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Critical current in semiconductor nanowire Josephson junctions in the presence of magnetic field

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Abstract. We study theoretically the critical current in semiconductor nanowire Josephson junction with strong spin-orbit interaction. The critical current oscillates with an external magnetic field. We reveal that the oscillation of critical current depends on the orientation of magnetic field in the presence of spin-orbit interaction. We perform a numerical simulation using a tight-binding model. The Andreev levels are calculated as a function of phase difference $\varphi$ between two superconductors. The DC Josephson current is evaluated from the Andreev levels in the case of short junctions. The spin-orbit interaction induces the effective magnetic field. When the external field is parallel with the effective one, the critical current oscillates accompanying the 0-$\pi$ like transition at the cusp of critical current. The distance of cusps increases gradually with increasing of the angle between the external and effective fields. The magnetic anisotropy of critical current is attributed to the spin precession due to the spin-orbit interaction.

1. Introduction

The spin-orbit (SO) interaction has attracted a lot of interest. In narrow-gap semiconductors, such as InAs and InSb, the strong SO interaction has been reported and many phenomena based on the SO interaction are investigated intensively, e.g., spin Hall effect [1]. The SO interaction has a great advantage also for application to the spintronic devices and to quantum information processing. InAs and InSb nanowires are interesting nanostructure for the application and studied in recent experiments, e.g., the electrical manipulation of single electron spin in quantum dots fabricated on the nanowires [2]. Nanowire-superconductor hybrid systems have been also examined to investigate the Majorana fermions induced by the SO interaction and the Zeeman effect [3].

In Josephson junctions, the supercurrent flows when the phase difference $\varphi$ between two superconductors is present. In this paper, we investigate theoretically the Josephson junction of semiconductor nanowires with strong SO interaction. The supercurrent in semiconductor nanowires has been reported by experiment groups [4, 5, 6, 7]. The Josephson effect with SO interaction has been studied theoretically for some materials, e.g., magnetic normal metals [8], where the combination of SO interaction and exchange interaction results in an unconventional current-phase relation, $I(\varphi) = I_0 \sin(\varphi - \varphi_0)$. The phase shift $\varphi_0$ deviates the ground state of junction from $\varphi = 0$ or $\pi$, which is so-called $\varphi_0$-state. The anomalous supercurrent is obtained at $\varphi = 0$. In previous studies, we have pointed out that the anomalous effect is attributed...
The nanowire along the $x$ direction is connected to two superconductors (Fig. 1). At $x < 0$ and $x > L$, the superconducting pair potential is induced into the nanowire by the proximity effect. We assume that the pair potential is $\Delta(x) = \Delta_0 e^{i\phi/2}$ at $x < 0$ and $\Delta(x) = \Delta_0 e^{-i\phi/2}$ at $x > L$, where $\phi$ is the phase difference between the two superconductors. In the normal region at $0 < x < L$, $\Delta(x) = 0$. When a magnetic field is applied to the junction, the Zeeman effect is taken into account in the nanowire. The magnetic field is not too large to break the superconductivity and screened in the superconducting regions. The Hamiltonian is given by $H = H_0 + H_{SO} + H_Z$ with $H_0 = p^2/(2m) + V_{\text{conf}} + V_{\text{imp}}$, the Rashba interaction $H_{SO} = (\alpha/h)(p_y \sigma_x - p_x \sigma_y)$, and the Zeeman term $H_Z = g\mu_B B \cdot \vec{\sigma}/2$, using effective mass $m$, $g$-factor $g$ ($< 0$ for InSb), Bohr magneton $\mu_B$, and Pauli matrices $\vec{\sigma}$. We neglect the orbital magnetization effect in the nanowire. $V_{\text{conf}}$ describes the confining potential forming the nanowire. $V_{\text{imp}}$ represents the impurity potentials.

We consider short junction, where the spacing between two superconductors is much smaller than the coherent length in the normal region, $L \ll \xi$. There is no potential barrier at $x = 0, L$. The Zeeman energy $E_Z = |g\mu_B B|$ and the pair potential $\Delta_0$ are much smaller than the Fermi energy $E_F$.

