Journal of Physics: Conference Series

PAPER • **OPEN ACCESS**

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To cite this article: Mariana Espinosa Aldama 2015 *J. Phys.: Conf. Ser.* **600** 012050

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Abstract.
The *gravity apple tree* is a genealogical tree of the gravitation theories developed during the past century. The graphic representation is full of information such as guides in heuristic principles, names of main proponents, dates and references for original articles (See under **Supplementary Data** for the graphic representation). This visual presentation and its particular classification allows a quick synthetic view for a plurality of theories, many of them well validated in the Solar System domain. Its diachronic structure organizes information in a shape of a tree following similarities through a formal concept analysis. It can be used for educational purposes or as a tool for philosophical discussion.

1. Introduction

The quest for the fundamental principles of the physical world its at an exciting moment. Obscure new phenomena, –commonly explained as dark matter, dark energy and inflation– has pushed into a crisis the standard paradigm of gravitation: General Relativity and the Concordance Cosmological Model. The plurality of theories available (some of them well validated) is a testimony to this critical condition. Nowadays, most professionals are ultra specialized, so is quite a job for the layperson to form a synthetic view of the hypothesis, concepts and theories being explored as well as their fruits. The *gravity apple tree* is a genealogical graphic representation of gravitational theories that illustrates the bulk of knowledge on this matter over the past 100 years.

This structured map is full of information. Theories, authors and dates are presented in green. Models for dark matter and dark energy are presented in orange and principles followed are in violet. Some selected principles are written in the middle of branches as they allow to distinguish groups of theories. The timeline grows upwards and radially according to the main article publication dates, included in the bibliography for completeness.

The theoretical framework of the tree is based on the work of philosopher Mario Bunge [1], who points out that foundations of physics are not rigid as building blocks, as such would turn our science into dogma. However, their provisional character does not render them nonexistent or less fundamental. Acknowledging the organic character of physics as a system of mutually dependent parts, I will focus on a hierarchical image and an axiomatic approach as it allows one to design this particular visual representation. I have applied a formal concept analysis that provides diagrams linked by a similarity criteria, which I then draw as tree branches [See 2]. The classification of theories is based on [3–8].

1.1. Background

The common early theoretical background is represented as grass, and is that of continuum mechanics. The formal background consists of logic, semantics, algebra, topology, analysis, and manifold geometry. The material background consists of protophysics, mainly the theory of local time and the general systems theory, and some elements of physical geometry [See 1].

The *gravity apple tree* has its base at the end of the 17th century, with the publication of Newton's *Principia* [9], represented by an apple. This theory, remained (and still does) as a paradigm for some 200 years, until 1905, when Special Relativity Theory joins the big explosion of physical knowledge (along with Quantum Theory and Maxwell's electrodynamics). Ten main branches emerge when different principles are followed. I will briefly explain them by groups, according to the agenda on gravitation.

2. Unification of Special Relativity and Gravitation

The Special theory of relativity has two mayor limitations. The first one is that it is restricted to non accelerated reference frames. The second one is that it does not include Newton's law in a self-consistent way. In Newton's theory gravitational force acts instantaneously; but according to the relativity theory, nothing can travel faster than light, which has a finite constant value [See 10, 11]. The birth of the first three branches, identified as the Physical Field branch, the Scalar Lorentz Covariat branch and the General Relativity branch (which keeps growing as the main trunk of the tree), represent the quest to solve this problems.

2.1. Physical Field Theories

Henri Poincaré (1904), Hermann Minkowski (1908) and Willem de Sitter (1905) considered gravitation should propagate with the velocity of light in a retarded way, and gravity to be a physical field, very much like the Maxwell-Faraday electromagnetic field with action at a distance in a flat four dimensional space-time [See 12].Their investigations failed to specify a unique result and the deviations for planetary motions computed by Minkowki were so small that they did not allow to make a firm decision about the law of gravity in the relativistic regime [See e.g. 13, 14].

The interest to develop the relativistic field theories after the second world war was in connection with the interactions between gravity and the meson, electromagnetic and pair fields. This was done mainly in two ways, with its own quantum approaches [15–20][See 14, 21].

2.2. Scalar Lorentz Covariant theories

To avoid action at a distance, the next natural approach was to consider a field law for gravitation, retaining the Laplacian scalar potential of gravity, extending the Poisson equation through a time derivative term. This dynamical scalar field, in contrast with a vector field, has no direction in space. Gunnar Nordstrom [22] propose a metric **g** constructed from a flat background metric η and a scalar dynamical field ϕ: $\mathrm{g} = \mathrm{f}(\phi)\eta$. Einstein and Fokker [23] applied the new methods of absolute differential calculus of Ricci and Levi-Civita to Nordstrom's theory, leading to a theory with kinematical effects where gravitational fields would slow clocks and alter the lengths of rods, concluding that it is actually only conformal to a Minkowsky space-time with the gravitational potential to be the conformal factor, and the presence of a gravitational field coincides with deviations of space-time from flatness [See 13, 22, 24–28].

