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Wavefront Measurement for Long Focal Large Aperture Lens Based on Talbot Effect of Ronchi Grating

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Abstract. We propose a wavefront measurement method based on Talbot effect. It is well known that the wavefront of large aperture long focal lens is difficult to be measured. Here a new method for wavefront measurement of large aperture long focal lens is presented, which is accomplished by fitting the slope of sub-wavefront in the whole aperture while the sub-wavefront slope is got by calculating the shifted moiré fringe numbers formed by Talbot effect of Ronchi grating in different position of the aperture. The formula of calculating the angle of subwavefront slope from the shifted moiré fringe numbers is derived. Error analysis demonstrates that this method can be applied to the wavefront testing with high accuracy.

1. Introduction

A presently widely used approach to measure the wavefront is the Shack-Hartmann sensor [1]. The optical wavefront is sampled by an array of microlenses. In the back focal plane of the lenses the displacement of the spots - generated by each microlens – gives a two-dimensional derivative field of the incident front. An instantaneous wavefront surface can be derived by computing the bary-center of each spot and performing a least-squares fit of the resulting measurements of the local slope to a basis of functions chosen to represent the surface. Wavefront measurement accuracy depends on the number of the microlens array and the detector. Interference method can also be used to wavefront measurement. While for long focal large aperture lens(D>400mm), interference method are not so effective. For large aperture reference plane or spherical plane is difficult to fabricate, and also large aperture collimating laser beam is hard to produce, large aperture interferometer is very expensive and large. We present a novel method for measuring the large aperture long focal lens.

The character of self-image of period object has been pointed out by general Talbot effect theory [2-5]. Two Ronchi gratings, which have the same grating period, are amounted with a Talbot distance and a very slight angle between their grating lines orientations. The first grating, illuminated by a thin beam, has its self-image at the location of second grating and moiré stripe appears behind the second grating. When the angle of the illuminated beam on the first grating changed slightly perpendicular to the grating lines, the moiré fringe will be observed to have an obvious shift. Therefore, the wavefront slope at different location in the aperture can be determined accurately by the shifted moiré fringe numbers. Then the wavefront can be tested by fitting the sub-wavefront slopes in all location of the aperture.
2. The principle and system of the measurement

The complex amplitude transmittance rate of the Ronchi grating $G(1)$ is presented by

$$g(x) = \sum_{n=-\infty}^{\infty} A_n \exp[i\frac{2\pi}{p} nx] \quad n = 0, \pm 1, \pm 2, \ldots$$  \(1\)

Where $p$ is the period of the Ronchi grating, as show in Figure 1. Assume the incident beam is a spherical wave:

$$u(x, y) = \frac{B_0}{|R|} \exp\left\{ i \frac{2\pi}{\lambda} \left\{ \frac{(x - R \sin \alpha)^2 + (y - R \sin \beta)^2}{2R} \right\} \right\} \times \sum_{n=-\infty}^{\infty} A_n \exp[i\frac{2\pi}{p} nx]$$  \(2\)

Where $R$ is the radius of the spherical wave, $\alpha$ and $\beta$ are the angle in $x$ direction and $y$ direction of the vector of incident beam, $B_0$ is the amplitude of the wave at the place of $R=1$ and $z$ is the propagation direction. The complex attitude of the wave after the grating can be expressed as:

$$u(x, y, 0) = u(x, y, R) \cdot g(x)$$

$$= \frac{A_0 B_0}{|R|} \exp\left\{ \frac{2\pi}{\lambda} \left\{ \frac{(x - R \sin \alpha)^2 + (y - R \sin \beta)^2}{2R} \right\} \right\} \times \sum_{n=-\infty}^{\infty} A_n \exp[i\frac{2\pi}{p} nx]$$  \(3\)

If only take $0$ and $\pm 1$ orders into account, the expression can be simplified as:

$$u(x, y, 0) = \frac{A_0 B_0}{|R|} \exp\left\{ \frac{2\pi}{\lambda} \left\{ \frac{(x - R \sin \alpha)^2 + (y - R \sin \beta)^2}{2R} \right\} \right\}$$

$$+ \frac{A_1 B_0}{|R|} \exp\left\{ \frac{2\pi}{\lambda} \left\{ \frac{(x - R \sin \alpha + \frac{\lambda R}{p})^2 + (y - \sin \beta)^2}{2R} \right\} \right\} + \frac{\lambda R}{p} \frac{\frac{\lambda^2 R}{2p^2}}$$

$$+ \frac{A_{-1} B_0}{|R|} \exp\left\{ \frac{2\pi}{\lambda} \left\{ \frac{(x - R \sin \beta - \frac{\lambda R}{p})^2 + (y - \sin \beta)^2}{2R} \right\} \right\} - \frac{\lambda R}{p} \frac{\frac{\lambda^2 R}{2p^2}}$$  \(4\)

**Figure 1.** A diagram for Talbot effect with illuminated spherical beam.

From equation (4), we can see that after the spherical wave passed through the grating $g(x)$, the
diffracted wave became three spherical waves. Let $A_1 = A_1^\dagger$, the intensity distribution at a distance $d$ behind the grating $G_1$ can be expressed as follows:

$$g_1(x,y,d) = u(x,y,d) - u^\dagger(x,y,d) = C_0 + C_1 \cos \left( \frac{2\pi}{p} \left( \frac{R}{d+R} x + \frac{Rd\sin \alpha}{(d+R)^2} \right) \right) \cdot \cos \left( \frac{2\pi}{\lambda} \left[ \frac{\lambda^2 R^2 + \lambda^2 R + \lambda^2 R^2}{2(d+R)^2} \right] \right) + O(x)$$  \hspace{1cm} (5)

when $2\pi/\lambda \lesssim \lambda^2 R^2/(2(d-R)p^2 - \lambda^2 R/2p^2) = 2\pi m (m=1,2,3,\ldots)$, we can obtain the Talbot image of the grating $G_1$ at the location giving by $d_T = \frac{2mp^2}{2mp^2 / R - \lambda}, m = 0, \pm 1, \pm 2, \ldots$.

