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To cite this article: Chaoxin Wang et al 2016 J. Phys.: Conf. Ser. 744 012006

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Active vibration isolation through a Stewart platform with piezoelectric actuators

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Abstract. A Stewart platform with piezoelectric actuators is presented for micro-vibration isolation. The Jacobian matrix of the Stewart platform, which reveals the relationship between the position/pointing of the payload and the extensions of the struts, is derived by the kinematic analysis and modified according to measured FRFs (frequency response function). The dynamic model of the Stewart platform is established by the FRF synthesis method to accommodate flexible modes of the platform. In active isolation, the LMS-based adaptive method is adopted and combined with the Jacobian matrix to suppress pure vibrations of the payload. Numerical simulations and experiments were conducted to prove vibration isolation performance of the Stewart platform subjected to periodical disturbances, and the results have demonstrated that considerable attenuations can be achieved.

1. Introduction

Micro-vibration usually refers to low-level mechanical vibration or disturbance, typically occurring at frequencies from less than 1 Hz to 1 kHz. It has significant influence on space missions such as Earth observation and laser communication by degrading the performance of sensitive devices [1-4]. Stewart platforms are widely used in precision systems to suppress vibrations. However, the kinematics and dynamics of Stewart platforms are complicated because the motions of all legs are coupled. The cubic configuration is weakly coupled in comparison to other configurations and it allows decoupled control and permits six-axis positioning/pointing of the payload [5]. In addition, the cubic configuration is of uniform stiffness and cross-coupling amongst actuators, simple kinematics as well as mechanical design. This is the reason that the cubic configuration is popular in the field of vibration control [6].

Geng proposed an active vibration isolation system with the Stewart platform and adopted robust adaptive filter algorithms for active vibration control [7]. Rahman proposed a six-axis orthogonal hexapod mount for vibration isolation, suppression and steering in space observational systems. The results show that the active stage reduces vibration propagation by additional attention in the mid frequency range 10-100 Hz compared to the passive-only cases [8]. Richard developed a vibration isolation and suppression system that utilizes hybrid isolation struts in a hexapod configuration. The active system was used to enhance vibration isolation performance at lower frequencies and to steer the payload [9]. Hanieh investigated an active interface wherein a six-degree-of-freedom Stewart
platform was used to actively increase damping of attached flexible systems. This active interface can
be used as a 6-dof positioning and steering device for space applications as well as micro-vibration
isolation [10]. Thayer developed a unique system that it had very soft axial stiffness (3-Hz corner
frequency) for active isolation and control. Experimental closed-loop control results have shown that
there are 20-25dB reduction in vibration in all six degrees of freedom across the bandwidth of interest
(5-20Hz) [6]. Agrawal adopted the multiple error least mean square algorithm and the clear box
algorithm for active vibration isolation using a Stewart platform. Based on the experimental result, it is
concluded that multiple error LMS is preferred for vibration isolation when a disturbance correlated
signal is available. In the absence of such a signal, the clear box algorithm is the method of choice [11].
Han investigated the dynamic model of a 6-axis hybrid vibration isolation system with a cubic
configuration by the Newton-Euler approach and presented numerical simulation results on vibration
isolation characteristics of the 6-dof hybrid platform in the frequency domain [12].

However, the vibration model of the Stewart platform still has room for improvement. Most of the
modelling methods such as New-Euler method, D’ Alembert principle, Kane’s method treat the struts,
the payload and the base as rigid bodies. In this paper, for the purpose of vibration analysis and control,
the strut is regarded as a beam while the payload and the base are taken as flexible bodies. The FRF
synthesis method is adopted to synthesize the substructures to derive FRFs of the Stewart platform.
Numerical simulations are given to prove vibration isolation performance of the Stewart platform
under the excitation of periodical disturbances. Experimental results are also given to demonstrate the
isolation performance of the Stewart platform.

