especially in the field of magnetic field influence on natural convection. Tagawa et al. presented work regarding the numerical model of natural convection in cubical enclosure for paramagnetic [10] and diamagnetic [11] fluids. They took Bai et al. [12] mathematical formulation of magnetic force as a basis for their own numerical investigation. The problem of natural convection in the strong magnetic field was carried out also in another geometries, ie. cylindrical [13], cylindrical annuli [14] or rectangular [15].

These days, the emphasis is placed, for example, on nanofluids behaviour in the strong magnetic field [16], the transition regime between laminar and turbulent convection [17] or magnetic field influence on forced convection. The last topic was at first undertaken by Ueno and Iwasaka in 1994 [18]. The results of experimental analysis displayed decrease in diamagnetic fluid flow and its total suppression under the influence of strong magnetic field. Ozoe in [19] modelled numerically the paramagnetic fluid flow (air) through the pipe surrounded in the middle by the single circular magnetic coil. Authors continued his efforts in the last papers in relation to flow structure for the various boundary conditions and geometries [20][21].

This paper, however, is focused on the heat transfer caused by the forced convection in the magnetic field represented by the Nusselt number in a form proposed in [22][23] for the various fluid properties (represented by Prandtl number). The low Reynolds number flows presented in this paper occurs in the small blood vessels called arterioles (Re<1). Nevertheless, they are big enough to not to treat them as Stokes flows. In the industry low Reynolds flows refer to the flows of fluids of high viscosity such as tars (petroleum tars for example) or honey.

2. Mathematical model

The mathematical model consisted of three basic conservation equations: continuity, momentum and energy. In order to obtain magnetic force distribution Biot-Savart's law was calculated.

The following assumptions were established for the continuity equation: the flow was incompressible, there were no external mass sources, the flow was stationary, laminar and three-dimensional. Taking these assumptions into account, the continuity equation could be presented in a form:

$$\nabla \cdot \vec{\upsilon} = 0\,, \tag{1}$$

where $\vec{\upsilon}$ denotes velocity vector (m/s).

With the consideration of above mentioned assumptions, adding to them that the gravitational and magnetic forces were treated as the external body forces, the momentum equation could be written as:

$$\rho\left(\vec{\upsilon} \cdot \nabla \vec{\upsilon}\right) = -\nabla p + \mu \nabla^2 \vec{\upsilon} + \vec{F}_{\mathrm{b}}\,, \tag{2}$$

where: ρ is the density (kg/m^3), p is the pressure (Pa), μ is the dynamic viscosity (Pa·s), $\vec{F}_{\mathrm{b}} = \vec{F}_{\mathrm{g}} + \vec{F}_{\mathrm{mag}}$ represents the body forces (N/m^3), $\vec{F}_{\mathrm{g}}$ is the gravitational force (N/m^3), $\vec{F}_{\mathrm{mag}}$ is the magnetic force (N/m^3).

To calculate the magnetic induction distribution around the single circular magnetic coil Biot-Savart's law was utilized with the following equation:

$$\vec{B} = \frac{\mu_m i}{4\pi} \oint_c \frac{d\vec{s} \times \vec{r}}{|r|^3}\,, \tag{3}$$

where: $\vec{B}$ is the magnetic induction vector (T), μ_m is the magnetic permeability (H/m), i is the current magnitude (A), $d\vec{s}$ is the infinitely small segment of the coil (m), $\vec{r}$ is the position vector (m).
The magnetic force acting on the paramagnetic fluid could be described in the form:

$$\vec{F}_{mag} = -\left(1 + \frac{1}{T_0\beta}\right)\frac{\chi_m \rho\beta(T - T_0)}{2\mu_m}\vec{\nabla}B^2\,, \tag{4}$$

where: $T_0 = (T_w - T_f)/2$ is the reference temperature (K), T_w is the heated wall temperature (K), T_f is the inlet fluid temperature (K), T is the local fluid temperature (K), β is the thermal expansion coefficient (K^{-1}), χ_m is the mass magnetic susceptibility (m^3/kg).
The following equation represented the energy budget coming from energy conservation law (the assumptions were as follows: viscous dissipation and species diffusion were negligible, flow was steady, without external heat source):

$$\vec{\upsilon} \cdot \nabla T = \frac{\lambda}{\rho c_p}\nabla T^2\,, \tag{5}$$

where: λ is the thermal conductivity (W/(m·K)), c_p is the specific heat (J/(kg·K)).

