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# Understanding dynamics of large-scale atmospheric vortices with moist-convective shallow water model

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# Understanding dynamics of large-scale atmospheric vortices with moist-convective shallow water model

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**Abstract.** Atmospheric jets and vortices which, together with inertia-gravity waves, constitute the principal dynamical entities of large-scale atmospheric motions, are well described in the framework of one- or multi-layer rotating shallow water models, which are obtained by vertically averaging of full "primitive" equations. There is a simple and physically consistent way to include moist convection in these models by adding a relaxational parameterization of precipitation and coupling precipitation with convective fluxes with the help of moist enthalpy conservation. We recall the construction of moist-convective rotating shallow water model (mcRSW) model and give an example of application to upper-layer atmospheric vortices.

## 1. Introduction

We study instabilities of idealised atmospheric upper-layer vortices of the type of cut-off lows frequently happening in the atmosphere, and the role of humidity, condensation and related moist convection upon their saturation. We use the simplest, albeit retaining all essential properties, moist-convective shallow shallow-water model and compare behavior of the vortices in "dry" and "moist-precipitating" environments, with the moisture being a passive tracer (M) and having a condensation sink (MP) which creates a moist-convective vertical flux in the latter. Adding evaporation source gives moist-precipitating-evaporating (MPE) configuration. The approach follows a similar study of atmospheric jets [1]. Condensation, having a switch character, is an essentially nonlinear phenomenon, and hence the techniques of linear stability analysis are inapplicable in moist-precipitating configurations. So we perform linear stability analysis of "dry" vortices, and then use the obtained unstable modes to initialize numerical simulations of both "dry" and moist-precipitating (and evaporating) saturation of the instability. We use pseudospectral collocation method in polar coordinates [2] for linear stability analysis, and well-balanced high-resolution finite-volume numerical scheme, adapted for mcRSW in [3], for nonlinear simulations.

## 2. The model and the vortex configuration

Equations of the two-layer mcRSW introduced in [3] read:

$$\begin{gathered}\frac{D\boldsymbol{v}_1}{Dt} + f\hat{z}\times\boldsymbol{v}_1 = -g\bigtriangledown_H(h_1+h_2),\ \frac{D\boldsymbol{v}_2}{Dt} + f\hat{z}\times\boldsymbol{v}_2 = -g\bigtriangledown_H(h_1+sh_2) + \frac{\boldsymbol{v}_1-\boldsymbol{v}_2}{h_2}\beta P_1 \\ \frac{Dh_1}{Dt} + \bigtriangledown.(h_1\boldsymbol{v}_1) = -\beta P_1,\ \frac{Dh_2}{Dt} + \bigtriangledown.(h_2\boldsymbol{v}_2) = +\beta P_1,\ \partial_t Q_1 + \bigtriangledown.(Q_1\boldsymbol{v}_1) = -P_1 + E.\end{gathered} \tag{1}$$

$D/Dt$ is material derivative, $\boldsymbol{v}_i$ - horizontal velocity in layer $i = 1, 2$, $f$ is Coriolis parameter, $h_i$ - layer thicknesses, $\theta_i$ - potential temperatures, $s = \frac{\theta_2}{\theta_1} > 1$ - stratification. The model is obtained from the primitive equations by vertical averaging between the material surfaces in isobaric pseudo-height coordinates and adding additional convective flux due to the latent heat release. This convective flux is linked to the water-vapor condensation in the lower humid layer $P_1$ via moist enthalpy conservation, and gives a sink (source) in the mass conservation equation in the lower (upper) layer. Due to the convective mass exchange this flux leads to the Rayleigh drag in the upper-layer momentum equation. Condensation provides a sink in the bulk moisture content $Q_1$ in the lower layer, while evaporation $E$ gives a source. Condensation is parameterised as follows: $P_1 = \frac{Q_1 - Q^s}{\tau} H(Q_1 - Q^s)$, where $H(.)$ is step function, and $Q^s$ is a saturation value. A parameterisation $E = \delta.|\vec{\mathbf{v}}|$ crudely corresponds to a vortex system evolving over the ocean.

