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A viscoelastic model to simulate soft tissue materials

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Abstract. Continuum mechanic theories are frequently used to simulate the mechanical behavior of elastic and viscous materials, specifically soft tissues typically showing incompressibility, nonlinear deformation under stress, fading memory and insensitivity to the strain-rate. The time dependence of a viscoelastic material could be better understood by considering it as composed by an elastic solid and a viscous fluid. Different types of mechanical devices can be constructed provided a particular configuration of elastic springs and dashpots. In this work our aim is to probe many of the soft tissue mechanical behavior, by considering a Kelvin's device coupled to a set of *in parallel* Maxwell's devices. Then, the resulting model composed of a long series of modified Kelvin bodies must span a broad range of characteristic times resulting in a suitable model for soft tissue simulation. Under driving static and dynamic stress applied to a *2-Dim* system, its time dependence strain response is computed. We obtain a set of coupled Volterra integral equations solved via the extended trapezoidal rule scheme, and the Newton-Raphson method to solve nonlinear coupled equations.

1. Introduction

Modeling the mechanical behavior of soft tissue are relevant for applications in surgical simulations in real time and fast precise calculations of tissue mechanical deformations [1, 2, 3, 4]. Soft tissue material response to stress achieves large deformations at the beginning of a relatively stress low level and subsequently stiffening at higher stress level. Their structural composition of collagen fibers distribution leads to a pronounced anisotropy [5, 6, 7, 8]. Almost all the biological soft tissues are mechanically less sensitive under different strain rates. Their stress-strain curve exhibit a hysteresis loop showing a nonlinear stress-strain relationship, and their hysteresis loop-area does not depend on the strain rate [9, 10, 11].

Viscoelastic materials time behavior are well understood wether considered as an elastic solid and a viscous fluid. Different types of mechanical devices are constructed provided a particular arrangement of elastic springs and dashpots [9]. Maxwell and Voigt models supply a logarithmic functional type relationship between strain-rate and frequency (inverse of the characteristic time), describing a decreasing curve for the former; while for the latter an increasing curve is observed. Kelvin's model strain-rate vs logarithm of the frequency describes a bell-shaped curve. None of these models are able to present a typical flat strain-rate vs frequency curve of living tissues. In this work our aim is to probe many of the features of the soft-tissue mechanical behavior, considering a Kelvin's device coupled to a set of in parallel Maxwell devices in *2 Dim*. The resulting model composed of a long series of modified Kelvin bodies will span a broad range of characteristic times, becoming a suitable model for soft tissue simulation[9]. In section 2 we present a mathematical formalism applicable to systems showing fading memory,

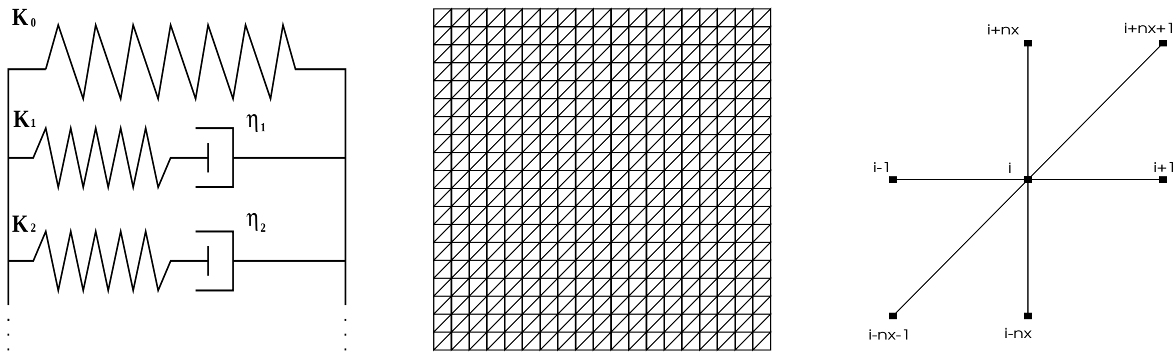


Figure 1. Left side: A single modified Kelvin device consisting of $N+1$ springs and N dashpots. In the middle: An structured network consisting of a set of modified Kelvin's devices. Right side: Neighbors connectivity graph for the $(i\,j)$ network link, where nx is the number of arbitrary elements in a row.

we include an alternative formulation suitable to treat nonlinear elastic materials. The physical formalism applied to an ensemble of modified Kelvin devices *N-Dim* network system is presented in section 3. Results considering a bi-dimensional network are shown in section 4. Conclusions and perspectives are presented in section 5.

