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Abstract. The Preisach formalism is used to model magnetic hysteresis loops in soft magnetic materials subject to tensile stress. The model uses the Stoner-Wohlfarth mechanism of coherent rotation and dispersion of easy axes to capture the vector response of the magnetization. The Preisach density is constructed as the weighed sum of normal probability density functions (pdf) for the regions of high and low induction. The model parameters reflect the effect of strain: increased pinning, modelled by the central pdf parameters; enhanced anisotropy dispersion modelled by the angular dispersion of easy axes. Upon removal of the tensile stress, compressive residual stresses give rise to effective demagnetizing fields leading to lower differential permeability with a two-peak profile. As deformation levels increase, the amplitude of and the relative distance between the two permeability peaks changes which is reflected in the side density parameters. Modelling results are in qualitative agreement with the experimental data. The potential and limitations of the model are discussed.

1. Introduction

Magnetization is coupled to strain via the magnetoelastic energy of the material. The resulting property is magnetostriction and is a measure of the deformation induced by a magnetic field as well as of the propensity of a material to deform under an applied field. Magnetization processes are influenced by imperfections and material defects such as voids and impurities (0D), dislocations (1D), grain boundaries and interfaces (2D) and of course cracks or pores (3D). 0D imperfections cause a localized variation of the anisotropy profile and an imbalance in the stress tensor around the point defect. 1D defects are abrupt changes in the regular ordering of atoms along the dislocation line while 2D defects are the result of the variation of the stress tensors at the interfaces and grain boundaries. The number of dislocations per unit volume is called dislocation density and they are measured per unit area in a plane. As dislocations propagate they may generate more dislocations assisted by other defects, grain boundaries, and surface irregularities.

The dislocation density increases and shifts with stress, affecting the magnetic response of a material. The effect of dislocations on the magnetic properties [1] of crystals has been studied both theoretically and experimentally. The results reveal a relationship between stresses, applied or residual, and macroscopic magnetic properties, such as the coercivity, the differential susceptibility, the remanence, the shape and the area of minor and major hysteresis loops, as well as microscopic processes such as domain wall mobility [2-3].

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In a rigid wall approximation model for domain wall motion [2] the inverse of the initial permeability and the coercive field are found to be proportional to the square root of the dislocation density. Experimental studies have shown that the motion of dislocations is more impeded by the 90° walls, also related to magnetostriction, than the 180° walls [4].

From the point of view of mechanical properties, stresses have been found to give rise to long range internal stresses, originating at the intergranular and intragranular level. The intergranular component is related to the grain-boundary structure and the incompatibility between grains, which can be considered as a soft mechanical region, and the high density of dislocations acting as hard mechanical region. As plastic strain increases, dislocations form tangles and forests of high density and compete against regions of lower dislocation density leading to intragranular internal stresses. A material in the plastic deformation region can be modeled as consisting small volumes under tensile stresses (hard regions) separated by larger volumes under compressive stresses (soft regions) [5]. These long range internal stresses, affect the magnetoelastic energy and the magnetization processes in the strained material.

Stress-dependent magnetization has been modeled at the atomic [6], the mesoscopic [2, 7-8] and the macroscopic level [9-11]. The Jiles-Atherton model [9] and the moving Preisach model [10-11] have established that the parameters of the characteristic Preisach density are related to microscopic parameters such as the dislocation density and the grain size. However, these models are scalar failing fail to capture the stress – related magnetization rotation and they do not take into account the effect of internal stresses. The model presented in [12] is a vector model based on the Preisach formalism which reproduces successfully the magnetization and permeability dependence on the applied field as a function of stress and whose properties and limitations are discussed in this work.

