Late time behaviour for Hard Parton Evolution using the Schwinger-Keldysh Formalism

To cite this article: Ben Meiring and W A Horowitz 2016 J. Phys.: Conf. Ser. 668 012116

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Late time behaviour for Hard Parton Evolution using the Schwinger-Keldysh Formalism

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Abstract. AdS/CFT computations have been used to describe the energy loss of QCD-like particles moving through a strongly coupled plasma, but little is understood regarding the initial conditions of these jets. We are investigating an analytic description of jet initial conditions through the time evolution of hard partons by means of the Schwinger-Keldysh Formalism. We have derived a general formula for the late time expectation value of the total 4-momentum in terms of the differential cross-section and the initial 4-momenta of the system. As a consistency check, we have found the same expression for $\lim_{x^0 \to \infty} \langle \int d^3 x T_{\mu \nu}(x) \rangle$ for an interacting scalar field theory to leading order.

1. Introduction
A good description of the collision process involving heavy nuclei (such as Pb or Au) is given in [1, 2]. In such a system we take interactions to begin at $\tau = 0$, when the nuclei collide. The ‘hard’ processes (those that involve a transfer energy of $\gtrsim 10$ GeV) occur at a time scale $\tau \sim Q^{-1}$ by the uncertainty principle. Interactions that are able to result in high transverse momentum (such as hard parton production) occur very early in the collision before the ‘softer’ processes that result in thermalization, which begin at $\tau \sim 0.2$ fm/c at an energy scale of $\sim 1$ GeV. Typically an expansion of QCD phenomena at weak coupling can be used to describe collisions with high momentum transfer [3].

1.1. The Expectation Value of the Energy Momentum Tensor
In the earliest moments of the collision ($\tau < 0.2$ fm/c), before the equilibration of a Quark Gluon Plasma (QGP) we assume that the the evolution of hard partons will be qualitatively similar to the evolution of partons that result from a proton-proton (p-p) collision. We aim to understand the $T_{\mu \nu} = \sum_i \partial_\mu \psi_i \partial_\nu \psi_i - g_{\mu \nu} \mathcal{L}$ (as defined in [4]) of hard partons resulting from collisions in a vacuum (similar to the evolution present after a proton-proton interaction). We will use the Schwinger-Keldysh real time formalism to perform a perturbative expansion at weak coupling for the expectation value of $T_{\mu \nu}$ due to the interaction of colliding particles. $(T_{\mu \nu}(x))$ in a theory with interaction Hamiltonian given by $H$, due to an initial state given by $|\text{in}\rangle$ is given by the expansion

\[
\langle T_{\mu \nu}(x) \rangle = \langle \text{in} | T \exp \left( i \int_{-\infty}^{x^0} dz_1 \hat{H}_1(z_1) \right) T_{\mu \nu}(x) T \exp \left( -i \int_{-\infty}^{x^0} dz_1 \hat{H}_1(z_1) \right) |\text{in}\rangle, \quad (1)
\]

where the $I$ subscripts indicate quantities in the interaction picture.
2. Results
We found a general result for the late time expectation value of the total momentum \( \int d^3x \hat{T}_{\mu 0} = \hat{P}_\mu \) for a system two colliding particles. For clarity we will only state the result of Equation 2 for the spatial momentum in the center of mass frame.

\[
\lim_{t \to \infty} \int d^2b \langle \vec{\hat{P}} \rangle = 0 = \int d\Pi_{pf} \left( \sum_f \vec{p}_f \right) \frac{d\sigma}{d\Pi_{pf}}^{(\text{in} \rightarrow \text{out})} \tag{2}
\]

where \( \langle \vec{\hat{P}} \rangle \) is the expectation value of the canonical momentum, \( \vec{p}_f \) are the eigenvalues of the final state particles, \( \int d\Pi_{pf} \) is an integral over the final state momentum phase space and

\[
\frac{d\sigma}{d\Pi_{pf}}^{(\text{in} \rightarrow \text{out})} = \int d^2b \left( \prod_{i=A,B} \int \frac{d^3k_i}{(2\pi)^3} \sqrt{2E_i} \int \frac{d^3\bar{k}_i}{(2\pi)^3} \sqrt{2\bar{E}_i} \right) \times e^{i(\vec{\bar{k}}^0_A - \vec{\bar{k}}^0_B)} \left( \text{in} \langle \{\bar{k}_i\} | T^\dagger | \{p_f\} \rangle \text{out} \right) \left( \text{out} \langle \{p_f\} | T | \{k_i\} \rangle \text{in} \right). \tag{3}
\]

Here \( \langle \text{out} \{p_f\} | T | \{k_i\} \rangle \text{in} \rangle = i\mathcal{M}(\{k_i\} \rightarrow \{p_f\})(2\pi)^4\delta^{(4)}(\sum_i k_i - \sum_f p_f) \) gives the transition amplitude for a set of momentum eigenstates \( |\{k_i\}\rangle \) at \( t = -\infty \) to evolve to momentum eigenstates \( |\{p_f\}\rangle \) at \( t = \infty \), \( \phi_i \) and \( \bar{\phi}_i \) are wavepackets, and \( \vec{\bar{k}}^0 \) gives the impact parameter. The definition of 3 is motivated by equation (4.76) in [4]. The interpretation of Equation 2 is that given an initial state \(|\text{in}\rangle\) with total spatial momentum in the centre of mass frame \( \sum \vec{p}_i = 0 \), the final state particles must distribute themselves in momentum space so as to keep the total momentum conserved.

2.1. The Elastic Collisional \( \lambda \psi_1^2 \psi_2^2 \) Model
We checked Equation 2 for a theory of two interacting scalar fields, \( \mathcal{L} = \frac{1}{2} \left( \partial \psi_1 \right)^2 - \frac{1}{2} m^2 \psi_1^2 + \frac{1}{2} \left( \partial \psi_2 \right)^2 - \frac{1}{2} m^2 \psi_2^2 - \lambda \psi_1^2 \psi_2^2 \), with initial state given by two particle wave packets of species \( \psi_1 \) and \( \psi_2 \). We find that in the center of mass frame, to leading order in the coupling \( \lambda \),

\[
\lim_{t \to \infty} \int d^2b \langle \vec{\hat{P}} \rangle = 0 = \int \frac{d^3p_\xi d^3q}{(2\pi)^6 2E_p 2E_q} \left( \vec{p} + \vec{q} \right) \frac{\lambda^2 \delta^{(4)}(p + q)}{\sum_f \lambda} \tag{4}
\]

As we would expect, the late time total momentum is given by a distribution of final state particles in the rest frame weighted by the differential cross-section.

3. Conclusion
We have found a consistent and general result for the late time behaviour of the total 4-momentum for any given quantum field theory in terms of the initial 4-momentum and the differential cross-section. We have verified the expression explicitly for the \( \lambda \psi_1^2 \psi_2^2 \) model. Going forward, we will attempt to find finite time results for the expectation value of the energy momentum tensor associated with the evolution of hard partons.

References
[4] Peskin M E and Schroeder D V