2.1. Dimensionless parameters
In presented analysis the non-dimensional parameters played role of measure in regard to the magnetic field influence on the phenomena in heat transfer. The definitions of applied parameters are listed in Table 1, with following symbols: υ_{avg} is the average inlet velocity (m/s), d is the pipe diameter (m), R is the pipe radius (m), T_b is the bulk temperature (K) [23], A is the heat transfer surface area (m^2).

Table 1. Dimensionless parameters

Number	Equation
Reynolds	$\mathrm{Re} = \frac{\rho \upsilon_{avg} d}{\mu}$
Prandtl	$\mathrm{Pr} = \frac{c_p \mu}{\lambda}$
Nusselt	$\mathrm{Nu} = -\frac{\left(\frac{\partial T}{\partial r}\right)_{r=R}}{T_b - T_w} \cdot 2R,\;$ where $T_b = \int_A T\upsilon dA \,/ \int_A \upsilon dA$

3. Numerical approach
The studied geometry was circular straight three-dimensional duct (pipe) shown schematically in figure 1. The length of the duct was l = 0.2 m and its diameter was equal to d = 0.01 m. It should be emphasized that the single circular magnetic coil's center was placed in the origin of global coordinate system. The coil was placed perpendicularly to the flow axis (in the XY plane, see figure 1). The diameter of the coil was always double of the pipe diameter. The plane consisting the coil divided the pipe in two equal parts. The wall of the first part was adiabatic and the wall of the second part was isothermally heated (Dirichlet boundary condition) with the constant temperature of 310 K. The parabolic velocity profile was assumed at the inlet and the constant value of pressure (equal to surrounding pressure) was assumed at the outlet (see figure 1).

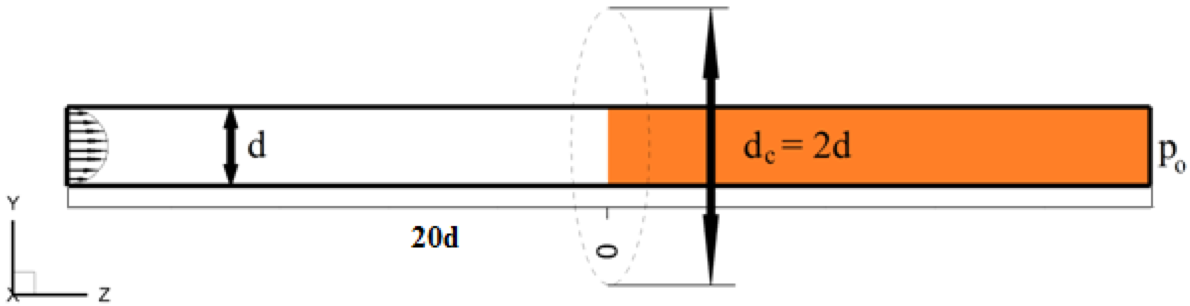

Figure 1. The studied geometry, boundary conditions and location of magnetic coil [20].