2. Preisach modeling

The Preisach formalism [13] treats the magnetization response of a magnet exposed to an externally applied field as the superposition of the responses of elementary hysteresis operators. The novelty of the model is that, departing from the classical thermodynamic treatment of hysteresis, it is based on the phenomenology of hysteresis and not on the physical properties of the material being studied. Hysteresis in magnetic materials is a highly non-linear phenomenon, almost synonymous to magnetism, the origins of which lie in the atomic structure of a material and the ensuing complex network of short- and long-range interactions. The etymology of the word suggests that in a system exhibiting hysteresis (means delay in Greek) the output, y, lags the input, x. Furthermore, the present output is a complicated non-linear function of the present input as well as of the past input values. From a stability point of view, in a system exhibiting hysteresis, for a given input value, there may be more than one possible equilibrium states depending on previous equilibrium states, ie, the history of the system. Therefore, a system with hysteresis is also a system with memory.

According to the Preisach formalism, the elementary hysteresis operator, $\gamma_{ab}$ (figure 1) is a rectangular loop with values +1 and -1 bounded by two critical fields, $a$ and $b$, with $a > b$, also known as the upper and lower switching fields, respectively, because the magnetization ‘switches’ to the +1 – state (or -1 – state) for $h>a$ (or $h<b$).

One of the nicest features of the Preisach formalism is the elegant inclusion of interactions: if there are no interactions, the upper and lower fields are equal, $a=b$; in the presence of interactions it becomes harder to switch to the +1 – state ($|a|>|b|$) or the -1 – state ($|a|<|b|$) depending on the direction of interactions (figure 1). The shift of the elementary loop is then equivalent to the interaction field $h_i = ((a+b)/2)$ while the half-width of the elementary hysteresis loop $h_c = ((a-b)/2)$ can be thought of as the analogous to the coercive field of the loop.

At this point, it is important to note that the elementary hysteresis operator $\gamma_{ab}$ does not represent a specific, physical entity of the material, such as a grain, a particle or a domain; it is a statistical variable representing the fields necessary to cause the switching of the magnetization of entities with identical coercivities and experiencing the same amount of interactions. The values allowed for $a$ and
$b$ define the Preisach plane (figure 1) over which a probability density function (pdf) is constructed, $\rho(a,b)$. The value of $\rho(a,b)$ for a given point $(a,b)$ is a weight representing the probability that a physical entity in the material is characterized by the given upper and lower switching fields. $\rho(a,b)$ can be inferred from minor loop measurements and is the footprint of the hysteresis processes inside a material.

Figure 1. Classical Preisach Model: the elementary hysteresis operator with upper and lower switching fields $a$ and $b$ (top left); the triangular Preisach plane, over which the density $\rho(a,b)$ is defined, bounded by $a=+h_s$, $b=-h_s$ and the interactions axis $h_i$; the staircase boundary separating regions of positive and negative magnetization for an alternating field sequence $+h_s, h_1, h_2, h_3, h_4$ with $h_1, h_3<0$, $h_2, h_4>0$.

The magnetization $M(H)$ of the material for a given applied field $H$ is then computed by:

$$M(H) = \iiint_{a>0} \rho(a,b)\gamma_{ab} H dadb \quad (1)$$

The Classical Preisach Model (CPM) described in equation 1 is an inherently scalar model accounting only for irreversible processes (switching) and has performed very well over the years for a number of systems and materials for which the following assumptions are valid or allowed: 1) a field $H$ results in the same change of magnetization $\Delta M$ regardless of the existing magnetization configuration, eg whether the field is applied at the remanent or the demagnetized state 2) the magnetization processes are one-dimensional (1D), confined along the axis of the applied field 3) reversible processes are decoupled from irreversible processes and can be added on later to the response computed with equation 1. As a result of the above assumptions, CPM i) fails to reproduce non-congruent minor loops, ie minor loops which depend on the magnetization configuration as well as on the applied field ii) cannot account for the off-easy axis contributions of the magnetization in non-perfectly oriented media iii) fails to capture the effect of magnetization rotation leading to reversible as well as irreversible processes.

To relax the first assumption, the moving model [14] has been proposed which, along the lines of the mean-field theory, introduces the idea of an effective field, $H_{eff}$, rather than the applied field $H$, being responsible for the state of $\gamma_{ab}$:

$$H_{eff} = H + \alpha M \quad (2)$$
where $\alpha$ is a constant or a function of the magnetization configuration $M$ on which $H$ is applied. The moving model resulted in more realistic loops, levying the congruency.