2.3. General Relativity (GR)

Einstein was in a race for discovery. After an initial paper in 1907, from 1912 untill 1916 when he finally published the complete theory of General Relativity (GR), he published some 25 articles related to his progress. The development of the theory can be divided into tree parts: (i) The

Journal of Physics: Conference Series **600** (2015) 012050 doi:10.1088/1742-6596/600/1/012050

understanding of the Strong Equivalence Principle (1907-1908); (ii) The choice of a Riemannian geometry and of the appropriate metric tensor field for the relativistic Newtonian potential (1912-1913); (iii) The search for the correct field equations for the metric tensor field (1913-19015). While working on the patent office, in 1907, Einstein suddenly realized that a falling man would not feel the difference between gravity and an accelerating force acting upon him. The equality between acceleration and gravity meant an equality of inertial mass and the gravitational mass. In Newton's theory, inertial mass and gravitational mass, even though seem equivalent, are not supposed to be the same thing. Inertial mass is a property of an isolated body in an empty universe, while gravitational mass is a property of a system of bodies. Einstein's strong version of the Equivalence Principle guided his research for a generalization of the relativity theory. The next step in his quest was to find a way to express such equivalent condition in a formal mathematical way where physics laws were the same in accelerated and rotating reference frames. Einstein struggled for some 5 years until his university friend, mathematician Marcel Grossmann, suggested the use of the non-euclidean geometry of Bernhard Riemann. He also adopted differential calculus and the notion of a metric tensor, developed also by Ricci and Levi-Civita in 1900. The metric tensor is a set of 10 functions of two tangent vectors defined on a manifold which produces a real number. A Riemannian manifold gives a positive value to every non-zero vector. Metric tensors are used to define and calculate the length of curves in the manifold. The Riemannian manifold and the metric tensor form a metric space with a distance function. Einstein and Grossmann's (1913) proposal ran into three serious problems: (i) It violated the principle of causality; (ii) the motion equation did not reduce to Poisson's equation for Newtonian gravity; (iii) the field equations violated the conservation of energy. In the summer of 1915, mathematician David Hilbert invited Einstein to give a series of lectures at the University of Gottingen. In november 1915 Hilbert sent a postcard to Einstein in Berlin, explaining he had derived the correct field equations by a variational principle using a Lagrangian function that was invariant under coordinate transformations. Einstein rushed to derive his equations the hard way and publish his results in december 1915. He published a polished complete review with the whole theory in march of 1916, which is the date we consider in the *gravity apple tree* [See 10, 29–31].

2.4. Prior Geometry

Scalar Lorentz Covariant theories enter in a much wider group of theories that still contemplate a flat non-dynamical space-time as background η [See 32–37]These theories with prior geometry are a subgroup of the metric theories, which is represented by the right and top side branches. Metric theories postulate that matter follows geodesics responding only to the metric $\mathbf{g}$ and not to any extra field. Extra fields can contribute to the metric's curvature, and can be generated by matter, but they cannot interact with it directly. Therefore, $\mathbf{g}$ is the only field that enters the equations of motion. Metric theories were developed during the 50s 60s and 70s, when It became clear the need for some neutral parameters that allowed comparison between theories and with experiments in the Solar System. Sir Arthur Eddington (1922) formulated the first post-newtonian parametrization (PPN). Ken Nordtvedt (1968, 1969), Clifford Will (1971) and Wei-Tou Ni (1972) have developed 9 new parameters that answer questions such as how much space curvature is produced by unit rest mass? Or how much gravity is produced by unit of kinetic energy, of gravitational potential energy, internal energy or by pressure?

3. For a classical unification

The idea to unifying gravity and electromagnetism was first put forward by Poincaré in his lecture on "New conceptions of matter" (1912). Four branches emerged from this unifying ideas exploring, mainly, mathematical alternatives to treat symmetries and measuring objects such as the affine connection and the metric tensor, obtaining much general geometries, and theories

Journal of Physics: Conference Series **600** (2015) 012050 doi:10.1088/1742-6596/600/1/012050

with gauge invariance, torsion, non-metricity and extra dimensions [38–49] [See 13, 14, 50–54].