Six sections are organized in this paper. The Jacobi matrix of the Stewart platform is derived in
Section 2, which reveals the relationship between the extensions of struts and the position-pointing of
the payload. Section 3 discusses the modelling of the Stewart platform. The base and the payload are
modelled by plate elements and the struts are regarded as beam elements. FRFs from the base to the
payload are obtained through the FRF-based model synthesis method and the results are verified by
FEM. Section 4 presents an FxLMS based adaptive control scheme. Simulation results are given to
illustrate isolation performance of the platform under periodical. Experimental results are given in
Section 5 and conclusions are summarized in Section 6.

2. Kinematic analysis of the Stewart platform

Determining the position and attitude of the payload in terms of the length of struts is the direct
problem in the design of a Stewart platform. On the contrary, determining the length of struts in terms
of given coordinates (x,y,z) and angles (α,β,γ) is the inverse problem.

![Coordinate systems of the base and payload](image)

Figure 1 Coordinate systems of the base and payload

The position and attitude of a Stewart platform is usually described by Euler angles, but this
method is not practical since the angles between a vector and coordinate axes have to be derived.
Hence, the Rodrigues’ rotation formula is employed because the rotation matrix can be obtained
through simple mathematical operations when a vector and its rotation vector are given [13]. The
govern equation is

$$
R = \cos(\theta)I + (1-\cos(\theta))rr^T + \sin(\theta) 
\begin{bmatrix}
0 & -r_z & r_y \\
r_z & 0 & -r_x \\
-r_y & r_x & 0
\end{bmatrix}
$$

(1)
where, $I$ is the unit matrix, and $r = \begin{bmatrix} r_x & r_y & r_z \end{bmatrix}$ is the dot product of a vector and its rotation vector. $	heta$ is the angle between two vectors.

The coordinate systems and vectors are defined in Figure 1. $O_p$ and $O_b$ are selected as reference points respectively on the payload and the base, on which two Cartesian systems $P$ and $B$ can be defined. The coordinate system $B$ is stationary and the system $P$ is set up on $B$. Moreover, $h = (x, y, z)$ is the vector from $O_b$ to $O_p$. The vectors from $O_p$ to the connecting points of the payload and the struts are $p_i$ ($i = 1, 2, 3, 4, 5, 6$). The vectors from $O_b$ to the connecting points of the base and the struts are $b_i$. The vectors from the connecting points of the base to the connecting points of the payload and the struts are $q_i$. The unit vectors from the connecting points of the base to the connecting points of the payload and the struts are $s_i$. The following relationship exists [14].

$$s_i = (R_{p_i} + h - b_i) / \| R_{p_i} + h - b_i \|$$

(2)

The Jacobi matrix $J$ relates the extension velocity $\dot{D}_i$ of the $i$th strut with the velocity vector $\chi = (v^T \omega^T)$. Defining $\omega = \dot{\theta}$, $v = h$, there exist $q_i = R_{p_i}$ and the result is

$$\dot{D}_i = s_i^T (v + \omega \times q_i) = \begin{bmatrix} s_i^T (q_i \times s_i)^T \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} = J\chi$$

(3)

According to Equation (3), the pure displacement in $X$, $Y$ and $Z$ directions can be calculated (The pure displacement is defined as the maximum displacement the payload can reach in one direction while the displacements in other directions are zero). The relationship between the extension of struts and the pure displacement of the Stewart platform in $X$, $Y$ and $Z$ directions are listed in Table 1, in which $s$ is the maximum extension of struts and $\Delta L_i$ is the extension of the $i$th strut.

<table>
<thead>
<tr>
<th>Item</th>
<th>$\Delta L_1$</th>
<th>$\Delta L_2$</th>
<th>$\Delta L_3$</th>
<th>$\Delta L_4$</th>
<th>$\Delta L_5$</th>
<th>$\Delta L_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_{pure}$</td>
<td>$\sqrt{3}s/2$</td>
<td>$s/2$</td>
<td>$-s$</td>
<td>$s/2$</td>
<td>$s/2$</td>
<td>$-s$</td>
</tr>
<tr>
<td>$y_{pure}$</td>
<td>$s$</td>
<td>$-s$</td>
<td>$0$</td>
<td>$s$</td>
<td>$-s$</td>
<td>$0$</td>
</tr>
<tr>
<td>$z_{pure}$</td>
<td>$s$</td>
<td>$s$</td>
<td>$s$</td>
<td>$s$</td>
<td>$s$</td>
<td>$s$</td>
</tr>
</tbody>
</table>