![Diagram](image1.png)

**Figure 2.** The building blocks of the model for stress dependent magnetization: the SW hysteresis operator accounting for interactions, the probability density function and the Preisach plane

The vector hysteresis model presented in [15] has been developed along the lines of the Preisach formalism. The scalar elementary hysteresis operator $\gamma_{ab}$ is replaced by a vector one, such as the Stoner-Wohlfarth mechanism of coherent rotation of the magnetization for an isolated, single domain, ellipsoidal particle with easy axis, $x$, along the long axis of the ellipse, and a transverse hard axis, $y$. The mechanism is conveniently depicted by the, so called, SW – astroid (figure 2) which is the locus of solutions to the equation

\[ h_x^{2/3} + h_y^{2/3} = 1 \]  

where $h_x$ and $h_y$ are the components of the input field $H$, normalized to the particle’s anisotropy field $H_k$ along the easy and hard axis, respectively. When a field $h$ is applied, the angle $\phi$ of the magnetization vector, $m$, to the easy axis, is the solution to the transcendental equation

\[ h_x \tan \phi - h_y \sin \phi = 0 \]  

which is the result of the minimization of the free energy equation for an isolated, ellipsoidal magnetic particle with uniaxial anisotropy under the influence of an applied field, $h$.

Interactions are included along the lines of the Preisach formalism, *i.e.* the SW operator, $\gamma_{ab}$, is allowed to shift along the easy axis. Then, the magnetization vector response $M$ for an applied field $H$ is given by

\[ M(H) = \int_{a \neq b} \rho(a,b) \gamma_{ab} H dadb \]  

\[ (5) \]
Equation 5 describes the response for a system where the magnetic domains share the same easy axis, ie a perfectly oriented system with uniaxial anisotropy. For a more realistic response, dispersion of orientations is added by superimposing the responses of angularly distributed perfectly oriented models:

$$
\mathbf{M}(H) = \int \rho(\theta) d\theta \int_{a>b} \rho(a,b) \eta_{ab} H \, dadb
$$

(6)

where $\rho(\theta)$ is a probability density function of angles of anisotropy axes.

Figure 3. Computed hysteresis loops along the easy (x) and hard (y) axis (left) and the transverse $m_y$ vs the longitudinal $m_x$ component of the magnetization (right)

Figure 3 demonstrates the vector properties of the model depicting the dependence of the magnetization along the easy and the hard axis for a major loop field sequence applied at a $\pi/6$ angle to the easy axis. The computed results are in agreement with experiments [15] and the loops predicted by this model are non-congruent [16-17].

3. Stress-dependent magnetization and Preisach modeling

Researchers from several laboratories have reported results on the effect of stress on material parameters such as the Barkhausen Noise, the coercivity, the remanence or the permeability of ferromagnets [4,18-19]. The results concern measurements in loaded and unloaded states. In the loaded case, the applied stress has the effect of an effective field assisting the magnetization processes along the direction of application. Barkhausen Noise, remanence and loop squareness increase with strain as the material enters the plastic deformation region but they decrease as the material approaches fracture. The increase is associated with the increase of dislocations but as the material approaches fracture, further domain wall movement is progressively hindered and rotations are more prominent.

Upon unloading, the Barkhausen Noise is lower with respect to the loaded case. As described in the introduction, the deformed material may be considered as consisting of a hard mechanical phase with high dislocation density and a soft phase with low dislocation density resulting in long range internal stresses, mostly compressive in nature, which play a predominant role upon the removal of the applied stress [18-19]. Remanence is found to decrease with strain which is an indication of the presence of transverse domains. Domain rotation and and non-180° wall motion become dominant as magnetization reversal mechanisms [19] while the double-peaked Barkhausen Noise envelop and permeability, characteristic of strained materials [figure 4], appears when the magnetization is perpendicular to the easy magnetization axis and is associated to 90° domain wall rotation [18].