Writing the "dry" two-layer system in polar coordinates it is easy to see that flow configurations in cyclo-geostrophic equilibrium, i.e. the equilibrium between centrifugal, Coriolis and pressure forces:

$$(\frac{V_1}{r} + f)V_1 = g\partial_r (H_1 + H_2), \quad (\frac{V_2}{r} + f)V_2 = g\partial_r (H_1 + sH_2) \tag{2}$$

are exact solutions corresponding to axisymmetric vortices. We are interested in isolated vortices satisfying (2) and having zero circulation at infinity, and work with alpha-Gaussian vortices with the following azimuthal velocity distribution, in non-dimensional variables:

$$V^*(r^*) = \pm\epsilon r^{*\frac{\alpha}{2}} e^{\frac{(-r^{*\alpha} + 1)}{2}}, \quad \alpha \geq 1. \tag{3}$$

Here the positive sign corresponds to cyclones and the negative one to anticyclones. Taking the profile (3) for $V_1$, supposing $H_1 + sH_2 = 0$, and taking $H_1 + H_2$ as the primitive of the l.h.s. of the first equation (2) calculated with (3), allows to find unambiguously $H_1$ and $H_2$ for a vortex with non-zero velocity uniquely in the upper layer.

## 3. Linear stability problem

Linearising the "dry" version ($P_1 = 0$) of (1) about the above-described vortex, and looking for solutions of the linearised equation in the form of azimuthal harmonics $\propto e^{i(l\theta - \omega t)}$, where $\theta$ is the polar angle, leads to a linear eigenproblem for the frequencies $\omega$. Eigenvalues with non-zero imaginary part correspond to instabilities. The resulting eigenproblem is discretised in radial direction following [2] and solved by the pseudo-spectral collocation method. Fig. 1 shows a typical most unstable mode resulting from the linear stability analysis. This mode corresponds to the classical baroclinic instability, and is used to initialise nonlinear simulations.

## 4. Nonlinear saturation of the instability. Comparison of dry and moist scenarios

Nonlinear saturation of the instability was studied by direct numerical simulations initialised with the main vortex configuration with superimposed most unstable mode of weak amplitude (several per cent). A useful diagnostics of the outputs of such simulations is provided by potential vorticity (PV) $q_i = \frac{\zeta_i + f}{h_i}$, $i = 1, 2$, which is a Lagrangian invariant of the dry system layer-wise, or PV anomaly (PVA) $q_i - f/H_i$, where $H_i$ are non-perturbed thicknesses of the layers. Here $\zeta_i == \partial_x v_i - \partial_y u_i$ ($i = 1, 2$) is relative vorticity. We present in Fig. 2, 3 the evolution of the PVA, respectively in upper and lower layers, during the saturation of the baroclinic instability of the upper-layer cyclone. As we see, the initial upper-layer vortex is streched due to the perturbation, elongates and forms secondary vortices of opposite sign, while their lower-layer counterparts are also being formed. The vortex finally splits in two secondary dipoles which run away in opposite directions. This scenario of "dipolar splitting" is known in literature [4],