Table 1 The relationship between the extension of struts and the pure displacement

3. Modelling of the Stewart platform
The mechanical structure of the Stewart platform with cubic configuration is shown in Figure 2. It can be seen that the platform includes three parts, i.e. the base, the struts and the payload. The struts in fact are piezoelectric actuators used to isolate vibration transmission from the base to the sensitive payload. The cubic configuration can be assumed unchanged in actuation due to the micro-extension of each strut.

![Figure 2 The Stewart platform](image1)

Figure 2 The Stewart platform  Figure 3 The simplified model of a piezoelectric strut

3.1 FRFs of the base and the payload
The dynamic models of the base and the payload are expressed by FRFs, as given in Equation (4), where the subscript $p$ denotes the payload and the subscript $b$ denotes the base.
\[
\begin{bmatrix}
X_{p,I} \\
X_{p,O}
\end{bmatrix} =
\begin{bmatrix}
H_{pI} & H_{pIO} \\
H_{pOI} & H_{pO}
\end{bmatrix}
\begin{bmatrix}
F_{p,I} \\
F_{p,O}
\end{bmatrix}
\begin{bmatrix}
X_{b,I} \\
X_{b,O}
\end{bmatrix} =
\begin{bmatrix}
H_{bI} & H_{bIO} \\
H_{bOI} & H_{bO}
\end{bmatrix}
\begin{bmatrix}
F_{b,I} \\
F_{b,O}
\end{bmatrix}
\] (4)

Note that \( \mathbf{F} \) represents the external force, of which the first subscript stands for the substructure and the second one for internal nodes or boundary nodes, for example, \( \mathbf{F}_{p,I} \) denotes the force acting on the internal nodes of the payload. The same notation applies to the displacement \( \mathbf{X} \) as well. Furthermore, \( \mathbf{H} \) indicates the mobility matrix, where the first subscript stands for the substructure and the following two subscripts stand for internal nodes or boundary nodes.

### 3.2 Modelling of the piezoelectric strut

The Stewart platform consists of six piezoelectric struts and twelve elastic hinges, which connect the struts with the base and the payload. Compared to the axial stiffness, the lateral stiffness of the hinges is relatively small. Therefore, in the modelling, the hinges are idealized as spherical joints, but the struts are described by beam elements (Euler-Bernoulli beam) in order to consider lateral flexibility.

#### 3.2.1 Axial impedance matrix of the piezoelectric strut

For longitudinal vibration in the Z direction (Figure 3), the following four-end-parameter model can be derived [15]:

\[
\begin{bmatrix}
F_x \\
Z_o
\end{bmatrix} =
\begin{bmatrix}
\cos(\beta L) & -\frac{1}{E A B} \sin(\beta L) \\
E A B \sin(\beta L) & \cos(\beta L)
\end{bmatrix}
\begin{bmatrix}
F_z \\
Z_o
\end{bmatrix},
\] (5)

where \( F_z, Z_o \) denote the input force and the input displacement, respectively, \( F_{zo}, Z_o \) denote the output force and the output displacement, respectively, \( E \) is the Young's modulus, \( A \) is the cross-section area, \( L \) is length of the strut, \( \rho \) is the density of mass, \( \beta \) is the wavenumber and \( \beta = \omega \sqrt{\rho / E} \).