Based on the above results, it is clear that a macroscopic hysteresis model should be able 1) to incorporate the effect of long-range stress-related interactions and 2) to account for the rotation of transverse domains 3) to account for the effect of compressive fields.
The vector model proposed for the modelling of stress dependent magnetization [12] is based on equation 6 but uses a weighed mixture of gaussians (figure 2) to construct the Preisach density, ie a central density, \( \rho_2 \), and two side densities, \( \rho_{1,3} \) [17]:

\[
\rho(a, b) = w_1 \rho_1(a, b) + w_2 \rho_2(a, b) + w_3 \rho_3(a, b)
\]

where \( w_1 + w_2 + w_3 = 1 \).

The central density, \( \rho_2 \), is the pdf of operators with \( a > 0, b < 0 \) (figure 2), accounts for the switching occurring around the coercivity and corresponds to the second and main peak in the permeability curve. The first side density, \( \rho_1 \), is the pdf for operators with \( a, b > 0 \), ie the interaction field is greater than the coercivity and opposite to the direction of the applied field. It accounts for the first peak observed in the differential permeability (figure 4), when the field is decreasing from positive saturation, before remanence. This peak may be related to the switching that occurs under the effect of strong stress-related magnetostatic interactions, before the applied field becomes zero. The third side density, \( \rho_3 \), has an odd symmetry to the first one, with \( a, b < 0 \), ie the interaction field is greater than the coercivity and opposite to the direction of the applied field. It is responsible for the third peak observed in the permeability before reaching saturation (descending curve). Approaching negative saturation, there are magnetic domains which experience positive magnetostatic interactions high enough to oppose their alignment with the field.

4. Results

The model described by equations 6 and 7 has been used to calculate hysteresis loops and the corresponding differential permeability curves as well as the derivative of the latter ones, for various sets of model parameters in order to reproduce the effect of plastic strain on: the maximum inductance, \( B_{\text{max}} \); the remanence, \( B_r \); the coercivity, \( H_c \); the value and the location of the primary and secondary peak of the differential permeability. Note that \( B_{\text{max}} \) is used instead of saturation magnetization, for practical purposes, in order to signify the magnetization value beyond which no further irreversible processes occur. Following the ceteris paribus principle, the role of the following parameters is examined: central distribution parameters, \( \mu_2 \) and \( \sigma_2 \), side distribution parameters, \( \mu_{1,3} \) and \( \sigma_{1,3} \), weights \( w_{1,2,3} \).

The results are compared qualitatively against experimental data obtained on commercial low carbon electrical steel laminations of 0.5mm thickness. Tensile stress has been applied at constant strain rates, namely 0.1, 0.5 and 1 mm/min, and the hysteresis loop of the sample was measured at various strain levels from 0 till fracture while the sample was loaded and after unloading. The hysteresis loop measurements were obtained using the in-house ac hysteresiograph at various frequencies from 0.1 to 1 Hz. The results are in arbitrary units (a.u.) because the hysteresiograph is not calibrated. The magnetic field and the tensile stress are applied along the length of the lamination; the output is measured along the same direction. The output of the hysteresiograph is a voltage pulse proportional to the differential permeability, \( \mu_{\text{diff}} \) of the sample, the integration of which yields the \( B(H) \) loop. The results presented in this work concern hysteresis loops at fields high enough to achieve \( B_{\text{max}} \); frequency of 0.1 Hz; strain rate of 0.5 mm/min; samples after unloading, in order to demonstrate and discuss the performance of the model in the unloaded case, where the effect of residual stresses is prominent.

Fig. 4 shows the effect of strain on the magnetization process. At zero strain, the hysteresis loop is more square and the maximum value of the permeability curve occurs near the coercivity, as expected. As the strain increases, the maximum inductance \( B_{\text{max}} \) and the remanence \( B_r \) decrease, the coercivity \( H_c \) increases, the maximum permeability \( \mu \) decreases and the permeability curve has two peaks: a primary peak around the coercivity and a secondary one as remanence is approached from saturation: switching occurs even before the field changes sign and the permeability changes in a non-monotonic way. This non-monotonic field dependence of the permeability, often referred to as negative permeability or ‘dip’ in permeability [6-7,11,21], is associated with residual stresses and will be
discussed in the following section. A useful tool in evaluating the effect of strain on permeability is its derivative $d\mu$ (figure 4).