The unstructured grid consisted of 176600 wedge-shaped elements with the usage of Gambit 2.3 software. The solver chosen for the computations was Ansys Fluent 14.5. A package of special user-defined functions was implemented into it including: three-dimensional parabolic velocity profile, distribution of magnetic field and magnetic field force, the momentum source terms, calculation of Nusselt number equation and bulk temperature equation.
The first upwind discretization scheme was used to solve the momentum and energy equations, together with the standard pressure interpolation scheme. In the case of difficulties with the convergence of continuity equation it was replaced with body force weighted scheme, which is helpful while solving the problems with large external body forces.
The residuals were set to 10^{-6} for continuity and momentum equations, and 10^{-8} for energy equation.
Applied in the calculations properties of the working fluid are listed in Table 2. The fluid serving as a reference for the computational analysis was the water solution (C_{mass} = 80%) of gadolinium nitrate hexahydrate ($Gd(NO_3)_3{\cdot}6H_2O$) with the molar concentration of C_{gado} = 0,8 mol/kg. The Prandtl number characterising this fluid was equal to Pr = 584. Addition of gadolinium nitrate hexahydrate to the water caused the change of magnetic susceptibility and gave the strong paramagnetic characteristics of fluid.

Table 2. Thermo-physical and magnetic properties of the reference working fluid at 298 K [3].

Property	Symbol	Unit	Value
Density	ρ	kg/m^3	1463
Dynamic viscosity	μ	Pa·s	$8.689 \cdot 10^{-2}$
Thermal expansion coefficient	β	K^{-1}	$0.52 \cdot 10^{-3}$
Volumetric magnetic susceptibility	χ	-	$3.38 \cdot 10^{-4}$
Magnetic permeability	μ_m	H/m	$4\pi \cdot 10^{-7}$
Thermal conductivity	λ	W/(mK)	0.397

Taking into account previous studies, Authors decided to focus on low Reynolds number flows. The Reynolds number for all of the studied cases was equal to Re = 0.84. The magnetic field induction in the center of the coil was in each computed case equal to 10 T.

4. Results and discussion

Computations were performed for the flow with and without magnetic field. However, taking into account large computational time for each case, the emphasis was placed on the flows influenced by the magnetic field. Number of analysed cases with the flow without magnetic field were reduced to four cases for Prandtl number equal to Pr = 75, Pr = 200, Pr = 300 and Pr = 584 and served as a database for comparison of the Nusselt number distribution along the heated wall between flows with and without the magnetic field (see figure 2). The dimensionless position h/d is a ratio between distance from the magnetic coil *h* and diameter *d*. All fluids had the same value of magnetic susceptibility.