The Bogoliubov-de Gennes (BdG) equation is written as

$$\begin{pmatrix} H - E_F & \Delta \\ \Delta^\dagger & -(H^* - E_F) \end{pmatrix} \begin{pmatrix} \psi_e \\ \psi_h \end{pmatrix} = E \begin{pmatrix} \psi_e \\ \psi_h \end{pmatrix}$$

(1)

with $\Delta = \Delta(x) \hat{g}$. $\psi_e = (\psi_{e+}, \psi_{e-})^T$ and $\psi_h = (\psi_{h+}, \psi_{h-})^T$ are the spinors for electron and hole, respectively. $\hat{g} = -i\sigma_y$. The energy $E$ is measured from the Fermi level $E_F$. The BdG equation determines the Andreev levels $E_n (|E_n| < \Delta_0)$ as a function of $\phi$.

The ground state energy of junction is given by $E_{gs}(\phi) = -1/2 \sum_n |E_n(\phi)|$, where the summation is taken over all the positive Andreev levels, $E_n(\phi) > 0$. The contribution from continuous levels ($|E| > \Delta_0$) can be disregarded in the case of short junctions [11]. At zero temperature, the supercurrent is calculated as $I(\phi) = (2e/h) |dE_{gs}/d\phi|$. The current is a periodic function for $-\pi \leq \phi < \pi$. The maximum (or absolute value of minimum) of $I(\phi)$ yields the critical current $I_c$.

The BdG equation in eq. (1) is expressed in terms of the scattering matrix [11]. The scattering matrix of electrons (holes) transport in the normal region is given by $\hat{S}_e$ ($\hat{S}_h$). $\hat{S}_e$ and $\hat{S}_h$ are
related to each other by $\hat{S}_e = \hat{S}_h$ for the short junctions. We denote $\hat{S}_e = \hat{S}$ and $\hat{S}_h = \hat{S}^*$. The Andreev reflection at $x = 0$ and $L$ is described by the scattering matrix $\hat{r}_{\text{he}}$ for the conversion from electron to hole and $\hat{r}_{\text{eh}}$ for that from hole to electron. The normal reflection can be neglected. The matrix coefficients of $\hat{r}_{\text{he}}$ and $\hat{r}_{\text{eh}}$, e.g., $\exp\{-i\arccos(E/\Delta_0) - i\varphi/2\}$ for $\hat{r}_{\text{he}}$ at $x = 0$, are calculated from the boundary condition at $x = 0$ and $L$. The SO interaction does not affect the Andreev reflection coefficients. The Andreev levels, $E_n(\varphi)$, are obtained from the product of $\hat{S}$, $\hat{r}_{\text{he}}$, and $\hat{r}_{\text{eh}}$,\[ \det(\mathbf{1} - \hat{r}_{\text{eh}}^* \hat{S}^* \hat{r}_{\text{he}} \hat{S}) = 0. \] Equation (2) is equivalent to the BdG equation (1).

To calculate the scattering matrix $\hat{S}$, we adopt the tight-binding model which discretizes a two-dimensional space ($xy$ plane). The edges of nanowire are represented by a hard-wall potential. The width of nanowire is $W = 12a$ with the lattice constant $a = 10$nm. The Fermi wavelength is fixed at $\lambda_F = 18a$, where the number of conduction channels is unity. The length of normal region is $L = 50a$. The on-site random potential by impurities is taken into account, the distribution of which potential is uniform. We set that the mean free path due to the impurity scattering is $l_{\text{mfp}}/L = 1$. The SO length is $l_{\text{SO}} = L = 0.2$ with $l_{\text{SO}} = k_{\text{so}}^{-1} = h^2/(ma)$. The magnetic field is $\mathbf{B} = Be\theta$ with the angle $\theta$ from the $x$ axis in the $xy$ plane.

3. Results

We consider a sample for the nanowire. For the magnetic field, we introduce a parameter, $\theta_B = E_Z L/(\hbar v_F)$, which means an additional phase due to the Zeeman effect in the propagation of electron and hole. Here, $v_F$ is the Fermi velocity in the absence of SO interaction.

