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L Assirati[1], N R Silva[2,1], L Berton[2], A A Lopes[2], O M Bruno[1,2]

[1] Scientific Computing Group, São Carlos Institute of Physics, University of São Paulo (USP), cx 369 13560-970 São Carlos, São Paulo, Brazil - www.scg.ifsc.usp.br
[2] Institute of Mathematics and Computer Science, University of São Paulo (USP), Avenida Trabalhador são-carlense, 400 13566-590 São Carlos, São Paulo, Brazil

E-mail: `assirati@usp.br`, `nubiars@icmc.usp.br`, `lberton@icmc.usp.br`, `alneu@icmc.usp.br`, `bruno@ifsc.usp.br`

Abstract. In image processing, edge detection is a valuable tool to perform the extraction of features from an image. This detection reduces the amount of information to be processed, since the redundant information (considered less relevant) can be disconsidered. The technique of edge detection consists of determining the points of a digital image whose intensity changes sharply. This changes are, for example, due to the discontinuities of the orientation on a surface. A well known method of edge detection is the Difference of Gaussians (DoG). The method consists of subtracting two Gaussians, where a kernel has a standard deviation smaller than the previous one. The convolution between the subtraction of kernels and the input image results in the edge detection of this image. This paper introduces a method of extracting edges using DoG with kernels based on the q-Gaussian probability distribution, derived from the q-statistic proposed by Constantino Tsallis. To demonstrate the method's potential, we compare the introduced method with the tradicional DoG using Gaussians kernels. The results showed that the proposed method can extract edges with more accurate details.

1. Introduction

Image processing is designated as any type of signal processing where the input is an image and the output can be another image or a set of features extracted from the input image. Once the computer vision involves the identification and subsequent classification of certain objects in a given image, edges detection is an essential tool in image analysis. When performing edge detection on an image, there is a reduction of the amount of information to be processed because the redundant information (considered less relevant) can be disconsidered.

The segmentation by edge detection is based on two important concepts: similarity and discontinuity. Thus, the algorithms look for points (or curves and contours) of the digital image where the intensity changes abruptly. This sudden change in intensity may occur for various reasons, as example, the orientation discontinuities in a surface and changes in brightness and illumination in a scene. Applications for the edge detection method are found in various fields of science: medicine [1], engineering and satellite images [2], robotics and machine vision [3].

There are several methods for edge detection, like: Canny, Sobel, Prewitt, and based on Gaussian masks (kernels), as Laplacian of Gaussian (LoG) and Difference of Gaussian (DoG) [4]. The DoG method generally uses classical Gaussians in its approach. But this work suggests the use of q-Gaussian for the composition of the mask that will be applied to the image to extract its edges. The q-Gaussian probability distribution comes from the q-algebra introduced by Tsallis. The composition of the DoG filter using q-Gaussian kernels is motivated by results obtained in other fields of science [5]. The potential of the introduced method is demonstrated by comparing with the traditional method for DoG. One can notice that the presented method is able to perform edge extraction with greater detail.

2. Laplacian of Gaussian vs Difference of Gaussians

Consider the one-dimensional Gaussian distribution: $f(x,\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2)}{2\sigma^2}\right)$, with $-\infty < x < \infty$, and $\sigma > 0$, where μ is the mean, σ is the standard deviation and σ^2 is the variance.

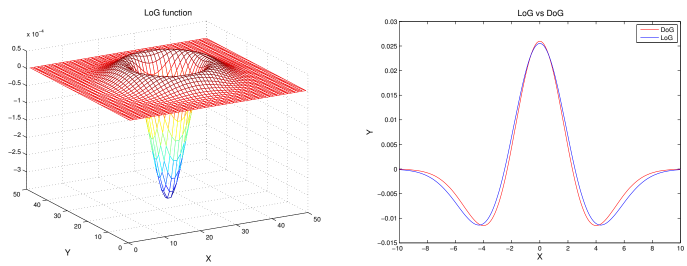
If we take the second derivative of the one-dimensional Gaussian function considering $\mu = 0$, we obtain the Ricker wavelet: $\psi(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \frac{1}{\sigma^4}(x^2 - \sigma^2) \exp \frac{-x^2}{2\sigma^2}$.