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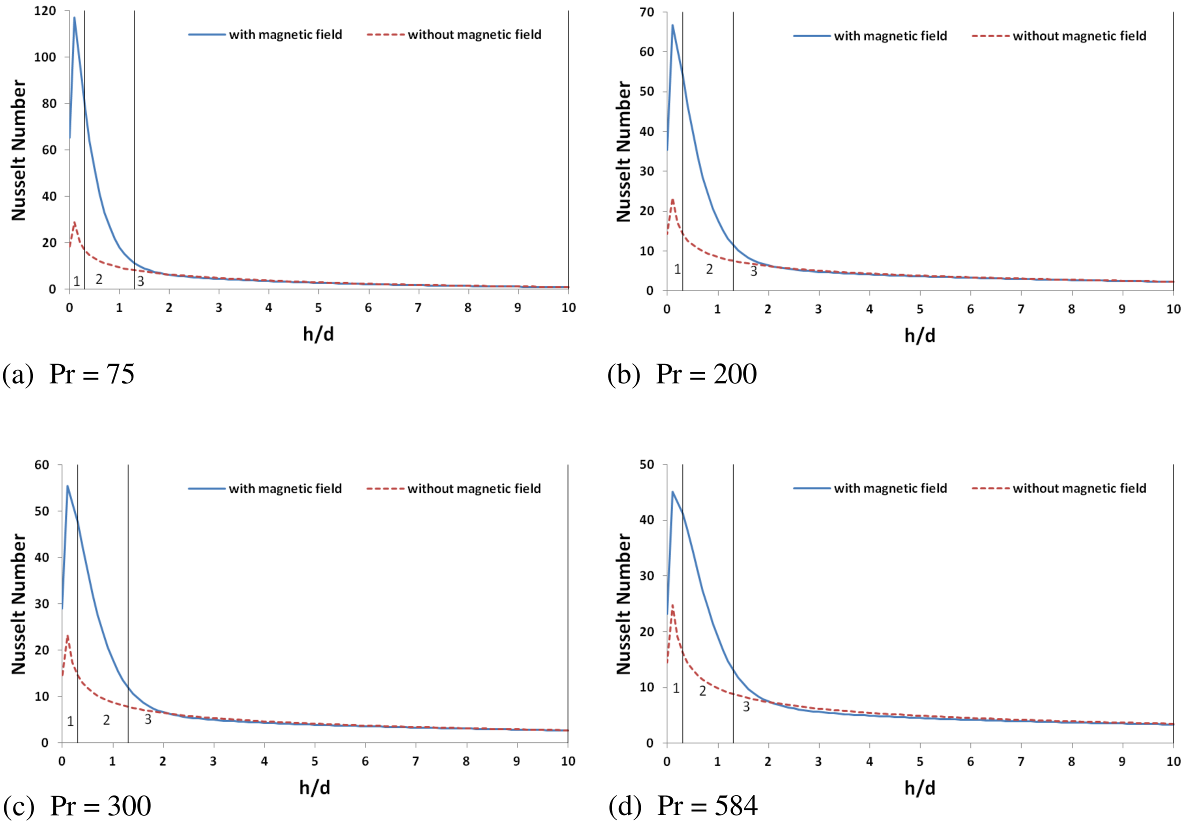
The following regions can be distinguished along the heated wall:
(1) inlet short region with the peak indicating the highest value of Nusselt number (section 1 – width of 0.3*d*),
(2) middle short region with the rapidly decreasing Nusselt number value (section 2 – width of 1*d*),
(3) outlet long region with slowly decreasing value of Nusselt number (section 3 – width of 8.7*d*).
The nomenclature shown in brackets will be used afterwards.
While studying figure 2 the most important conclusion is that the magnetic field possessed the positive impact on the flow heat transfer for all of the studied cases. The maximum Nusselt in section 1 number obtained for the case of Prandtl number Pr = 75 (figure 2(a)) for the variant with the magnetic field was over four times higher than for the variant without magnetic field. However, the disproportion between the values of Nusselt number with and without magnetic field decreased as the Prandtl number was increasing. In section 2 the Nusselt number value decreased faster in the case with applied magnetic field. In section 3 the course of Nusselt number for both cases is very similar. This effect is connected with the fact of declining impact of magnetic field.



(a) Pr = 75 (b) Pr = 200

(c) Pr = 300 (d) Pr = 584

Figure 2. Comparison between the Nusselt number calculated for the flow with and without magnetic field for various values of Prandtl numbers: (a) Pr = 75, (b) Pr = 200, (c) Pr = 300 and (d) Pr = 584.

Figure 3 presents the Nusselt number distributions along the heated wall for various values of Prandtl number for the flows under the influence of magnetic field. It was divided in the subfigures to prevail clarity. Obviously, the highest Nusselt numbers were obtained for the lowest values of Prandtl number. However, this tendency did not apply for the whole distribution. While the maximum value of Nusselt number decreased with the increase in the Prandtl number, the minimum value of it increased in section 3 (what cannot be distinguished from these figures). In Figure 3(d) the differences between various cases almost disappeared.

