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The generalized uncertainty principle as quantum gravitational friction

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The generalized uncertainty principle as quantum gravitational friction

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Abstract. In this article we present a dissipative Schrödinger–Langevin–like Hamiltonian which incorporates implicitly the deformed commutation relations which are linear in particle momenta due to a generalized uncertainty principle. This result is based on interpreting the deformation parameter as quantum gravitational friction on the configuration space.

1. Introduction

After Mead [1], who was the first who pointed out the role of gravity on the existence of a fundamental measurable length, a considerably ammount of effort has been devoted to study the modification of the Heisenberg uncertainty principle, known as Generalized Uncertainty Principle (GUP), together with the consequences it leads to [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. The specific form of the GUP and its associated commutation relations, together with the physical consequences they lead to have been recently studied [19, 20]. Moreover, there has been recent interest in a particular version of the GUP [22, 21, 23], which predicts not only a minimum length but also a maximum momentum [14, 15, 16].

As shown in [19], the effects of this GUP can be implemented both in classical and quantum systems by defining deformed commutation relations by means of

$$x_i = x_{0i} \; ; \; p_i = p_{0i} \left[1 - \gamma p_0 + 2\gamma^2 p_0^2\right], \tag{1}$$

where $[x_{0i}, p_{0j}] = i\hbar\delta_{ij}$ and $p_0^2 = \sum_{j=1}^{3} p_{0j}p_{0j}$ and $\gamma = \gamma_0/m_p c$, being γ_0 an adimensional constant.

These GUPs give place to deformed commutation relations which are linear and/or quadratic in particle momenta, depending on the theories which are consistent with the corresponding GUP (theories with a GUP were shown to be equivalent to doubly special relativity theories in certain sense [24]). When acting on massive particles, this minimum–legth and maximum–momentum new form of the GUP has been shown to be equivalent to a gravitationally–induced damping process in an Ohmic environment at zero temperature [25]. Therefore, both minimum length and maximum momentum give place to quantum gravitational friction for massive particles. In this work, a dissipative Schrödinger–Langevin–like Hamiltonian which incorporates implicitly this specific GUP will be introduced.

2. Quantum gravitational friction

Let us consider a non–relativistic Hamiltonian of the form $H = \frac{p^2}{2m} + V(x)$. To simplify notation p_0 will be denoted as p (only 1D systems will be considered). If the deformed commutator

$$[x, p] = i\hbar \left[1 - \frac{\gamma_0}{m_p c} p + \mathcal{O}\left(\gamma_0^2\right)\right] \tag{2}$$

is introduced, the corresponding Heisenberg equations of motion, $\dot{A} = \frac{i}{\hbar}[H, A]$ (where A is any observable) read $\dot{p} = -V'(x)$ and $m\dot{x} = p\left[1 - \gamma_0 \frac{p}{m_p c}\right]$. Therefore, the evolution equation in the configuration space is

$$m\ddot{x} - \frac{2\gamma_0}{m_p c} V'(x) m\dot{x} + V'(x) = 0. \tag{3}$$

Moreover, after defining

$$\bar{\alpha}(x) \equiv -\frac{2\gamma_0}{m_p c} V'(x), \tag{4}$$

Eq. (4) reads

$$m\ddot{x} + m\bar{\alpha}(x)\dot{x} + V'(x) = 0, \tag{5}$$

which resembles a Langevin equation with a position–dependent friction.

This equation can be derived from the so called system + bath approach to open systems [27] in the following way. We split the total Hamiltonian including system, bath and their couplings, in three parts as

$$H = H_s + H_b + H_{sb}, \tag{6}$$

where

$$H_s = \frac{p^2}{2m} + V(x) \tag{7}$$

stands for the Hamiltonian of the isolated system,

$$H_b = \frac{1}{2} \sum_i \left(\frac{p_i^2}{m_i} + m_i \omega_i^2 x_i^2 \right) \tag{8}$$

is the Hamiltonian for the bath, which acts as a resevoir, and can be represented as a set of harmonic oscillators, and

$$H_{sb} = \sum_i \left[\frac{f^2(x) d_i^2}{m_i \omega_i^2} - 2 d_i f(x) x_i \right] \tag{9}$$

expresses an interaction term between the isolated system and the bath, where d_i are appropriate coupling constants. When the system–bath coupling is linear, that is, for $f(x) = x$, the total Hamiltonian is known as the Caldeira–Leggett model [26, 27], widely used in the field of condensed matter physics. Following a standard approach [27], the bath degrees of freedom can be eliminated. Moreover, if Ohmic dissipation (memory free) and a nonlinear sistem–bath coupling is considered, the corresponding Langevin equation reads (after dropping a term involving noise)