Let \( a_{d11} = \cos \beta L, a_{d12} = -\sin \beta L / \beta EA, a_{d21} = \beta EA \sin \beta L \) and \( a_{d22} = \cos \beta L \), the impedance matrix \( Z_z \) can be derived from Equation (5):

\[
Z_z = \begin{bmatrix}
-a_{d11} & \frac{1}{a_{d12}} \\
-a_{d21} & \frac{a_{d22}}{a_{d12}}
\end{bmatrix}
\] (6)

#### 3.2.2 Lateral impedance matrix of the piezoelectric strut

For lateral vibration in the X direction (Figure 3), the transfer matrix of a uniform Ruler-Bernoulli beam can be expressed as follows [15]:

\[
\begin{bmatrix}
X \\
\Theta \\
M_z \\
F_{L, z}
\end{bmatrix} =
\begin{bmatrix}
S(\lambda x) & T(\lambda x) & U(\lambda x) & V(\lambda x) \\
\lambda V(\lambda x) & S(\lambda x) & T(\lambda x) & U(\lambda x) \\
E \lambda I U(\lambda x) & E \lambda I V(\lambda x) & S(\lambda x) & T(\lambda x) \\
E \lambda I T(\lambda x) & E \lambda I U(\lambda x) & \lambda V(\lambda x) & S(\lambda x)
\end{bmatrix}
\begin{bmatrix}
X \\
\Theta \\
M_z \\
F_{L, z}
\end{bmatrix}
\] (7)

where \( \lambda = \sqrt{\bar{m} \omega^2 / (EI)} \), \( EI \) denotes the bending stiffness and \( \bar{m} \) is the line mass density, and \( S, U, V \) and \( T \) are defined as:

\[
S(\lambda x) = \frac{\cosh \lambda x + \cos \lambda x}{2}, \quad U(\lambda x) = \frac{\cosh \lambda x - \cos \lambda x}{2}, \quad V(\lambda x) = \frac{\sinh \lambda x - \sin \lambda x}{2}, \quad T(\lambda x) = \frac{\sinh \lambda x + \sin \lambda x}{2}
\] (8)

\[4\]
Substitute the boundary conditions $M_{z,\theta} = 0$ and $M_{z,f} = 0$ (spherical hinge) into Equation (7) to eliminate the variables $\Theta$ and $M_z$ for condensation of freedom, we can have

$$
\begin{bmatrix}
X_d \\
F_{a,\theta, \theta}
\end{bmatrix}
= 
\begin{bmatrix}
\frac{a_{i1}}{a_{i2}} - \frac{a_{i3}a_{i31}}{a_{i2}} & \frac{a_{i4}a_{i34}}{a_{i2}} \\
\frac{a_{i2}}{a_{i3}} - \frac{a_{i4}a_{i32}}{a_{i3}} & \frac{a_{i4}a_{i33}}{a_{i3}}
\end{bmatrix}
\begin{bmatrix}
X_d \\
F_{a,\theta}
\end{bmatrix}
\tag{9}
$$

Therefore, the impedance matrix $Z_\theta$ can be deduced from Equation (9). Equation (10) gives the matrix impedance and its elements are computed according to $a_{d11} = a_{i1} - \frac{a_{i3}a_{i31}}{a_{i2}}$, $a_{d12} = a_{i4}a_{i34}$, $a_{d21} = a_{i1} - \frac{a_{i3}a_{i31}}{a_{i3}}$, and $a_{d22} = a_{i3}a_{i32}$.

Similarly, the impedance matrix $Z_y$ can be derived, and it is also given in Equation (10). For the piezoelectric strut, $Z_x = Z_y$.

$$
Z_y = 
\frac{a_{d11}}{a_{d21}} - \frac{1}{a_{d12}}, \quad Z_x = 
\frac{1}{a_{d12}}
\tag{10}
$$

3.2.3 Complete impedance matrix of the piezoelectric strut

Given the impedance matrices of the strut in X, Y and Z directions, the complete impedance matrix can be obtained by assembling the impedance matrices in the local coordinate system. Denote the forces and displacements by

$$
F_d = \begin{bmatrix}
F_d \\
F_{a,\theta}
\end{bmatrix}, \quad X_d = \begin{bmatrix}
X_d \\
X_{a,\theta}
\end{bmatrix}
\tag{11}
$$

then the impedance matrix can be expressed by a partition matrix:

$$
\begin{bmatrix}
F_d \\
F_{a,\theta}
\end{bmatrix}
= 
\begin{bmatrix}
Z_{d11} & Z_{d12} \\
Z_{d21} & Z_{d22}
\end{bmatrix}
\begin{bmatrix}
X_d \\
X_{a,\theta}
\end{bmatrix}
\tag{12}
$$