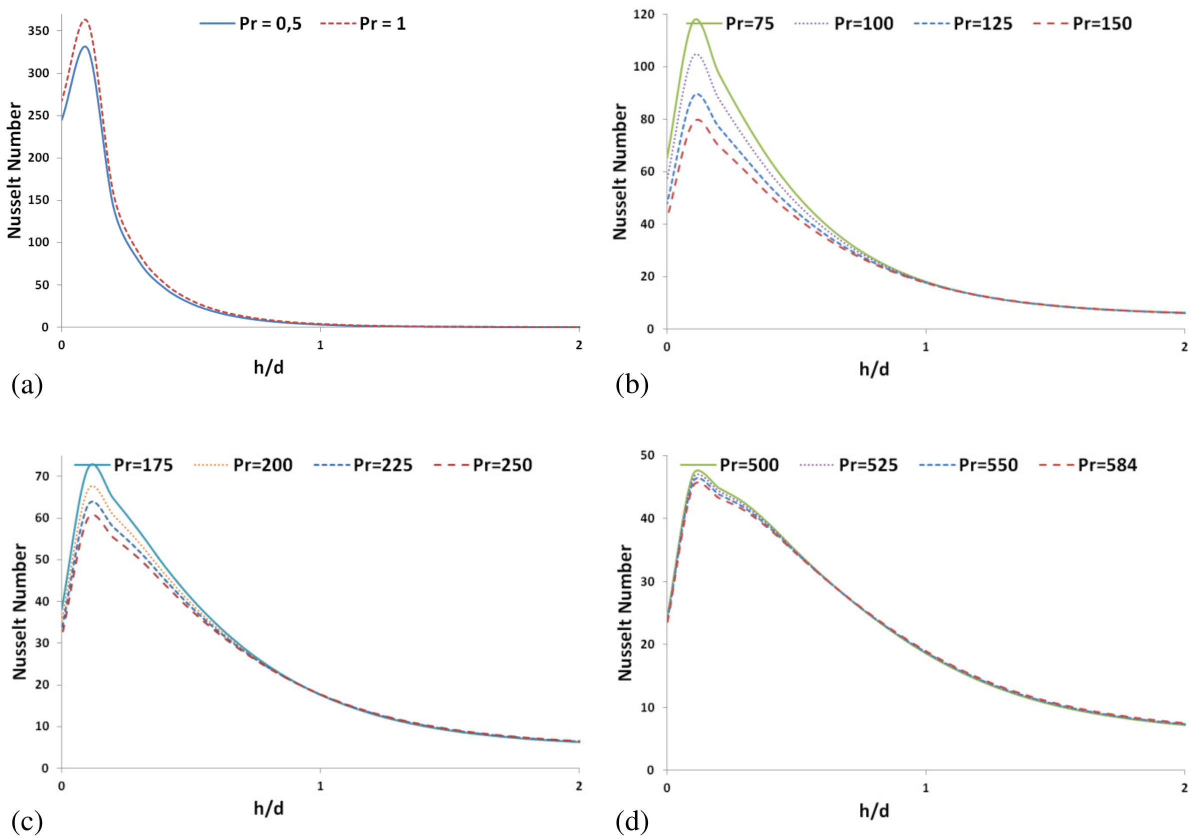


Figure 3. Distribution of the Nusselt number for the fluid flow under the influence of strong magnetic field for various values of Prandtl number: (a) Pr = 0.5 and Pr = 1, (b) Pr = 75, Pr = 100, Pr = 125 and Pr = 150, (c) Pr = 175, Pr = 200, Pr = 225 and Pr = 250, (d) Pr = 500, Pr = 525, Pr = 550 and Pr = 584.

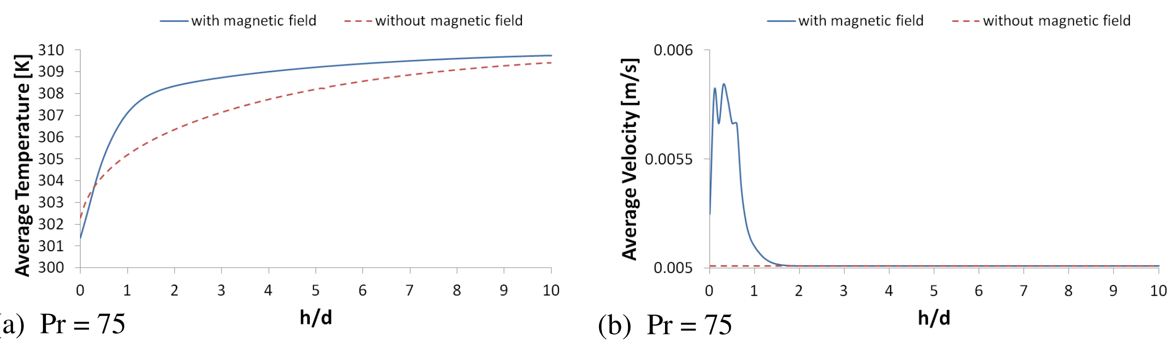
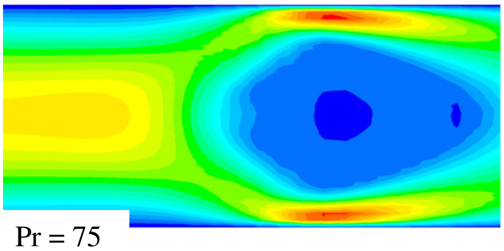
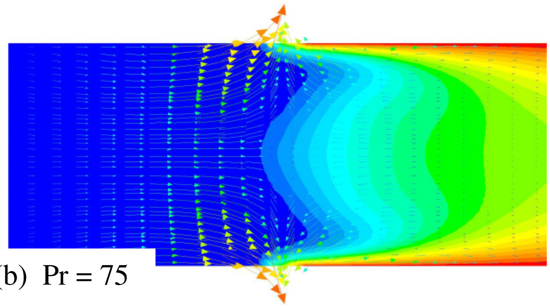