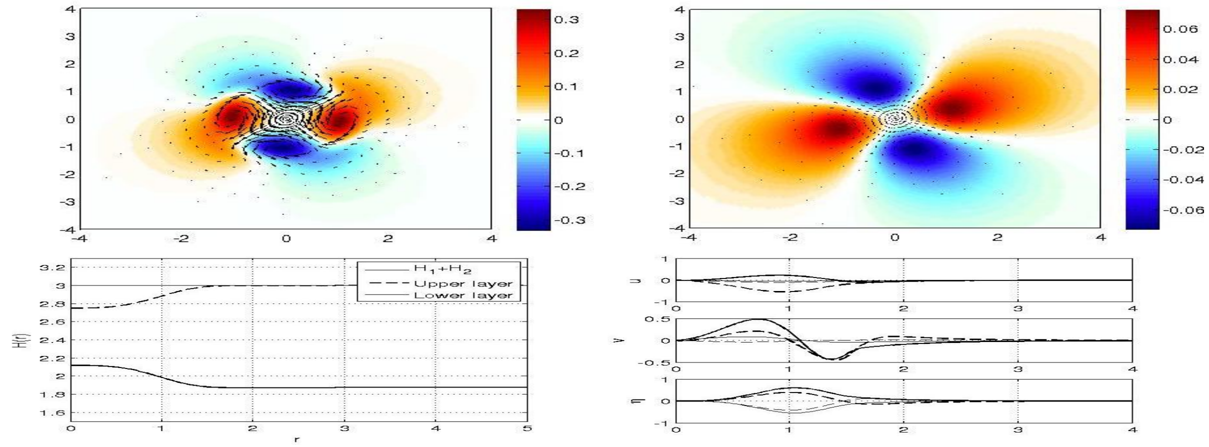


**Figure 1.** Most unstable mode. Top: pressure and velocity in upper (left) and lower (right) layers. Bottom: $H_1$ and $H_2$ (left), radial structure of the mode in upper (thick) and lower (thin) layers. Dashed (Solid): imaginary (real) real parts. ($\alpha = 4$, $l = 2, \epsilon = 0.08$, $s = 1.37$).

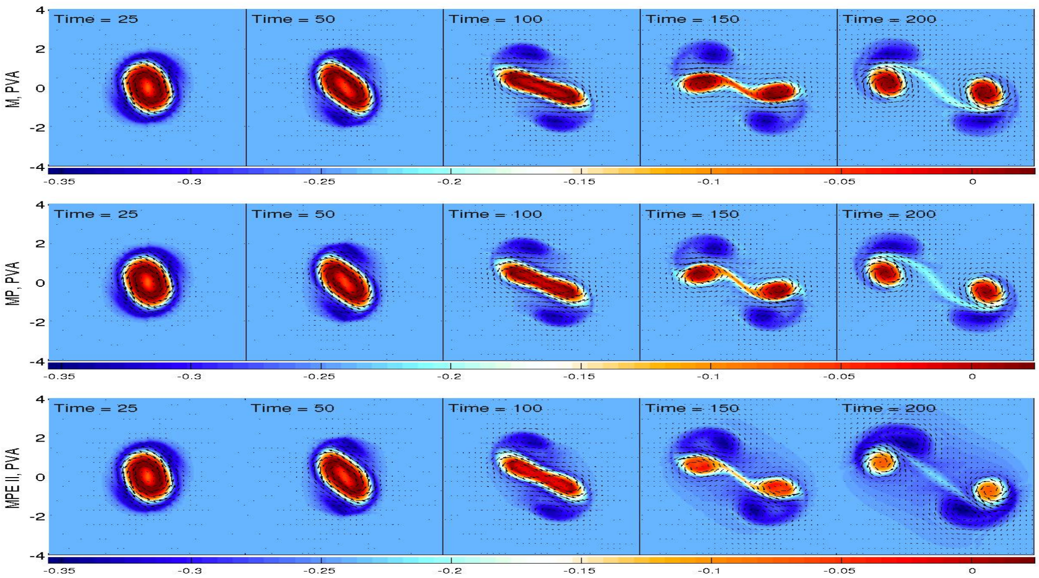


**Figure 2.** Evolution of PVA in the upper layer during the evolution of the upper-layer cyclone

and thus serves as a benchmark of the "dry" scenario (M). Yet, our simulations clearly show that precipitation, especially in conjunction with evaporation substantially enhances the cyclonic partners of the dipoles, especially in the lower layer, and thus modifies the scenario. The changes are spectacular in the precipitating and evaporating environment.