$$m\ddot{x}(t) + m\alpha \left[f'(x)\right]^2 \dot{x}(t) + V'(x) = 0. \tag{10}$$

Therefore, we conclude that

$$-\bar{\alpha}(x) = m\alpha \left[f'(x)\right]^2, \tag{11}$$

or

$$-\frac{2\gamma_0}{m_p c} V'(x) = \alpha \left[f'(x)\right]^2. \tag{12}$$

Thus, one can define a quantum gravitational friction as

$$\alpha_{QG} \equiv \frac{2\gamma_0}{m_p c}. \tag{13}$$

Moreover, after absorbing the negative sign by the d_i constants, we note that the non–linear coupling between the system and the bath is given by

$$f(x) \equiv \int_0^x \sqrt{V'(y)}dy. \tag{14}$$

As the noise function (not considered here) is the responsible of the appearance of thermal effects, we can conclude that the deformed commmutator derived from Eq. (1) can be written as

$$[x,p] = i\hbar\left(1 - \frac{\alpha_{QG}}{2}p\right) \tag{15}$$

and think of the deformation as a consequence of a gravitationally–induced damping process due to an Ohmic environment at zero temperature.

Therefore, from the configuration space perspective, the dynamics driven from H_s using deformed canonical commutation relations (linear in p), is completely equivalent to that obtained from the system + bath approach applied to the total Hamiltonian $H = H_s + H_b + H_{sb}$ when the deformation parameter γ is interpreted as a friction coefficient and a specific non–linear system–bath coupling is introduced.

Moreover, by means of the GUP–induced deformed canonical commutator, a general non–relativistic Hamiltonian of the form $H = \frac{p^2}{2m} + V(x)$ transforms into (see, for example [19])

$$\begin{aligned} H &= \frac{p^2}{2m} + V(x) - \frac{\gamma_0}{m m_p c}p^3 + \mathcal{O}(\gamma_0^2) \\ &= \frac{p^2}{2m} + V(x) - \frac{\alpha_{QG}}{2m}p^3 + \mathcal{O}(\gamma_0^2). \end{aligned} \tag{16}$$

Thus, in light of the previous interpretation, the second term of Eq. (16) can be taken as a dissipative one.

A different (but equivalent in certain sense) formulation of the linear GUP as a quantum gravitational friction effect can be stated as follows. In a recent work [28] we derived a generalization of the so–called Schrödinger–Langevin or Kostin equation for a Brownian particle interacting with a heat bath. This generalization is based on a nonlinear interaction model providing a state-dependent dissipation process exhibiting multiplicative noise. The idea of the GLE is to introduce a dissipative potential, V_d, in the time–dependent Schrödinger equation such that the evolution of the average values for the position, velocity and acceleration observables satisfy the classical equation of motion, which results to be a (generalized) Langevin equation [29]. But, as we shown, the GUP leads to a generalized Langevin equation with a position–dependent coupling that turns out to be $f(x) = \int_0^x \sqrt{V'(y)}dy$. Therefore, the formalism developed in Ref. [28] is suitable for the linear GUP case. In fact, following [28] the following damping potential for the GUP can be derived:

$$V_d = -\frac{\alpha_{QG}}{2}\tilde{S} = -\alpha_{QG}\, p\, V(x), \tag{17}$$

where

$$\tilde{S} \equiv \left(\frac{df}{dx}\right)^2 S - 2\int S\frac{df}{dx}\frac{d^2 f}{dx^2}dx \tag{18}$$

is the coupling–dependent phase of the wavefunction.

Therefore, the GUP–dissipative Hamiltonian equivalent to that of Eq. (16) can be written as

$$H = \frac{p^2}{2m} + V(x)\,[1 - \alpha_{QG}\, p] + \mathcal{O}(\gamma_0^2) \tag{19}$$

Notice, that, although Eq. (19) predicts the correct Langevin–like equation derived from the deformed commutation relations, the correct dynamics for the phase space variables can not be obtained from it. This situation is similar to that found within the well known Caldirola–Kanai effective Hamiltonian approach [30].

3. Conclusions

In this work we have argued that the deformation parameter which enters in a generalized uncertainty principle which is linear in particle momenta can be interpreted as a friction coefficient form the configuration space point of view. Moreover, a dissipative Schrödinger–Langevin–like Hamiltonian which incorporates implicitly the deformed commutation relations due to the mentioned generalized uncertainty principle has been introduced.

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