Figure 4. Distribution of the average temperature and average velocity for the Prandtl number equal to 75.



(a) Pr = 75



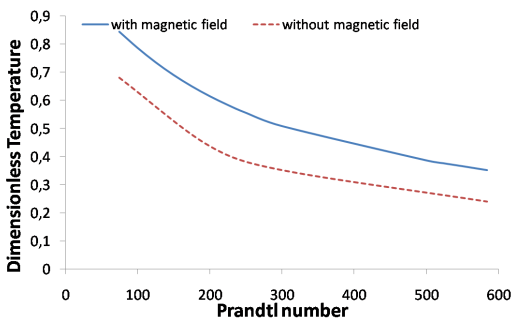
(b) Pr = 75

Figure 5. Enlargement of flow structure caused by the magnetic field in the area close to the magnetic coil: (a) velocity field, (b) temperature field and magnetic force vectors.

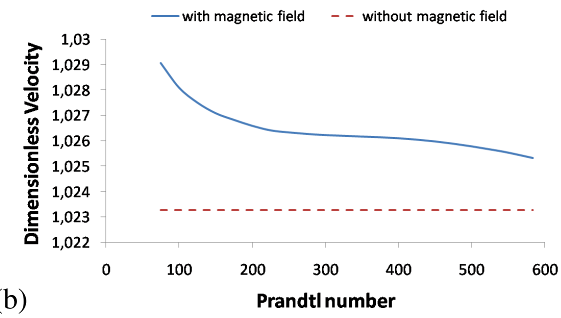
Figure 4 presents the distribution of average temperature (figure 4(a)) and velocity (figure 4(b)) along the heated wall for Pr = 75. It can be seen that the presence of magnetic field significantly increased both averaged values in the area close to the magnetic coil. In the case of temperature distribution this effect remained until the end of the duct.

Figure 5 presents the enlargement of flow structure close to the coil location under the influence of magnetic field. In the velocity field (figure 5(a)) three zones could be distinguished: acceleration zone near the wall, deceleration zone in the middle and recirculation zone in between. The temperature distribution (figure 5(b)) followed the velocity distribution with a similar "M" shaped profile. All of these changes were caused by the magnetic force acting on fluid (vectors shown in figure 5(b)). One should pay attention to the change of magnetic force direction, that was happening when the fluid crossed the reference temperature.

Figure 6(a) presents the change of the flow dimensionless bulk temperature ($T_d = (T_b - T_f)/(T_w - T_f)$) versus Prandtl number for the case without and with magnetic field. Figure 6(b) shows the change in dimensionless velocity ($\upsilon_d = \vec{\upsilon} / \upsilon_{avg}$) versus Prandl number for above mentioned cases. The enhancing and accelerating effect of magnetic field could be observed.



(a)



(b)

Figure 6. Distribution of the dimensionless bulk temperature (a) and dimensionless velocity (b) versus Prandtl number for the case with and without magnetic field.

