Analytical Expressions for the Hard-Scattering Production of Massive Partons

To cite this article: Cheuk-Yin Wong 2016 J. Phys.: Conf. Ser. 668 012097

View the article online for updates and enhancements.

Related content
- Partons in the chiral periphery of the nucleon
  Carlos G Granados
- Collinear limits in QCD from MHV rules
  Tom G. Birthwright, E.W. Nigel Glover, Valaya V. Khoze et al.
- Effects of Surface Emitting and Cumulative Collisions on Elliptic Flow
  Liu Jian-Li, Wu Feng-Juan, Zhang Jing-Bo et al.
Analytical Expressions for the Hard-Scattering Production of Massive Partons

Cheuk-Yin Wong
Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA

Abstract. We obtain explicit expressions for the two-particle differential cross section \( E_cE_a d\sigma(AB \to c\kappa X)/dc d\kappa \) and the two-particle angular correlation function \( d\sigma(AB \to c\kappa X)/d\Delta \phi d\Delta y \) in the hard-scattering production of massive partons in order to exhibit the “ridge” structure on the away side in the hard-scattering process. The single-particle production cross section \( d\sigma(AB \to cX)/dy_{cT} dcT \) is also obtained and compared with the ALICE experimental data for charm production in pp collisions at 7 TeV at LHC.

1. Introduction
Knowledge of massive quark production processes in pp collisions provides useful insight to guide our intuition in heavy quark production in nucleus-nucleus collisions. Analytical expressions for these processes summarize important features and essential dependencies so as to facilitate the uncovering of dynamical effects wherever they may occur. Similar analyses in massless quark production have led to new insights in the dominance of the hard-scattering process over a large \( p_T \) domain and have paved the way for locating the boundary between the hard-scattering process and the flux-tube fragmentation process in high-energy pp collisions [1, 2].

Accordingly, we would like to obtain \( E_cE_a d\sigma(AB \to c\kappa X)/dc d\kappa \) for the two-particle differential cross section in the production of massive partons \( c \) and \( \kappa \). From such a general result, we integrate out the transverse momenta and obtain the two-particle angular correlation function \( d\sigma/d\Delta \phi d\Delta y \) where \( \Delta \phi = \phi_\kappa - \phi_c \) and \( \Delta y = y_\kappa - y_c \), exhibiting analytically the “ridge” structure on the away side at \( \Delta \phi \sim \pm \pi \) in the hard-scattering process. We subsequently examine \( d\sigma(AB \to cX)/dy_{cT} dcT \) for the single-particle spectrum and compare with ALICE experimental data for charm production in pp collisions at 7 TeV at LHC [3].

2. Hard Scattering Integral for \( E_cE_a d\sigma(AB \to c\kappa X)/dc d\kappa \)
In the parton model, the hard-scattering cross section for \( AB \to c\kappa X \) is given by [4]

\[
d\sigma(AB \to c\kappa X) = \sum_{ab} \int K_{ab} dx_a d\alpha_T dx_b d\beta_T G_{a/A}(x_a, \alpha_T) G_{b/B}(x_b, \beta_T) d\sigma(ab \to c\kappa),
\]

where \((x_a, \alpha_T)\) and \((x_b, \beta_T)\) represent the momenta and \( G_{a/A} \) and \( G_{b/B} \) the structure functions of the incident partons \( a \) and \( b \) respectively, and \( K_{ab} \) is the correction factor which can be obtained perturbatively [5] or it can also be approximated nonperturbatively [6]. The quantity \( d\sigma(ab \to c\kappa) \) is the cross section element for the process \( ab \to c\kappa \),

\[
d\sigma(ab \to c\kappa) = \frac{1}{4[(a \cdot b)^2 - m_a^2 m_b^2]^{1/2}} |T_{fi}|^2 \frac{d^3c}{(2\pi)^3} \frac{d^3\kappa}{(2\pi)^3} (2\pi)^4 \delta^4(a + b - c - \kappa).
\]

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
Here, we normalize the Dirac fields by $\bar{u}u = 2m$. The quantity $|T_{ij}|^2$ is related to $d\sigma/dt$ by

$$|T_{ij}|^2 = 16\pi[\hat{s} - (m_a + m_b)^2][\hat{s} - (m_a - m_b)^2]\frac{d\sigma(ab \rightarrow ck)}{dt}. \quad (3)$$