In the two-dimensional the Gaussian distribution becomes:

$$f(x,y,\sigma) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{(x^2+y^2)}{2\sigma^2}\right), -\infty < x, y < \infty, \sigma > 0. \tag{1}$$

The Laplacian of Gaussian LoG is a multidimensional generalization of the Ricker wavelet. To obtain it we need to take the two-dimensional Laplacian of the Gaussian distribution:

$$LoG(x,y) = -\frac{1}{\pi\sigma^4}\left[1 - \frac{x^2+y^2}{2\sigma^2}\right] \exp -\frac{x^2+y^2}{2\sigma^2} \tag{2}$$



(a) (b)

Figure 1. (a) LoG Function, $\sigma = 2.5$, (b) Difference of Gaussians vs Laplacian of Gaussians 1D.

However, in practice the Laplacian of Gaussian (LoG), Figure 1(a), is approximated by the Difference of Gaussians function (DoG) since this reduces the computational costs for two or more dimensions. The DoG is obtained by performing the subtraction of two Gaussian kernels where a kernel must have a standard deviation slightly lower than the previous one. Figure 1(b) compares the LoG function with $\sigma = 2.5$ with the DoG function using kernels with $\sigma_1 = 2.5$ and $\sigma_2 = 2.15$. The convolution of the DoG filter with the input image generates the edge detection for this image.

3. q-Gaussian

In 1988, Tsallis proposed the non additive statistical mechanics, entitled "Q-statistic" [6]. This theory suggests that different systems require different tools of analysis, appropriated to the particularities of this system. The informational tool entropy, applied to the information theory by Shannon [7] is defined as: $S(x) = -\sum_{x=0}^{W} p(x) \log p(x)$, where $p(x)$ is the occurrence probability, and W is the total number of probabilities.

The generalization proposed by Tsallis gives the definition of the q-entropy: $S_q(x) = \frac{1}{q-1}\left(1 - \sum_{x=0}^{W} p^q(x)\right)$, where $p(x)$ is the occurrence probability, W is the total number of probabilities and q is an adjustable parameter, freely variable. The correct choice of certain q parameters can evidence important characteristics of the system. When $q \to 1$, one retrieves the standard entropy.

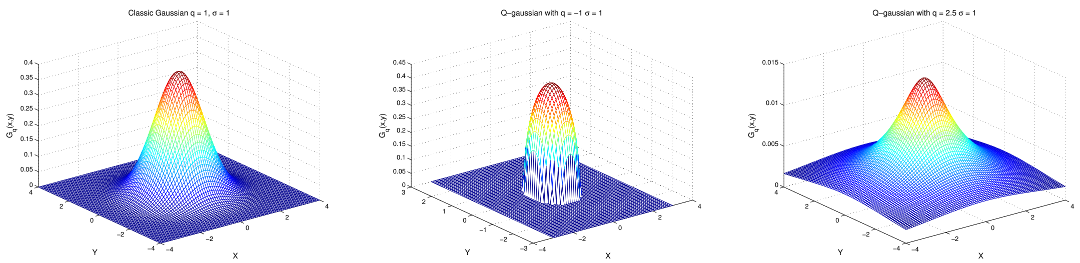
The q-Gaussian probability distribution comes from the maximization of the Tsallis entropy under appropriate constraints [8]. Again, when $q \to 1$, one retrieves the Gaussian distribution. The q-Gaussian is defined as:

$$G_q(x) = \frac{1}{C_q\sqrt{2\sigma^2}} \exp_q \frac{-x^2}{2\sigma^2}, \tag{3}$$

with $\exp_q(x) = [1 + (1-q)x]^{\frac{1}{1-q}}$ and

$$C_q = \begin{cases} \frac{2\sqrt{\pi}\Gamma(\frac{1}{1-q})}{(3-q)\sqrt{1-q}\Gamma(\frac{3-q}{2(1-q)})} & \text{if } -\infty < q < 1 \\ \sqrt{\pi} & \text{if } q = 1 \\ \frac{\sqrt{\pi}\Gamma(\frac{3-q}{2(q-1)})}{(3-q)\sqrt{q-1}\Gamma(\frac{1}{q-1})} & \text{if } -\infty < q < 1\,. \end{cases}$$