An important characteristic of the saturation of the baroclinic instability is related inertia-gravity wave emission. The sources of inertia-gravity waves in the atmosphere are not well quantified, in spite of the important dynamical role they are playing, e.g. in the transport of energy and momentum. The wave activity is diagnosed with the help of wind divergence field. It has a peak during initial stretching of the vortex, but no enhancement related to precipitation

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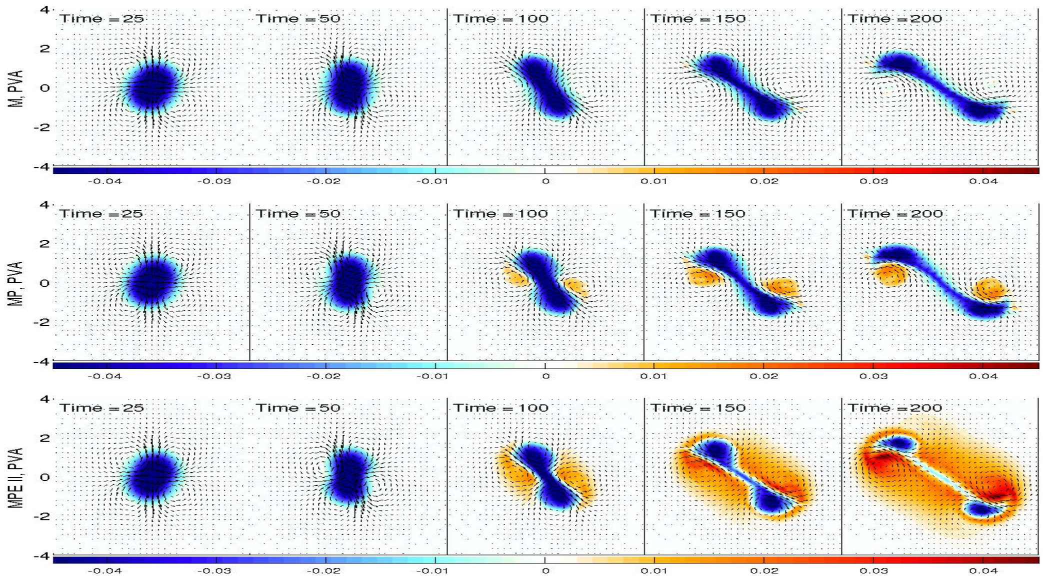


**Figure 3.** Same as in Fig. 2, but for the lower layer.

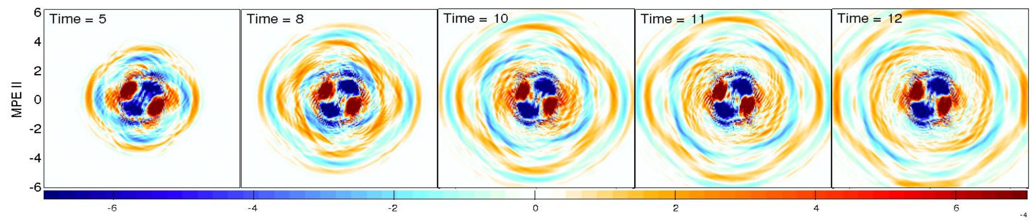


**Figure 4.** Wave emission in the MPE II environment ($\alpha = 4, \epsilon = 0.08, s = 1.37, H_0 = 3, H_2/H_1 = 0.6, \gamma = 0.05$. Propagation of a packet of inertia-gravity waves is clearly visible.

events. The wave-field corresponding to the maximum of wave emission is presented in Fig. 4

## 5. Summary and conclusions

Thus, we performed a linear stability analysis of shielded upper-layer atmospheric vortices and used the results of this analysis to initialise fully nonlinear numerical simulations of the saturation of their dominant instability in "dry", moist-precipitating and moist-precipitating and evaporating scenarios. We have found that moist effects significantly change the saturation scenario, leading to spectacular enhancement of secondary lower-layer vortices.

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