2. Systems with fading memory

In this section we reproduce an established one dimensional model, with a more convenient notation for our purposes [9]. Consider a single mechanical device consisting of $(N+1)$ springs and N dashpots set up in parallel (left side of Figure 1). A force is supposed to cause an uniaxial deformation on springs identified as stress and an strain (proportional to the spring constant K) and denoted by σ and u; respectively. Meanwhile into a dashpot, at any instant, it causes an strain rate $\dot{\alpha}$ (proportional to the viscosity η). Denoting the spring constants as K_o and K_n, the dashpot viscosity by $\eta_n\,(n=1,2,\ldots,N)$, its inelastic mechanical response is an internal variable denoted by $\dot{\alpha}_n(t)$. For the viscous stress we have $\sigma^v(t)=\sum_{n=1}^{N}\eta_n\,\dot{\alpha}_n(t)$. To complete the constitutive hypotheses is assumed a linear, elastic, stress-strain spring response. Since $u(t)$ is the strain on the spring, $K_o u(t)=\sigma(t)-\sigma^v(t)$, thus $\sigma(t)=\tilde{K}u(t)-\sum_{n=1}^{N}K_n\alpha_n(t)$ where $\tilde{K}=K_o+\sum_{n=1}^{N}K_n$ is the initial modulus, and the internal variables must satisfy the detailed balance evolution equation, $u(t)=\tau_n\dot{\alpha}_n(t)+\alpha_n(t)$, with $\alpha_n(0)=0\,;\,n=1,2,\ldots N$ and $\tau_n=\eta_n/K_n$ being the relaxation times . Then, $\alpha_n(t)=u(t)-\int_0^t\exp[-(t-s)/\tau_n]\,\dot{u}(s)ds$, and we find

$$\sigma(t)=\int_0^t G(t-s)\,\dot{u}(s)ds \quad , \quad G(t)=K_o+\sum_{n=1}^{N}K_n\,\exp(-t/\tau_n)\,. \tag{1}$$

As a straightforward extension including nonlinear elastic response, we replace the viscous strain by a stress-like set of internal variables $Q_n(t)=K_n\alpha_n(t)\,;\,n=1,\ldots,N,$ so the stress response $\sigma(t)=\tilde{K}u(t)-\sum_{n=1}^{N}Q_n(t)$. By defining the nondimensional relative moduli becomes $\gamma_o=K_o/\tilde{K},\ \gamma_n=K_n/\tilde{K}\,;\,n=1,\ldots,N,\ (\gamma_o+\sum_{n=1}^{N}\gamma_n=1)$ and the initial store energy function as $\mathcal{U}(u) = 1/2\, u\, \tilde{K}\, u$, the model is recasted in the following form:

$$\sigma(t) \;=\; \tfrac{\partial}{\partial u}\mathcal{U}(u) \;-\; \textstyle\sum_{n=1}^{N} Q_n \quad , \quad \tfrac{\partial}{\partial u}\mathcal{U}(u) \;=\; \tfrac{\tau_n}{\gamma_n}\dot{Q}_n + \tfrac{1}{\gamma_n} Q_n \,. \tag{2}$$

chemical energy rate canceling [9], the equation of motion is given by

$$m\ddot{q}(t) + \frac{\partial \mathcal{U}}{\partial\, q} \;-\; F^e \;= 0\,, \tag{3}$$

where F^e is the external force on the device. From the results on section 2, this equation of motion yields a typical equation with fading memory [9],

$$m\ddot{q}(t) + \int_0^t ds\, G(t-s)\dot{q} \;-\; F^{(e)} = 0\,. \tag{4}$$

Now we propose the following model for an ensemble of l connected modified Kelvin's devices, by generalizing the storage energy as,

$$\mathcal{U} = 1/2 \sum_{(i,j)=1}^{l} K^o_{ij}\, q_i q_j + 1/2 \sum_{(ij)=1}^{l} \sum_{n=1}^{N} K^n_{ij}(q_i - \alpha_{i,n})(q_j - \alpha_{j,n})\,.$$