4. Generalizations of GR

Metric theories with additional dynamical fields were suggested separately by five groups of scientists, represented by: Willy Scherrer (1941)[55] a german mathematician working at the university of Bern; Pascual Jordan (1947) [56], in Princeton; Yves Thiry (1948) [57, 58] in Paris; and Robert H. Dicke and his Phd student Carl H. Brans (1961), also at Princeton. Scherrer's approach came form "wave mechanics"; Jordan's and Thiry's group projected Kaluza's five dimensional space-time back into the fourth dimension, associating ϕ with a variable gravitational "constant" G, according with Eddington's and Dirac's Large Numbers Hypothesis [59, 60]. This hypothesis states that the strength of gravity is inversely proportional to the age of the universe, and its mass being proportional to the square of the universe's age implies a a particular cosmology. [See 61]. Dicke's and Brans' theory considered strongly Mach's Principle, which basically states that "the inertial forces observed locally in an accelerated laboratory may be interpreted as gravitational effects having their origin in distant matter accelerated relative to the laboratory" [62] [See 62, 63].This theory expects to prove only a weak version of the Equivalence Principle. The formulation presented by [64, 65] is a generalization of the scalar-tensor theories; Jacob Bekenstein (1977) [66] proposed a Variable Mass Theory (VMT) where rest masses of elementary particles are allowed to vary in space-time via the scalar field, while Barker (1978) [67] proposed a constant G. An even greater generalization is that of Hordenski (1974) [68]. See also [69–71] for vector-tensor theories.

5. Towards quantum gravity

There are two main ways to approach the quantum field theory. One, supposes a fictitious background space, separating the gravitational field into the sum of two components: the background and a quantum field. This would be the approach taken by String Theory [72] which departs from Kaluza's-Klein Theory. The second one is to reconstruct the Quantum Field Theory with no background space, where the space itself is quantified. Charles Misner proposed that calculations could be performed by summing over all possible space-times, applying the ADM formalism (named after its developers: Richard Arnowitt, Stanley Deser and Charles Misner) [73]; and along John Wheeler [74] thought space-time must had a foam-like structure at very small scales . Wheeler and Bryce Dewitt (1967) [75] proposed a wave function over geometries which expresses the probability of having one space-time geometry rather than another. Loop quantum gravity is a mathematical description that uses a novel formulation of GR due to Abhay Ashtekar [76] where the Ashtekar connection field replaces Einstein's gravitational potential in terms of elementary quantum excitations called loops [See 77].

6. New Phenomena

The top set of branches is related to the unexplained new phenomena: (i) The homogeneity of the early universe as interpreted by the measurements of the Cosmic Microwave Background radiation (CMB) and its supposed transition from a curved into a flat structured universe, which are commonly explained by invoking an inflationary period. (ii) The present accelerated expansion of the universe, called dark energy. (iii) The dark matter problem, which refers to the discrepancies between the predictions of Newton's laws and the actual dynamics observed in astronomical systems, as well as the failure of general relativity to explain the observed gravitational lenses and the structure formation, amongst other anomalies. Besides the top set of branches that includes the standard model LCDM, the F(R) theories, and the *MONDian* theories, we can find other alternative theories at the extremes of the main branches, such as John Moffat's Modified Gravity (MOG)[31, 78] better related to Brans-Dicke theory or Mannheim's Conformal Gravity, more related to Wey'ls geometry [See 7, 79–81].

6.1. Dark Matter
A first natural hypothesis to the flat rotational galactic curves was to maintain Newton's law valid, as well as Einstein's GR, and suppose the existence of undetected baryonic matter. This approach produced a number of models for dark entities such as baryonic MACHO's which have been ruled out, since the amount of baryons needed does not match with the early nucleosynthesis model, the observed abundances of elements and the baryon acoustic oscillations. Other alternatives are scalar fields, axions, neutrinos, massive neutrinos and other non-baryonic particles with seemingly ad hoc properties. Hot, warm or cold dark matter models, depending on velocity, are tested on computer simulations of cosmic and galactic structure expecting to tune the right properties that match the observations [See e.g. 82–86] The standard model has given more weight to the very precise data obtained from the CMB and in some sense, has worked its way down from cosmology towards galactic domains, having here most of its problems to matching theory and data. Nevertheless, only direct detection and manipulation of these particles will prove the theory right, since it seems to be almost unfalsifiable [See 81, 87, 88].

6.2. The MONDian Branch
In contrast to the standard model, the top right sided branch emerged from a phenomenological theory that gave priority to galactic phenomena and supposes the possibility that Newton's laws may not be valid at some point. This branch follows the heuristic insights of Mordehai Milgrom's Modified Newtonian Dynamics (MOND) where modifications of the inertial or gravitational laws are solved by the addition of fields [See e.g. 89–96]. The guiding line for *MONDian*theories is the observation that the dynamics depends on the acceleration regime that a particular system experiences, which depends on the surface density of the system. Below a critical acceleration known as Milgrom's fundamental acceleration $\sim 10^{-10}\mathrm{m/s^2}$ [See 97], systems are affected in such a way that the gravitational acceleration a test particle experiences at such sufficiently large distance is greater than its Newtonian value [See 98]. Most *MONDian* theories do not fulfill the strong version of the equivalence principle; therefore, it is conjectured that gravitational dynamics of the systems are to be affected by the field of their host system. This external field effect has not been observed [See 6, 99, 100].