Since the impedance matrix in Equation (12) is expressed in the local coordinate system, it needs to be converted to the global coordinate system. Let $Z_{d,i}$ and $Z_{b,i}$ be the impedance matrices in the local and global systems, respectively and $R$, the rotation matrix associated with the $i$th strut, the impedance matrix of the $i$th strut in the global coordinate system can be given as

$$
\begin{bmatrix}
F_{b,i} \\
F_{a,\theta,i}
\end{bmatrix}
= 
\begin{bmatrix}
R & O_{a,3} \\
O_{b,3} & R
\end{bmatrix}
\begin{bmatrix}
Z_{d_{21}} & Z_{d_{22}} \\
Z_{d_{11}} & Z_{d_{12}}
\end{bmatrix}
\begin{bmatrix}
R & O_{a,3} \\
O_{b,3} & R
\end{bmatrix}
\begin{bmatrix}
X_{b,i} \\
X_{a,\theta,i}
\end{bmatrix}
= 
\begin{bmatrix}
Z_{d_{21}} & Z_{d_{22}} \\
Z_{d_{11}} & Z_{d_{12}}
\end{bmatrix}
\begin{bmatrix}
X_{b,i} \\
X_{a,\theta,i}
\end{bmatrix}
\tag{13}
$$

3.3 FRF synthesis via elastic media[16]

The adopted synthesis method in this paper is based on the assumption that two substructures are connected by elastic elements. In this paper, all the beams described by Equation (13) are regarded as elastic elements. Hence, FRFs from the base to the payload can be obtained by synthesizing the two substructures via the beam elements.

Figure 4 The Stewart platform and its dynamic model
In Figure 4, A represents the payload and B represents the base, and they are modelled by plate elements. There are three connecting points and one inner point on Substructure A and Substructure B, respectively. The piezoelectric struts are modelled by beam elements. A and B are connected by the beams. FRFs of all substructures can be computed by FEM.

In Figure 4, $i$ represents the inner point, $c$ represents the connection point, $\bar{c}$ is the displacement of the connection point on A, $\bar{c}$ is the displacement of the connection point on B.

For two connecting points, the forces and displacements should satisfy $f = ZDx$, where $ZD$ is the impedance matrix, $f = [f_A, f_B]^T$ is the force vector, $x = [x_A, x_A, x_B]^T$ is the displacement vector.

According to the conditions of displacement continuity and force equilibrium, there exist

$$
\begin{cases}
X_A = x_A, \quad X_B = x_B, \quad X_c = x_c
\end{cases}
$$

where $[f_A, f_B]^T$ and $[x_c, x_c]^T$ are the external force and displacement vectors of the connecting points, respectively.

FRFs of Substructures A and B can be obtained according to Equation(4), the boundary conditions are given by Equation(14), and the mechanical impedance of the strut is given by Equation(13). Therefore, the synthesized FRFs from A to B can be written as

$$
\begin{bmatrix}
X_A \\
X_B \\
X_c
\end{bmatrix} =
\begin{bmatrix}
H_{ii} & H_{iA} & H_{iC} \\
H_{iA}^T & H_{AA} & H_{AC} \\
H_{iC}^T & H_{AC}^T & H_{CC}
\end{bmatrix}
\begin{bmatrix}
F_A \\
F_B \\
F_c
\end{bmatrix}
$$

where $X, H$ and $F$ represent displacements, FRFs and forces associated with the Stewart platform, $I$ represents the inner point, $C$ represents the connecting point, $X_i$ is the displacement vector of the inner point on the A, $F_i$ is the force vector of the inner point on the A, $H_{iA}$ is the FRF matrix between the inner point on the A and the connecting point on the B. $H_{AC}$ and $H_{AC}^T$ is the FRF between the driving forces of the struts and the displacements of the inner points on A.