Figure 2 shows the phase difference $\varphi_0$ at the minimum of $E_{\text{gs}}$ when the magnetic field is increased and rotated. In the absence of SO interaction, $\varphi_0$ takes only 0 or $\pi$ exactly and clear 0-$\pi$ transition happens (see Ref. [10]). In the presence of SO interaction, $\varphi_0$ is deviated from 0 and $\pi$, where the anomalous Josephson current is obtained. When the magnetic field is in the $y$ direction ($\theta = \pi/2$), the transition between $\varphi_0 \approx 0$ and $\varphi_0 \approx \pi$ takes place around $\theta_B = \pi/2, 3\pi/2, \cdots$. The transition points are shifted gradually to large $\theta_B$ with decreasing of

![Figure 2](image-url)
angle $\theta$. At $\theta \approx 0$, the transition is not observed in Fig. 2.

The transition points correspond to the positions of cusps of critical current. Figure 3(a) exhibits the critical current when the magnetic field increases. The critical current $I_c$ oscillates as a function of $\theta_B$. The distance of cusps becomes long when the direction of magnetic field is tilted from the $y$ axis. In the case of the parallel magnetic field to the nanowire ($\theta = 0$), $I_c$ has no cusp in accordance with no transition. The critical current at $\theta = 0$ decreases with increase of $\theta_B$ although $\phi_0$ is almost fixed at zero in Fig. 2(b). Figure 3(b) shows $I_c$ when the magnetic field is rotated in the $xy$ plane. The strength of magnetic field is fixed. We find the transition with the angle $\theta$ in Fig. 2(a). The critical current also oscillates as a function of $\theta$. At small magnetic field ($\theta_B < \pi/2$), $I_c$ changes monotonically. The oscillation with $\theta$ can be obtained when the magnetic field is $\theta_B > \pi/2$. If the state of junction at $\theta = \pi/2$ is $\phi_0 \approx \pi$ (0), the critical current shows one cusp (two cusps) in Fig. 3(b). Therefore we can estimate the state at $\theta = \pi/2$ from the magnetic anisotropy of critical current.

4. Conclusions and Discussion
We have studied the DC Josephson effect in the semiconductor nanowire with strong SO interaction. We have examined a numerical simulation using the tight-binding model in the case of short junction. The combination of SO interaction and Zeeman effect in the nanowire results in the anomalous Josephson effect. The critical current oscillates as a function of magnetic field. In the presence of SO interaction, the oscillation of critical current depends on the magnetic field orientation. The oscillation period is the shortest when the magnetic field is perpendicular to the nanowire. For a parallel magnetic field to the nanowire, the 0-$\pi$ (like) transition and the cusp of critical current are not found.

In this numerical model, we have considered the Rashba interaction. In the quasi-one-dimensional nanowire, the effective magnetic field induced by the Rashba interaction is in the $y$ direction. The magnetic anisotropy of critical current is understood intuitively by the spin precession in the propagation of electron and hole. When the external magnetic field is parallel to the effective SO field (the $y$ direction), the spin quantization axis is fixed in that direction. The electron and hole receive the additional phase in the propagation. On the other hand, when
the external field is in the $x$ direction, the spin quantization axes for electron and hole are not parallel with each other since the effective fields for electron and hole are antiparallel. The spin of electron and hole forming the Andreev bound state is rotated, which rotation cancels out the phase $\theta_B$ due to the Zeeman splitting. As a result, the critical current oscillation disappears. In the case of general SO interaction, the effective field would be deviated from the $y$ axis. By measuring the magnetic anisotropy of critical current oscillation, we can evaluate the direction of effective field due to the SO interaction.

Acknowledgments
We acknowledge financial support by the Motizuki Fund of Yukawa Memorial Foundation. We acknowledge fruitful discussions about experiments with Professor L. P. Kouwenhoven, A. Geresdi, V. Mourik, K. Zuo of Delft University of Technology. T.Y. is a JSPS Postdoctoral Fellow for Research Abroad.

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