We consider the simplified case with $m_a = m_b = 0$ and treat $a_T, b_T$ as small perturbations. The cross section element is then

$$d\sigma(ab \rightarrow ck) = \frac{s_{ab}d\sigma(ab \rightarrow ck)}{2\pi}\frac{d^3c d^3\kappa}{E_c E_\kappa}\delta^4(a + b - c - \kappa), \quad (4)$$

where $\hat{s} = s_{ab} = (a + b)^2$ that is different from $s = s_{AB} = (A + B)^2$. We get

$$\frac{E_c E_\kappa d\sigma(AB \rightarrow cKX)}{d^3c d^3\kappa} = \sum_{ab} \int K_{ab}dx_adx_bG_{a/A}(x_a,a_T)G_{b/B}(x_b,b_T)\frac{\hat{s}}{2\pi}\frac{d\sigma(ab \rightarrow cK)}{dt} \delta^4(a + b - c - \kappa). \quad (5)$$

We consider a factorizable structure function with a Gaussian intrinsic transverse momentum distribution,

$$G_{a/A}(x_a,a_T) = G_{a/A}(x_a)\frac{1}{2\pi\sigma^2}e^{-a_T^2/2\sigma^2}. \quad (6)$$

Upon integrating over the transverse momenta $a_T$ and $b_T$, we obtain

$$\frac{d\sigma(AB \rightarrow cKX)}{dy_T dx_T d\phi_c dy_N d\phi_N} = \sum_{ab} \int K_{ab}dx_adx_bG_{a/A}(x_a,a_T)G_{b/B}(x_b,b_T)\frac{e^{(G_{c+\kappa T}^2)}}{2\pi(4\sigma^2)}\frac{\hat{s}}{2\pi}\frac{d\sigma(ab \rightarrow cK)}{dt} \delta(a + b - c - \kappa). \quad (7)$$

To carry out the integration over $x_a$ and $x_b$, we write out the momenta in the infinite momentum frame,

$$a = \left(x_a\frac{\sqrt{s}}{2} + a_T^2 + a_T^2\right), \quad a_T, \quad x_a\frac{\sqrt{s}}{2} = \left(a_T^2 + a_T^2\right)\frac{2x_a\sqrt{s}}{a_T}, \quad (8)$$

$$b = \left(x_b\frac{\sqrt{s}}{2} + b_T^2 + b_T^2\right), \quad b_T, \quad -x_b\frac{\sqrt{s}}{2} = \left(b_T^2 + b_T^2\right)\frac{2x_b\sqrt{s}}{b_T}, \quad (9)$$

$$c = \left(x_c\frac{\sqrt{s}}{2} + c_T^2 + c_T^2\right), \quad c_T, \quad x_c\frac{\sqrt{s}}{2} = \left(c_T^2 + c_T^2\right)\frac{2x_c\sqrt{s}}{c_T}, \quad (10)$$

$$\kappa = \left(x_\kappa\frac{\sqrt{s}}{2} + \kappa_T^2 + \kappa_T^2\right), \quad \kappa_T, \quad -x_\kappa\frac{\sqrt{s}}{2} = \left(\kappa_T^2 + \kappa_T^2\right)\frac{2x_\kappa\sqrt{s}}{\kappa_T}, \quad (11)$$

where $x_c$ and $x_\kappa$ can be represented by $y_c$ and $y_\kappa$

$$x_c = \frac{m_{cT}}{\sqrt{s}}, \quad x_\kappa = \frac{m_{\kappa T}}{\sqrt{s}}. \quad (12)$$