### 3.4 Verification of the synthesized model

In order to verify the FRF synthesized method, an FE model of the Stewart platform is built. In the model, the distance between the payload and the base is 0.098m. Plate elements are used to discretize the base and the payload and the material is aluminum. The struts are modelled by beam elements and the length of each strut is 0.169m. The points connecting the struts with the payload and the base are located on a circle of radius 0.138m and the connection is simulated with ball joints. The base plate is fixed at three points, which are on a circle of radius 0.12m. Forced responses of the Stewart platform to a unit force are analyzed in the frequency range 1-500Hz with resolution of 1Hz. The unit force is loaded on the base. FRFs from the center of the base to the center of the payload are computed and plotted in Figure 5.
For the purpose of comparison, FRFs computed by the synthesis method are also plotted in Figure 5. It can be seen that the curves given by the two different methods coincide well with each other. Therefore, the synthesis method is effective and can be used to establish a dynamic model of the Stewart platform for further investigation of vibration isolation.

4. Adaptive control

4.1 FxLMS-based adaptive control

Figure 6 gives the diagram of an adaptive control system, where $\hat{H}(z)$ is the identified control channel model, $d(n)$ is the disturbance signal, $y(n)$ is the output of the control channel, $e(n)$ is the control error, $e(n)=d(n)-y(n)$, $\hat{d}(n)$ is the observed disturbance signal, $W(z)$ is the controller for the FIR filter followed by the saturation unit to limit the output amplitude of the controller. The coefficient of the controller $W(z)$ is $\theta=\{\theta_0, \theta_1, ..., \theta_{n-1}\}^T$, which can be adjusted according to

$$\theta(n+1)=S_u\left(\theta(n)+\mu\frac{e(n)p(n)}{\gamma+||p||^2}\right)$$

(16)

where $p(n)=[z(n), z(n-1), ..., z(n+1-\alpha)]^T$, $S_u$ is the first derivative of the Sigmoid function, $0<\mu<1$, $\gamma>0$, $u(n)=S(\theta^T(n)r(n))$ is the control signal, $r(n)=[\hat{d}(n), \hat{d}(n-1), ..., \hat{d}(n+1-\alpha)]^T$. In Figure 6, $\hat{d}(n)$ is the observed disturbance signal, which is used as the reference signal. It should be pointed out that the control channel model is depended on specified responses to be controlled.

![Figure 6 The adaptive control diagram](image)

4.2 Numerical simulation

Suppose a vertical force is exerted on the center of the base, and the force is composed of two components, i.e. $10\sin(60\pi t)+5\sin(140\pi t)$ N. Use the adaptive control method given by Equation (16) and the FRFs given in Section 3.3 to simulate the effectiveness of active suppression. The results are plotted in Figure 7. As can be seen, attenuations of acceleration responses in the X, Y, Z directions are prominent. To give a detailed comparison, RMS values of the responses are calculated. Before control, the RMS values are respectively 4.38mg, 1.09ug and 0.337ug in the Z, Y, X directions. After control, the values become respectively 0.0506mg, 0.027ug and $4.48\times10^{-4}$ug.

![Figure 7 Comparison of responses before/after control](image)
5. Experiments

5.1 The experimental system and background vibration

The experimental system is shown in Figure 8. It can be seen that the Stewart platform is mounted on a rigid table, which is placed in a plexiglass container filled with soft sand and supported by rubber isolators. Three accelerometers, denoted by $P_1$, $P_2$ and $P_3$, are mounted at the center of the payload to measure translational vibrations in the Z, X and Y directions. Acceleration response signals are conditioned by a charge amplifier, which is connected respectively to a controller and a signal analysis system. The controller is responsible for processing the acceleration signals and sending command signals to the power amplifier of the struts. The rigid table is excited by a shaker, which responds to a signal source embedded in the signal analysis system. Isolation with the plexiglass container and the rigid table is to reduce the influence of ground vibration on the platform as much as possible.

![Figure 8 Configuration of the experiment and the experimental platform](image)

The recorded background vibration is shown in Figure 9, which indicates that the amplitude of vibration is less than 10$\mu$g, and the peak of PSD is 28.1$\mu$g$^2$/Hz in the frequency range of 5-130Hz. This level of vibration is enough for the experiment.