Figure 5 depicts the calculated hysteresis and permeability curves for various values of angular dispersion $\phi$ while keeping all other model parameters constant. This parameter takes into account the effect of easy axes dispersion which is expected to increase with strain. As the angular dispersion increases the maximum inductance $B_{\text{max}}$ and the remanence $B_r$ as well as the maximum permeability $\mu$ decrease which is in agreement with the experimental evidence. However, the coercivity of the computed loops decreases which indicates that angular dispersion is not sufficient to explain the effect of strain on the magnetization process.

Next, the effect of the mean values of the side densities, $\mu_1$ and $\mu_3$, is examined. Figure 5 shows the calculated results for side densities centered at +/- 0.1 $\mu_2$, 0.2 $\mu_2$, 0.3 $\mu_2$ from the central one, $\mu_2$. $B_{\text{max}}$ and $H_c$ are practically unaffected but the squareness of the hysteresis loop at the remanence and the coercivity as well as $\mu$ decrease as the side densities move away from the central density to stronger fields. Furthermore, the secondary peaks of the permeability are enhanced and occur at stronger fields. This is consistent with the effect of increasing strain.

Varying the variance of the side densities, it is possible to fine tune the peak heights in the permeability curve: narrower distributions lead to enhanced secondary peaks with respect to the primary one. Finally, the weights of the densities control the relative heights between the primary and secondary permeability peaks.

Figure 4. From top to bottom: Measured hysteresis loop $B(H)$; differential permeability, $\mu$; derivative of the differential permeability measurement, $d\mu$, for the descending curve at various strain levels.
Figure 5. Calculated hysteresis loops (left) and permeability curves for the descending branch (right) at angular dispersions $0 < \phi_1 < \phi_2 < \phi_3$.

Figure 6. Calculated hysteresis loops (left) and permeability curves for the descending branch (right) for side densities located at $+/-0.1\mu_2$, $0.2\mu_2$, $0.3\mu_2$ with respect to the central density; the respective contour plots of the characteristic density $\rho(a,b)$ are shown at the bottom.
The results presented so far concern the effect of each model parameter while the rest are kept constant. Figure 7 presents the results obtained when all parameters are varied in order to reproduce the phenomenology observed experimentally with increasing strain (figure 4). At zero strain, the effect of the side densities is negligible and the angular dispersion is lowest. In order to generate the curves for increasing strain levels, $\varepsilon_1 < \varepsilon_2 < \varepsilon_3$, the mean value of the central density $\mu_2$ increases, $\mu_1$ and $\mu_3$ move away from $\mu_2$ towards higher fields, the ratio $w_1/w_2$ increases and so does the angular dispersion.

5. Discussion

The parameters of the central distribution, $\mu_2$ and $\sigma_2$, control the coercivity and the squareness at the coercivity respectively and as a consequence they control the value and location of the primary peak of the differential permeability. Therefore, we can claim that the mean $\mu_2$ is related to the switching via 180° domain wall movement which is associated to pinning. As strain increases, the dislocation density increases, grains become smaller and as a result pinning increases. The parameter $\sigma_2$ is related to the mixture of exchange and magnetostatic interactions, ie short-range vs long-range interactions in the material.

The effect of angular dispersion is very strong (figure 5) affecting the maximum induction, the loop squareness parameters, the coercivity, and the values of the primary and secondary permeability peaks.
All of the above decrease with increasing dispersion which underlines the important role of rotations. The increase in strain and the build-up of residual stresses leads to smaller grains with different crystalline anisotropy axes. The applied magnetic field forces the domain magnetizations to rotate towards its direction in order to minimize their energy. This leads to lower induction levels and lower squareness at the remanence and coercivity.