The two delta functions in Eq. (7) can be integrated to yield

$$\frac{d\sigma(AB \rightarrow cdx)}{dy_T dx_T d\phi_c dy_N d\phi_N} = \sum_{ab} K_{ab}x_aG_{a/A}(x_a)x_bG_{b/B}(x_b)\frac{e^{(G_{c+\kappa T}^2)}}{2\pi(4\sigma^2)}\frac{d\sigma(ab \rightarrow cK)}{dt}, \quad (13)$$

where

$$x_a = x_c + \kappa_T^2 + \kappa_T^2, \quad x_a^2 = \frac{m_{cT}}{\sqrt{s}} + \frac{m_{\kappa T}}{\sqrt{s}} - \frac{b_T^2 + b_T^2}{x_a}, \quad (14)$$

$$x_b = x_\kappa + c_T^2 + c_T^2, \quad x_b^2 = \frac{m_{cT}}{\sqrt{s}} + \frac{m_{\kappa T}}{\sqrt{s}} - \frac{a_T^2 + a_T^2}{x_b}. \quad (14)$$

The above explicit formula gives the cross section for the production of $c$ and $\kappa$, when the elementary cross section $d\sigma(ab \rightarrow cK)/dt$ is given explicitly in terms of its depending variables.
3. \(c\kappa\) angular correlation \(d\sigma(AB \rightarrow c\kappa X)/d\Delta\phi d\Delta y\)

We can represent \(c\) and \(\kappa\) by \((y_c, \phi_c)\) and \((y_c + \Delta y, \phi_c + \Delta \phi)\), respectively. After averaging over \(y_c\) and \(\phi_c\), and integrating over \(c_T, \kappa_T\), the correlation function (13) from the process \(ab \rightarrow c\kappa\) is

\[
\frac{d\sigma(AB \rightarrow c\kappa X)}{d\Delta\phi d\Delta y} = K_{ab} x_a G_{a/A}(x_a) x_b G_{b/B}(x_b) d\sigma(\Delta\phi),
\]

where \(\delta(\Delta \phi) = \frac{1}{(4\pi\sigma^2)} \int_0^{\infty} c_T dc_T \int_0^{\infty} \kappa_T dk_T \exp\left\{-\frac{c_T^2 + 2c_T\kappa_T \cos \Delta \phi + \kappa_T^2}{4\sigma^2}\right\} \frac{d\sigma(ab \rightarrow c\kappa)}{dt}.
\]

The above analytical expression assumes a simple form for \(a=b\) and \(c=\kappa\). The structure function can be represented in the form \(x_a G_{a/A}(x_a) \times (1 - x_a)^{2n}\) for which the two-particle angular correlation function becomes

\[
\frac{d\sigma(AB \rightarrow c\kappa X)}{d\Delta\phi d\Delta y} \sim A \left[ 1 - \frac{2m_c}{\sqrt{s}} \right] (\cosh y_c + \cosh(y_c + \Delta y)) + 2 \left( \frac{m_c}{\sqrt{s}} \right)^2 [1 + \cosh(2y_c + \Delta y)] \delta(\Delta \phi).
\]

If we consider \(d\sigma(ab \rightarrow c\kappa)/dt\) to be approximately of the form

\[
\frac{d\sigma(ab \rightarrow c\kappa)}{dt} = \frac{A}{(1 + c_T/m_c^2)^n/2},
\]

where \(n=4\) from pQCD, then the integration over \(c_T\) and \(\kappa_T\) in Eq. (16) gives \(\delta(\Delta \phi)/A\) as shown in Fig. 1. The correlation function has maxima at \(\Delta \phi \sim \pm \pi\) and a minimum at \(\Delta \phi = 0\). It is relatively flat in \(\Delta y\) because \(m_c/\sqrt{s} \ll 1\). This gives the distribution in the form of a ridge structure on the away side at \(\Delta \phi \sim \pm \pi\).

4. Production of massive quarks by gluons

We consider the process \(gg \rightarrow c\bar{c}\), where analytical expressions \(d\sigma/dt\) have been obtained earlier by Cambridge [7] and Glück, Owen, and Reya [8]. In the notation of [8], the cross section is

\[
\frac{d\sigma(gg \rightarrow c\bar{c})}{dt} = \frac{\pi \alpha_s^2}{64\pi^2} \left[ 12 M_{as} + \frac{16}{3} M_{tt} + \frac{16}{3} M_{uu} + 6 M_{st} + 6 M_{su} - \frac{2}{3} M_{tt} \right].
\]