![Figure 9 Background vibration](image)

5.2 Modification of the Jacobi matrix

The theoretical relationship between the extensions of the struts and the translational displacements are listed in Table 1. However, the Jacobi matrix needs to be refined since the experimental model is not strictly uniform or symmetrical. FRFs associated with the pure displacements in the X, Y, Z axes were measured and plotted in Figure 10. It can be seen that vibration in the primary direction is at least 15dB larger than that in other directions. Based on these measurements, the Jacobi matrix was refined and given in Table 2.

![Figure 10 Pure vibration in the three axes](image)
Table 2 The refined Jacobi matrix

<table>
<thead>
<tr>
<th>Item</th>
<th>ΔL₁</th>
<th>ΔL₂</th>
<th>ΔL₃</th>
<th>ΔL₄</th>
<th>ΔL₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>x&lt;sub&gt;pure&lt;/sub&gt;</td>
<td>0.3s</td>
<td>0.6s</td>
<td>-0.8s</td>
<td>0.6s</td>
<td>0.7s</td>
</tr>
<tr>
<td>y&lt;sub&gt;pure&lt;/sub&gt;</td>
<td>0.8s</td>
<td>-1.25s</td>
<td>-0.2s</td>
<td>0.9s</td>
<td>-1.25s</td>
</tr>
<tr>
<td>z&lt;sub&gt;pure&lt;/sub&gt;</td>
<td>0.7s</td>
<td>1.1s</td>
<td>1s</td>
<td>0.7s</td>
<td>1.1s</td>
</tr>
</tbody>
</table>

5.3 Active vibration isolation under periodical disturbances

The Stewart platform should have the ability to suppress periodical disturbances. In the experiment, disturbances composed of different harmonics were tested. Here, only one circumstance is presented as an example. Figure 11 illustrates the results of control of responses composed mainly of 30Hz and 70Hz components, which are induced by vibrations in the base. It can be seen that the dominant peaks in the spectra are substantially suppressed. Attenuations of the two peaks are listed in Table 3. In addition to the tonal responses, the natural vibration of the base at about 14Hz is also excited by the shaker. Moreover, in the X-axis response spectrum, there is a component at 140Hz, which is caused by the weak nonlinearity of the shaker or the platform.

![Figure 11](image)

Table 3 Attenuations in the three axes (reference = 1 m/s)

<table>
<thead>
<tr>
<th>Frequency(Hz)</th>
<th>Item</th>
<th>Z(dB)</th>
<th>Y(dB)</th>
<th>X(dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>Without control</td>
<td>-91.72</td>
<td>-78.41</td>
<td>-101.13</td>
</tr>
<tr>
<td>70</td>
<td>With control</td>
<td>-113.42</td>
<td>-102.67</td>
<td>-114.78</td>
</tr>
<tr>
<td>70</td>
<td>Attenuations</td>
<td><strong>21.70</strong></td>
<td><strong>24.26</strong></td>
<td><strong>13.65</strong></td>
</tr>
<tr>
<td>30</td>
<td>Without control</td>
<td>-92.13</td>
<td>-97.98</td>
<td>-90.62</td>
</tr>
<tr>
<td>30</td>
<td>With control</td>
<td>-113.96</td>
<td>-108.49</td>
<td>-113.88</td>
</tr>
<tr>
<td>30</td>
<td>Attenuations</td>
<td><strong>21.83</strong></td>
<td><strong>10.51</strong></td>
<td><strong>23.26</strong></td>
</tr>
</tbody>
</table>

6. Conclusions

A Stewart platform with piezoelectric actuators is presented in this paper and the modelling, dynamics as well as vibration control are investigated by simulation and experiment. The FRF based modelling method takes the platform as a flexible system to consider inherent modes of the structure. Control of the displacements of the payload is based on the Jacobi matrix, which relates the extensions of struts with the position/attitude of the payload and is refined according to FRF measurements. The FxLMS based adaptive cancellation is employed in the active isolation to attenuate periodical responses of the
payload induced by vibration in the base. Simulation and experiments were conducted to verify the effectiveness of active isolation of micro-vibrations with the Stewart platform.

References