The side densities represent the effect of the build-up of the internal demagnetizing field due to residual stress. As residual stresses increase, the magnetoelastic energy, which is a function of the magnetostriction and the elastic properties of the material, increases. To counterbalance this increase in energy, an effective mean field opposing the applied field is established. Increased mean values, $\mu_1$ and $\mu_3$, result in lower coercivity, remanence and permeability peak value. The secondary permeability peaks are enhanced and shift away from the zero field as these parameters increase suggesting that the internal demagnetizing fields are stronger as strain increases.

Finally, the weights, $w_1, w_2, w_3$, of the three pdfs are related to the degree magnetic domains are affected by internal demagnetizing fields and the strength of these fields with respect to their coercive fields and other interactions they experience.

Based on the above analysis, the mechanisms and factors affecting the stress-dependent magnetization processes are three: i) increased pinning ii) dispersion of anisotropy axes iii) internal demagnetizing fields. The first is more important at low strain levels when dislocations increase and propagate fairly easily. This explains why at low strain levels the most prominent increase in coercivity is observed [4]. As strain increases and the material is well into the plastic deformation region, the other two mechanisms prevail leading eventually to deterioration of magnetic structure and material failure. These observations hold for both loaded and unloaded materials but the effect of residual stresses is more obvious in the later case.

Micromagnetic calculations using the Landau-Lifshitz equation to study the effect of dislocations and the orientation of the crystallographic axes on the magnetization have shown that magnetoelastic energy becomes more important as dislocations increase enforcing the magnetic dipoles to align along preferable directions corroborating our results [8]. Along the same lines, RFIM modeling has shown the effect of an internal demagnetizing field due to long-range dipolar interactions [6] aiming to minimize the energy of the material. These interactions are enhanced in high inductance regions by the residual compressive stresses causing rotation.

The model presented reproduces the double peak observed in the permeability the origins of which are still under investigation by several researchers. The interest is focused on the finding that the permeability becomes negative approaching remanence. Measurements of the Preisach density on samples deformed under tensile stress, designed to be used with a moving Preisach model [14] have associated the dip in susceptibility with negative pdf values. Negative pdf values have also been measured in the FORC analysis [20]. Monte Carlo simulations have shown that the negative susceptibility around zero field is indeed associated to dislocations because of spin fluctuations [7]. Finally, negative Barkhausen jumps may be possible if the increase in the local Zeeman energy is associated with a local magnetic configuration with the magnetization opposing the applied field [21]. In our approach, the phenomenology is reproduced using a weighed sum of densities where the central density represents the coercivity and magnetostatic or exchange interactions of the material and the side density accounts for the effect of the internal demagnetizing fields established with increasing strain.
The identification of this model is still under investigation. Preliminary results show that $B_{\text{max}}$ is controlled by angular dispersion, $H_c$ by the central density $\rho_2$. The identification of the other parameters should be based on the differential permeability curve rather than the hysteresis curve because the side densities determine the position and relative heights of the permeability peaks. The derivative of the differential permeability is a useful metric in that direction since it is related to the location of the permeability ‘dip’.

6. Conclusion
A vector model utilizing the Stoner-Wohlfarth mechanism of coherent rotation and the statistical approach to interactions of the Classical Preisach Model is proposed for the modelling of the hysteresis phenomenology in stress dependent magnetization. The effect of residual stresses is modelled by a weighed mixture of pdfs allowing for nonzero density values in the triangular areas of the Preisach plane where the interaction fields are stronger than the coercive fields on the underlying assumption that the residual, mostly compressive, stresses establish a long-range demagnetizing field. The modelling results are in qualitative agreement with experimental findings corroborating the results of other modelling approaches. An interesting remark is that the negative permeability observed in strained materials approaching the remanent state of magnetization is controlled by the side densities which represent the long-range demagnetizing fields. The model parameters are related to material parameters but the identification should be based on the differential permeability curve along with its derivative as well as on the hysteresis loop.

7. References