Upon writing out the above quantity as a function of \(c_T, \bar{c}_T, \bar{y} = (y_c + y_{\bar{c}})/2\) and \(\Delta y = y_c - y_{\bar{c}}\), we get the heavy-quark pair production cross section

\[
\frac{d\sigma(AB \rightarrow c\bar{c} X)}{dy_c c_T d\phi_c} \sim A K_{ab} (1 - x_a)^{2n} (1 - x_b)^{2n} \frac{e^{-c_T^2/4s}}{2\pi(4\pi\sigma^2)} \frac{d\sigma(gg \rightarrow c\bar{c})}{dt},
\]

where

\[
\frac{d\sigma(gg \rightarrow c\bar{c})}{dt} = \frac{\pi \alpha_s^2}{4^5 m_c^2} \cosh^4 \bar{y} \left\{ \left[ \frac{12}{\cosh^2 \bar{y}} + \frac{64}{3} \cosh 2\bar{y} - 24 \right] + \left( \frac{m_c}{m_{c\bar{c}}} \right)^2 \left[ \frac{64}{3} + \frac{24 \sinh^2 \bar{y}}{\cosh^2 \bar{y}} - \frac{8}{3} \right] \right. \\
\left. + \left( \frac{m_\bar{c}}{m_{c\bar{c}}} \right)^4 \left[ - \frac{64 \cosh 2\bar{y}}{3} + \frac{8}{3} \frac{1}{\cosh^2 \bar{y}} \right] \right\},
\]

\(x_a = 2m_c \cosh \bar{y} \Delta y/2\), and \(x_b = 2m_{\bar{c}} \cosh \bar{y} \Delta y/2\). This shows the back-to-back correlation of \(c_T\) and \(\bar{c}_T\) and the relatively flat distribution as a function of \(\Delta y\).
5. Single-particle charm production

We need to integrate over $\kappa$ in Eq. (5) to get the single-particle distribution of $c$. Neglecting intrinsic $p_T$ and integrating over $\kappa$, we get

$$E_c d\sigma(AB \rightarrow cX) = \sum_{ab} \int dx_a dx_b G_{a/A}(x_a) G_{b/B}(x_b) \frac{s d\sigma(ab \rightarrow cd)}{d\Omega} \delta(\hat{s} + \hat{t} + \hat{u} - m_c^2 - m_k^2). \tag{21}$$

The integral over $x_a$ and $x_b$ can be evaluated by the saddle point method [1], and we get for $\hat{y} = y_c \approx 0$,

$$E_c d\sigma(AB \rightarrow cX) \propto K_{ab}(1 - x_a) y_a^{1/2}(1 - x_b) y_b^{1/2} \frac{1}{\sqrt{x_c}} \frac{d\sigma(gg \rightarrow cc)}{dt} \tag{22}$$

where $K_{ab}$ [6] contains $1/M_{cT}^4$, $m_c^2/m_6^2$, and $m_4^2/m_5^2$. We examine the ALICE charm production cross section data in $pp$ collisions at 7 TeV [3] shown in Fig. 2. If we parametrize the data at mid-rapidity as $d\sigma/dy_{cT}d\eta_T \sim a/(1 + p_T^2/m_0^2)^n/2$, we find that the ALICE data can be fitted by a set of parameters given by

$$d\sigma/dy_{cT}dp_T|_{y_T=0} = a/(1 + p_T^2/m_0^2)^n/2$$

Table: Data $a \cdot \mu b/(GeV/c)^{-2}$, $n$, $m_0 (GeV)$

<table>
<thead>
<tr>
<th>$D^0$</th>
<th>1600</th>
<th>5.8</th>
<th>3.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D^+$</td>
<td>780</td>
<td>5.9</td>
<td>3.5</td>
</tr>
<tr>
<td>$D^{*+}$</td>
<td>808</td>
<td>5.7</td>
<td>3.5</td>
</tr>
</tbody>
</table>

6. Conclusion

We present analytical expressions for the hard-scattering production of massive quarks in order to guide our intuition, point out essential dependencies, and summarize important features. They will facilitate future comparisons with experimental data and pave the way for a better understanding of particle production processes.

The research was supported in part by the Division of Nuclear Physics, U.S. Department of Energy under Contract DE-AC05-00OR22725.

References