The same way as the classical Gaussian has a Two-Dimensional version, we can derive the multidimensional generalization to q-Gaussian. The bi-dimensional q-Gaussian is defined by $G_q(x,y) = \frac{\exp_q -(x^2+y^2)/(2\sigma^2)}{2C_q^2\sigma^2}$. It is important to note that curves with the same parameter σ can have its shape changed adjusting the q parameter, adapting it to the peculiarities of the problem in which it is employed. Figures 2(a), 2(b) and 2(c) show some representatives of the family of 2D q-Gaussian.



(a) $q = 1$, $\sigma = 1$ (b) $q = -1$, $\sigma = 1$ (c) $q = 2.5$, $\sigma = 1$

Figure 2. Q-Gaussian Function: Same σ, but different shapes.

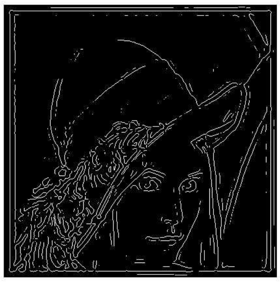
4. Method and Results

This study introduces the use of DoG method using q-Gaussian kernels as an alternative to traditional use of Gaussian kernels in edge detection. Following the metric proposed by the DoG filter, standard deviations σ_1 and σ_2 are setted, with σ_2 smaller than σ_1. After the filter having the appropriate size, we should set it with the input image in gray scale. After the convolution we identify the edges by using the "zero cross" detector.

The results for edge detection using the method DoG with q-Gaussian kernels show up rich in detail when compared to the method DoG with Classic Gaussian kernels because the q-Gaussian probability distribution have the adjustable parameter q. This parameter allow us to define the degree of detail that we seek in our detection. Figures 3(a), 3(b) and 3(c) show results obtained from q-Gaussian using $\sigma_1 = 0.2$ and $\sigma_2 = 0.1$.

(a) $q = 1$

(b) $q = -1$



(c) $q = 2.5$

Figure 3. (a)DoG with Classic Gaussian kernels, (b) and (c) DoG with q-Gaussian kernels.

5. Conclusions

The results presented in this work show that using q-Gaussian kernels proves to be an excellent alternative to the classical Gaussian kernels. Compared to the DoG filter with kernels using the normal distribution of probabilities, we note that we gain in details of edge detection. That is because in addition to the variable parameter σ, responsible for more or less blurring (Gaussian blur), we also have the entropic index q, variable and responsible for the shape of q-Gaussian, being able to get more details that the traditional approach when both have the same blur. The extensiveness or not extensiveness of the entropy depends on the system characteristics. Thus, it can be extended for certain values of q. In this point, we can apply this concept to our work. By using the q-Gaussian method, the entropic index q allow us adjust the function used in the filter to get the details and results that are more relevant.

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References

[1] Gudmundsson M, El-Kwae E and Kabuka M 1998 *Medical Imaging, IEEE Transactions on* **17** 469–474
[2] Augusto G, Goltz M and Demísio J 1984 1044–1045
[3] Jain R, Kasturi R and Schunck B 1995 *Machine vision*
[4] Gonzalez R and Woods R 2011 *Digital Image Processing* (Pearson Education) ISBN 9780133002324
[5] Soares I J A 2013 *Desenvolvimento de uma base de funções paramétricas para interpolação de imagens médicas.* Ph.D. thesis University of São Paulo - Faculdade de Filosofia, Ciências e Letras de Ribeirão Preto
[6] Tsallis C 1988 *Journal of Statistical Physics* **52** 479–487
[7] Shannon C E 1948 *The Bell System Technical Journal* **27** 379–423–623–656
[8] Tsallis C 2011 *Entropy* **13** 1765–1804