The equation of motion is straightforward, for $(i,\, j) \;=\; (1, 2, \ldots,\, l)$,

$$m_i\ddot{q}_i(t) + \textstyle\sum_{j=1}^{l}\int_0^t ds\, G_{ij}(t-s)\dot{q}_j \;-\; F_i^{(e)} = 0 \quad , \quad G_{ij}(t) = \tilde{K}_{ij}\left[\gamma_o^{ij} + \textstyle\sum_{n=1}^{N}\gamma_n^{ij}\exp\left(-t/\tau_n\right)\right]\,, \tag{5}$$

with $\gamma_o^{ij} = K^o_{ij}/\tilde{K}_{ij}$ and $\gamma_n^{ij} = K^n_{ij}/\tilde{K}_{ij}$. Although $\mathcal{U}(q)$ may be a nonlinear function of strain, the relaxation process is linear (a quasi-linear viscoelastic theory) .

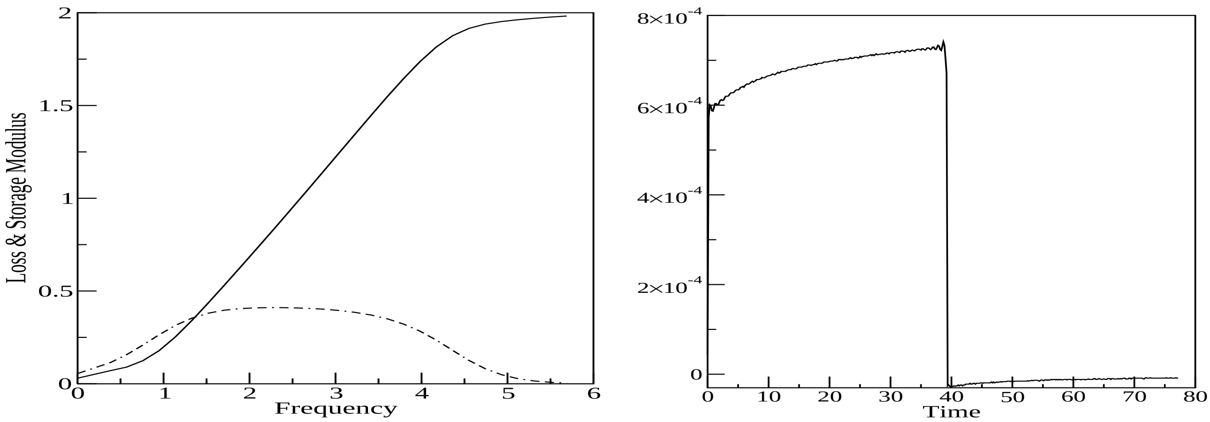
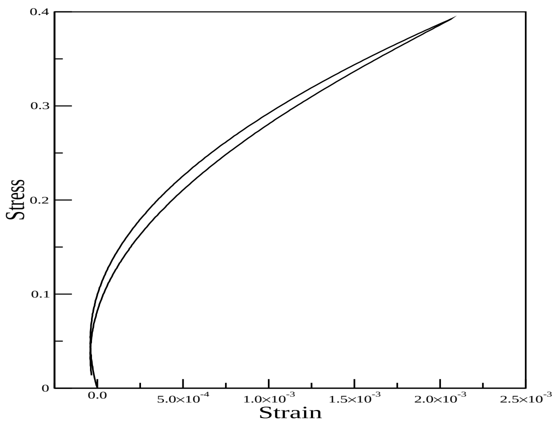


Figure 2. Left side: Loss and storage modulus as a function of the strain-frequency. The storage modulus is represented by continuous lines, the loss modulus is in long dotted line showing a flat curve reproducing soft tissue insensitivity to the strain-rate. Right side: A typical creep viscoelastic behavior for the network system.