6.3. F(R)
The top middle branch, which explores higher order invariants in the Einstein-Hilbert's action, has two clear periods of growth. Only the second one appears in the *gravity apple tree* for reasons of space and clarity. Nevertheless, it is interesting to note that as early as 1919, Hermann Weyl and Arthur Eddington (1923) explored functions of curvature, mainly as a scientific mathematical curiosity, without any theoretical or experimental reason, but that of a better understanding. In this same spirit, Einstein explored what is now known as the Palatini variation of the action, where the metric and the connection are considered independent variables, and so one can vary the action with respect to both of them. Even though both variational principles lead to the same equations when the Lagrangian is linear in R, this is not true for functions of higher order. [See 101]. Motivations started coming in the 1960s when Utiyama showed that if the Einstein-Hilbert action had higher order curvature invariants with respect to the Ricci scalar, it could be renormalizable. These corrections seemed to be only relevant for very strong gravity (near black holes) or at Planck scales as in the early universe. It is in 1980 when Starobinsky[102] applies this option to model a curvature driven inflation scenario avoiding singularities. [See 103].

6.4. Extended Gravity (EG)
More recently, extensions to the Lagrangian have been added in order to explain galactic anomalies without the hypothesis of dark matter. The proposal of Hernández and Mendoza

has qualitatively described the dynamics of all sort of symmetric galactic systems [104–106], including dwarf spheroidal galaxies, as well as gravitational lenses and extended binary star systems [C.f. 107, 108]. Extended gravity is also compatible with the current acceleration of the universe and provides a good solution for the initial values problem [109–111]. This approach relates to the F(R) theories since it tries to preserve the same principles of GR such as Einstein's Equivalence Principle, the Principle of Relativity, the geodesic motion of particles trough space-time, but incorporates as basic empirical principles Milgrom's acceleration constant and a surface density scale. EG does not necessarily predict the external field effect, though finer comparisons await a more finished theory including a generalized poisson equation [See 112, 113].

7. Remarks

The *gravity apple tree* is a visual tool that facilitates reflexions about foundations on physics, components of theories, and the image of science. With its aid we can easily illustrate progress and relations between theories; distinguish with different colors metric from non-metric theories, or use circles to make valid theories stand out from unviable ones; we can use arrows to show how theories are recycled or merged, and it can be easily illustrated how new objectives and new phenomena have revitalized most of the paths explored.

This type of visualizations are not intended to confirm facts, but as a learning tool. The image of the science in question projected by the *gravity apple tree* may be misleading in some senses. For example, it does not [yet] reflect how popular theories are. It does not include all the theories, and for any unintended omissions I apologize. Nevertheless, the image of such plurality of theories is shocking and invites to some philosophical reflections. Plurality of theories is a natural scientific way, which is to explore all the possible options available, mostly, just for scientific spirit. But the image of science provided by such plurality and the fact that many of these theories can be proved viable in the Solar System takes us to the natural philosophical question of how to rely on the truthfulness of our theories from a realistic and monistic point of view. This point of view generally considers this kind of plurality as a crisis and expects a revolution to happen soon. The key toward the next revolution is not just a new great hypothesis, or the look for the best method, since, as we have seen, approaches and methods are plural as well. Crucial experiments may serve to discard some theories, but scientists are ingenious and will look for ways to go around such problems to save their theories as much as possible. Simplicity of concepts will always be preferred, as well as the power to unify lower level principles. Nicholas Maxwell [114] claims that a strategy that always succeeds is to devise neutral observational terms between competing theories, which can be used to describe phenomena that constitute crucial experiments. Nature is the inevitable dictator. It constrains our theories through measurements of phenomena, observations and experiments. In this sense, the PPN have been with out a doubt the best way to prove a theory valid in the Solar System. Such kind of formalisms are to be considered for new theories in the low acceleration domain. For example, according to [115] it is possible to calculate the γ PPN in the *MONDian* regime, just where the third law of Kepler is substituted by the Tully-Fisher law. Amazingly, observations of individual galaxies, galaxy groups and galaxy clusters all imply that $\gamma \sim 1$, much in coincidence with the observed value in the Solar System where the third law applies. PPN type formalisms are being currently developed towards comparing theories for the cosmological scenarios. These might be called the Post Friedmanian Parameters. Many questions have been answered the past century, and much about curved spaces, acceleration and inertia. The actual question for gravitation is how matter actually curves space-time. This can only be known by astronomical observations.

8. Acknowledgements

The author acknowledges support granted by the Universidad Nacional Autónoma de México and by CONACyT (357910).

Journal of Physics: Conference Series **600** (2015) 012050 doi:10.1088/1742-6596/600/1/012050

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