5. Conclusions

The influence of magnetic field on the low Reynolds number flow was analysed, together with an effect of Prandtl number. Attention was paid to the understanding of heat transfer changes in accordance with the changes in studied parameters. The performed analyses showed that applying the magnetic force to low Reynolds number flow of paramagnetic fluid in the circular duct influenced the flow structure and the heat transfer. Regarding the flow structure, the characteristic "M-shape" profile was obtained, whereas an increased heat transfer rate was observed in all of the studied cases. However, the magnitude of this increment was dependent on Prandtl number value.

The analysis of the Nusselt number distribution for the flow with the magnetic field indicated that its changes along the heated wall were more rapid than in the case without magnetic field, what could be seen especially in the cases with lower Prandtl number.
The influence of magnetic field and increasing Prandtl number on flow bulk temperature and velocity was shown. In the case with applied magnetic field both of these parameters increased significantly.

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References

[1] Wrobel W 2011 Analiza wpływu silnego pola magnetycznego na procesy konwekcyjne płynów paramagnetycznych w układzie zamkniętym współosiowych powierzchni cylindrycznych, PhD Thesis, AGH University of Science and Technology
[2] Bednorz J G and Muller K A 1986 *Z. Phys. B. Con. Mat.* **64** 189
[3] Bednarz T 2004 Numerical and experimental analyses of convection of paramagnetic fluid in a cubic enclosure, PhD Thesis, Kyushu University
[4] Braithwaite D, Beaugnon E and Tournier R 1991 *Nature* **354** 134
[5] Wakayama N I and Sugie M 1996 *Physica B: Condens. Matter* **216** 403
[6] Wakayama N I and Wakayama M 2000 *Jpn. J. Appl. Phys.* **39** 262
[7] Wakayama N I 1991 *Chem. Phys. Lett.* **185** 449
[8] Uetake H, Nakagawa J, Horota N and Kitazawa K 1999 *J. Appl. Phys.* **85** 536
[9] Ikezoe Y, Hirota N, Nakagawa J and Kitazawa K 1998 *Nature* **393** 749
[10] Tagawa T, Shigemitsu R and Ozoe H 2002 *Int. J. Heat Mass Tran.* **45** 267
[11] Tagawa T, Ujihara A and Ozoe H 2003 *Int. J. Heat Mass Tran.* **46** 4097
[12] Bai B, Yabe A, Qi J, Wakayama N I 1999 *AIAA J.* **37** 1538
[13] Fornalik-Wajs E, Filar P, Wajs J, Roszko A and Pleskacz L 2014 *J. Phys. Conf. Ser.* **530** 012041
[14] Wrobel W A, Fornalik-Wajs E and Szmyd J S 2012 *J. Phys. Conf. Ser.* **395** 012124
[15] Kraszewska A 2015 An analysis of thermo-magnetic convection of paramagnetic fluid in rectangular enclosure Conference: *Konferencja „Innowacyjne pomysły młodych naukowców: Nauka – Startup – Przemysł”* Book of Abstracts 47
[16] Roszko A and Fornalik-Wajs E 2015 *Trans. IFFM* **128** 29
[17] Kenjeres S, Wrobel W A, Pyrda L, Fornalik-Wajs E and Szmyd J S 2014 *Flow Turbul. Combust.* **92** 371
[18] Ueno S and Iwasaka M 1994 *J. Appl. Phys.***75**1.356686
[19] Ozoe H 2005 Magnetic Convection (Imperial College Press)
[20] Pleskacz L and Fornalik-Wajs E 2014 *J. Phys. Conf. Ser.* **530** 012062
[21] Pleskacz L, Fornalik-Wajs E and Roszko A 2014 *ZN PRz Mechanika* **31** 425
[22] Papoutsakis E, Ramkrishna D and Lim H C 1980 *Appl. Sci. Res.* **36** 13
[23] Weigand B and Eisenschmidt K 2012 *Int. J. Therm. Sci.* **54** 89