4. Numerical results

For a two dimensional network of size $nx \times ny$ structured as in the middle of Figure 1, where each link $(i\,j)$ being one modified Kelvin device with preset relaxation times τ_n, a single relative moduli $\gamma_o^{i\,j}$ and $\gamma_n^{ij} = (1-\gamma_o^{i\,j})/N$; for $n = 1, 2, \ldots, N$ (independent of i and j). We consider the connectivity between neighboring elements (six), as shown in the right side of Figure 1. In the overdamped regime ($m_i \ddot{q}_i \approx 0$), and by integrat by parts Eqn. 5, we obtain the following expression for the i-th particle: $\sum_{j=1}^{6} \left[G_{i\,j}(0)\,\delta r_{i\,j} - \int_0^t \tilde{K}_{i\,j} \sum_{n=1}^{N} \frac{\gamma_n^{i\,j}}{\tau_n} e^{-(t-s)/\tau_n} \delta r_{i\,j}\, ds \right] = \vec{F}_i^{\,e}$. The summation is carried over all six neighbors: $j_1 = i - nx - 1$, $j_2 = i - nx$, $j_3 = i - 1$, $j_4 = i + 1$, $j_5 = i + nx$, $j_6 = i + nx + 1$, with $i = 1, \ldots, ny$; $\delta\, r_{i\,j}$ are the relative displacements (chosen to be quadratic [9]) between neighbors $(i\,j)$ and we set $\tilde{K}_{i\,j}$ to be unity . The resulting coupled system of Volterra integral equations are nonlinear. These are solved via extended trapezoidal rule scheme, and the Newton-Raphson method to solve nonlinear equations [13]. We consider $N = 6$ dashpots with different $\tau_n = 3^n$ and $\gamma_o = 0.25$, for a network of 20×20 links.

We compute: *a)* the response of the system to a dynamic oscillatory strain $F_i^e = \cos\omega\, t$, *b)* the energy stored due to the applied strain (storage modulus) and *c)* the dissipation energy as determined by the strain-rate (loss modulus). In Figure 2, left side, it is shown both the loss and storage modulus behavior vs the inverse of the relaxation time, in the range $(0.0, 6.0)\,Hz$. The loss modulus is represented by the curve in long dotted lines showing a plateau that resembles insensitivity to the strain-frequency. The storage modulus is represented by and increasing continuous curve until it reaches a constant value; delimiting the viscous region. Next we computed the systems response to a cyclic suddenly applied stress (step function). The procedure results in an instantaneous elastic deformation followed by a delayed time-dependent deformation; i.e. the creeping effect. This is shown in Figure 2, right side, where the applied stress is hold for $\approx 80\,sec$, while the strain response is pre-set to be linear. Then it is observed an instantaneous elastic deformation $\approx 6 \times 10^{-4}$ units, followed by a delayed time-deformation in the range $(0, 40)\,sec$ and a delayed time-recovery in a range of $(40, 80)\,sec$. Finally, we considered the response of the system to a cyclic stress linearly increasing and decreasing with slope $\pm 1/2\pi$ during successive time intervals and considered pre-set quadratic deformations. In Figure 3, left side, is plotted the deformation history curve for the elastic viscous deformation. Next, in Figure 3, right side, it is presented a typical intensity color stress deformation for the present network. The plot shown that the relative strain distributed on the network increases from the blue to the red color.