If we place another grating $G_2$ which has the same period as $G_1$ at the place of Talbot image of $G_1$, moiré fringe will appear. Equation (6) is the complex amplitude transmittance rate of grating $G_2$:

$$g_2(x,y) = \sum_{n=-\infty}^{\infty} A_n \exp \left[ \frac{2\pi i}{p} \left( x \cos \theta + y \sin \theta \right) \right].$$  \hspace{1cm} (6)

Where $\theta$ is the angle of the grating line between $G_1$ and $G_2$. Intensity distribution of the moiré fringe can be expressed as follows:

$$f(x,y) = g_1(x,y,d) \cdot g_2(x,y) = C_0 D_0 + C_1 D_0 \cos \left( \frac{2\pi}{p} \left( \frac{R}{d_T + R} x + \frac{Rd_T}{d_T + R} \sin \alpha \right) \right) + C_0 D_1 \cos \left( \frac{2\pi}{p} \left( x \cos \theta + y \sin \theta \right) \right)$$

$$+ \frac{1}{2} C_1 D_1 \cos \left( \frac{2\pi}{p} \left( \frac{R}{d_T + R} \cos \theta \right) x + \frac{Rd_T}{d_T + R} \sin \alpha + y \sin \theta \right)$$

$$+ \frac{1}{2} C_1 D_1 \cos \left( \frac{2\pi}{p} \left( \frac{R}{d_T + R} - \cos \theta \right) x + \frac{Rd_T}{d_T + R} \sin \alpha - y \sin \theta \right)$$  \hspace{1cm} (7)

So we obtain a corresponding relationship between $\alpha$ and $n$ ($n$ is the shifted moiré fringe numbers), which is given by:

$$\frac{2\pi}{p} \cdot \frac{Rd_T}{d_T + R} \sin \alpha = 2n\pi$$  \hspace{1cm} (8)

That is,

$$\sin \alpha = \frac{(d_T + R)p}{Rd_T} \cdot n$$  \hspace{1cm} (9)

For the long focal length measurement: $d_T \ll R$, equation (9) can be simplified as:

$$\sin \alpha \approx \frac{p}{d_T} \cdot n$$  \hspace{1cm} (10)

If we can get the shifted moiré fringe numbers $n$, the angle of the beam vector $\alpha$ can be calculated. Figure 2 shows the principle of the measurement system. For large aperture lens, an x-y direction scanning system was used to measure the slopes of the subwavefront at different location in the aperture.

The image of the moiré fringe can be mapped to the CCD detector by the image lens, through the frame grabber, the digital image of moiré fringe was obtained. The shifted moiré fringe numbers can be accurately calculated by image process such as correlation processing. Using scanning method, we can get the slope of subwavefront at each scanning point during the aperture, the wavefront can be reconstructed by fitting algorithm such as least-squares fit [7]. As Figure 3 shows.
3. Results

In our experiment, moiré fringe is mapped to a CCD camera by image system, and its digital image acquisition is realized by frame grabber. The shifted moiré fringe numbers can be accurately calculated by images processing.

From equation (10), $p$ and $z_T$ are invariable, the accuracy of $\alpha$ depends on the accuracy of $n$, the shifted moiré fringe numbers.

$$\cos \alpha \cdot \Delta \alpha = \frac{p}{z_T} \cdot \Delta n \quad (11)$$

When $\alpha << 1^\circ$, $\cos \alpha \approx 1$. In our experiment, $p = 0.01 \text{mm}$, $z_T = 100 \text{mm}$, assume the accuracy of shifted moiré fringe numbers can reach $\Delta n = 0.01$, then

$$\Delta \alpha = \frac{p}{z_T} \cdot \Delta n = \frac{0.01}{100} \cdot 0.01 = 10^{-6} \text{ rad} \quad (12)$$

We can see that the moiré fringe is sensitive to the variation of the angle of wavefront vector. If decrease the grating period and increase the Talbot distance, the accuracy would be improved. By calculating the shifted moiré fringe, we can obtain the wavefront slope very accurately.
Figure 4 shows the moiré fringe we get in two orthogonal directions. By calculating the shifted moiré fringe number in orthogonal directions, the space vector of the subwavefront can be decided.

4. Conclusion
In this paper we present a new method for the wavefront measurement of long focal large aperture lens. Theoretical analysis shows that this method can measure the wavefront with a high accuracy. For large aperture lens, the whole aperture is divided into many subaperture, the vector of each subaperture can be tested individually by scanning method. Accuracy as this method is, it is time consuming. This method is not suitable for the instantaneous wavefront measurement.

References
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