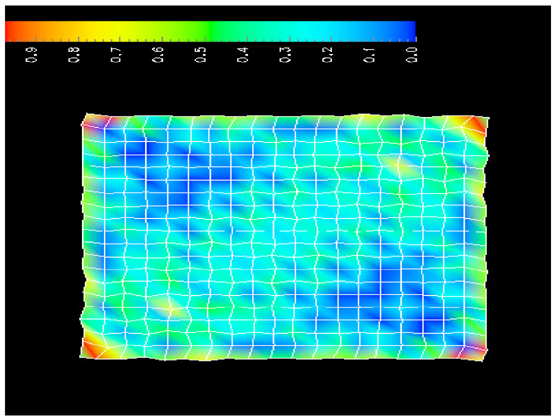


Figure 3. Left side: A typical hysteresis curve for *2-Dim* network system. Right side: A color intensity stress-deformation for a 20 × 20 square network.

5. Conclusions and perspectives

This model reproduces soft tissue mechanical behavior as it is numerically verified by simulated mechanical test. Storage-loss modulus, creeping and hysteresis are common features of viscoelastic mechanical behavior. It is remarkable that the model is capable to compute efficiently nonlinear elastic deformations in real time. It seems that methods based on continuum mechanics are more realistic than their spring-mass viscoelastic counterpart. However, the latter can be faster than the former, therefore more suitable for real time applications. It is curious that quasi-linear viscoelastic formulation is based on the assumption that the viscous stress is related to linearly superposed strain rates. Similar constitutive relations are also useful in a continuum mechanics formulation. There is not an exhaustive comparison between this two alternative approaches, when attempted in applications relying on advantages and precision [14]. The model presented here is appropriated to describe the viscoelastic behavior of bubbles foams, useful when studying microfluidics [15].

Acknowledgments

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References

[1] Bro-Nielsen M (1998) Finite element modeling in surgery simulation. Proc. of the IEEE: Special issue on virtual and augmented reality in medicine **86**(3) 524-30.
[2] Mosegaard J (2003) LR-spring-mass model for cardiac surgical simulation. Medicine meets virtual reality, **12** 256-58.
[3] Teschrer M, Girod S, Girod B (2000) Direct computation of nonlinear soft-tissue deformation. Vision modeling and visualization VMV'000, Saarbucken, Germany, November 22-24.
[4] Keeve E, Girod S, Pfeifel P, Girod B (1996) Anatomy-based facial tissue modeling using the finite element method. Proc. IEEE visualization, San Francisco, CA. USA.
[5] Yamamoto E, Hayashi K and Yamamoto N (1999) Mechanical properties of collagen fascicles from the rabbit pattelar tendom. ASME J. Biomech. Eng. **121** 124-31.
[6] Hasegawa M, and Azuna T (1974) Wall structure and static viscoelasticities of large veins. J. Jap. College Angiol. **14** 87-92.
[7] Chuong C J, and Fung Y C (1984) Compressibility and constitutive equation of arterial wall in radial compression experiments. J. Biomech, **17** 35-40.
[8] Woo S L Y, Lubock P, Gomez M A, Jemmott G F, Kuei S C and Akeson W H (1979) Large deformation nonhomogeneus and directional properties of articular cartilage. J. Biomech. **12** 437-46.
[9] Fung Y C (1994) A first course in continuum mechanics. Prentice Hall, Englewood Cliff, New Jersey.
[10] Fung Y C, Fronek K and Patitucci P (1979) Pseudoelasticity of arteries and the choice of its mathematical expression. Am. J. Physiol. **237** H620-631.
[11] Tong P and Fung Y C (1976) The stress-strain relationship for the skin. J. Biomech. **9** 649-57.
[12] The stability of the system is guaranteed if all the principal minors, including the determinant of the full matrices are positives.
[13] Press W H, Teukolsky S A, Vetterling W T, and Flannery B P (2002) Numerical Recipes in C++ (The art of Scientific Computing), Cambridge University Press.
[14] De Pascalis R, Abrahams I D, Parnell W J (2014) On nonlinear viscoelastic deformations: a reappraisal of Fungs quasi-linear viscoelastic model. Proc. R. Soc. A **470** 20140058.
[15] Espinoza Ortiz J S, Belich H, Orlando M T D, Lagos R E (2014) Topological rearrangements and flow simulation of dry ordered foams. Int. J. Model. Simul. Sci. Comput. **05**, 1441007.