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Neutrinos and rare isotopes

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Abstract. The close connection between neutrino physics and the physics explored at rare isotope facilities is explored. The duality between the Hamiltonian describing the self-interacting neutrino gas near the proto-neutron star in a core-collapse supernova and the BCS theory of pairing is elucidated. This many neutrino system is unique as it is the only many-body system driven by weak interactions. Its symmetries are discussed.

As you read the title of this contribution to the proceedings of the conference celebrating contributions of Taka Otsuka to nuclear physics on the occasion of his 60th birthday, you may wonder what neutrinos have to do with exotic nuclei. The answer follows from an examination of the scientific motivations of the physics enterprise with rare isotopes. Exploring physics with rare isotopes would impact many areas of inquiry. It would contribute to our understanding of nuclear structure by exploring the limits of nuclear existence, new shapes and new collective behaviors. Studies of nuclear astrophysics with rare isotope facilities encompass a broad range of subjects including searches for the origin of elements, investigating rare isotopes present in explosive phenomena in astrophysical settings, and understanding neutron star crusts. Rare isotopes can be viewed as laboratories to study fundamental symmetries where symmetry violations are sometimes amplified. Last, but not the least, research at rare isotope facilities would lead to societal applications. Neutrinos connect all these venues, forming an intellectual bridge between them.

One topic where the connection between rare isotope physics and neutrino physics is manifest is the nucleosynthesis of various elements. A complete understanding of element nucleosynthesis requires input from both areas. Not only the knowledge of neutrino properties such as masses and mixing angles, but also the spin-isospin response of a broad range of nuclei from stable isotopes to rare ions are the crucial components of a successful description of the element nucleosynthesis. This synergy is perhaps most obvious for the ν-process, the nucleosynthesis via neutrino-induced nucleon emission [1]. The most quoted example of the ν-process nucleosynthesis is the production of ^{19}F by neutrino capture on ^{20}Ne in the outer shells of the supernovae, which accounts for the entire observed abundance of ^{19}F. Of course the ν-process in more broadly operational. For example, recent work nicely ties the ν-process nucleosynthesis yields of ^{11}B and ^{7}Li in the He shells of supernovae to the neutrino properties [2]. Matter-enhanced neutrino oscillations governed by the mixing between the first and the third neutrino generations is operative at matter densities that exist in those outer shells of a supernova. This boosts the ν-process nucleosynthesis yields of ^{11}B and ^{7}Li for the inverted neutrino mass hierarchy, but not for the normal one.

The cosmic site of the r-process nucleosynthesis, driven by a succession of rapid neutron captures on heavy seed nuclei, is still unknown [3], but the neutrino-driven wind in the core-collapse supernovae is among the possible sites. Recent hydrodynamical simulations of the neutrino-driven wind do not seem to always result in the necessary extreme conditions [4]. However, since our theoretical understanding of core-collapse supernovae is still evolving, it is not possible to rule out the neutrino-driven wind as a possible site. Numerical modeling of core-collapse supernovae has made significant recent progress, in particular two- and three-dimensional models were successfully implemented unveiling a complex interplay between neutrino physics, thermonuclear reactions and turbulence [5, 6, 7]. Core-collapse supernovae are neutrino-dominated dynamical systems [8] and one crucial ingredient to understand the r-process nucleosynthesis they may host is a better knowledge of neutrino physics. A second crucial input is a better working understanding of the spin-isospin response of a broad range of nuclei.

There is another, more formal, connection between neutrino physics and the nuclear many-body problem. The sheer number of neutrinos ($\sim 10^{58}$) emitted from the cooling proto-neutron star following the core collapse necessitates inclusion of neutrino-neutrino interactions in the description of the neutrino transport in supernovae. Unlike the one-body Hamiltonian of the matter-enhanced neutrino oscillations where neutrinos interact with a mean-field (generated by the background particles other than neutrinos), the Hamiltonian describing the many-neutrino gas in a core-collapse supernova contains both one- and two-body terms, making it technically much more challenging. Inclusion of the neutrino-neutrino interaction terms leads to very interesting collective effects [9, 10]. In contrast to the many-body systems studied in condensed-matter physics where the systems are primarily driven by electromagnetic interactions, and in nuclear physics, where the systems are primarily driven by strong interactions, the many-neutrino system in the core-collapse supernovae is the only many-body system in nature driven by the weak interactions. In addition, the latter system contains many more particles than the other two. Partially motivated by these features many-body aspects of the supernova neutrinos recently started to receive more attention [11, 12, 13].

There is a further analogy between supernova neutrino many-body system and the pairing problem in nuclear physics. To demonstrate this analogy let us consider only two flavors of neutrinos: electron neutrino, ν_e, and another flavor, ν_x which mixes with the electron neutrinos. For simplicity, we will also ignore the antineutrinos, which are of course present in supernovae[1]. Introducing the creation and annihilation operators for a neutrino with three momentum $\mathbf{p}$, we can write down the generators of the neutrino flavor isospin algebras [11]:

$$\begin{aligned} J_+(\mathbf{p}) &= a_x^\dagger(\mathbf{p}) a_e(\mathbf{p}), \quad J_-(\mathbf{p}) = a_e^\dagger(\mathbf{p}) a_x(\mathbf{p}), \\ J_0(\mathbf{p}) &= \frac{1}{2}\left(a_x^\dagger(\mathbf{p}) a_x(\mathbf{p}) - a_e^\dagger(\mathbf{p}) a_e(\mathbf{p})\right). \end{aligned} \tag{1}$$

The integrals of these operators over all possible values of momenta generate the global flavor isospin algebra. Using the operators in Eq. (1) the Hamiltonian for a neutrino propagating through matter takes the form

$$H = H_\nu + H_{\nu\nu} = \left(\sum_p \frac{\delta m^2}{2p} \hat{B} \cdot \vec{J}_p - \sqrt{2} G_F N_e J_p^0\right) + \frac{\sqrt{2} G_F}{V} \sum_{\mathbf{p},\mathbf{q}} (1 - \cos\vartheta_{\mathbf{pq}}) \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}} \tag{2}$$

where the auxiliary vector quantity $\hat{B}$ is given by

$$\hat{B} = (\sin 2\theta, 0, -\cos 2\theta), \tag{3}$$

N_e is the background electron density and δm^2 is the difference between squares of the masses associated with mass eigenstates. In the above equations θ is the mixing angle between electron

[1] Inclusion of antineutrinos requires a second set of SU(2) algebras. For three flavors two sets of SU(3) algebras are needed.

neutrino and the other neutrino flavor and $\vartheta_{\mathbf{pq}}$ is the angle between neutrino momenta $\mathbf{p}$ and $\mathbf{q}$. In writing this equation a term proportional to identity is omitted as such terms do not contribute to the neutrino oscillations. Note that the presence of the $(1-\cos\vartheta_{\mathbf{pq}})$ term in the integral above is crucial to recover the effects of the Standard Model weak interaction physics in the most general situation. In the idealized case of an isotropic neutrino distribution and a very large number of neutrinos, this term may average to a constant and the neutrino-neutrino interaction term in the Hamiltonian simply reduces to the Casimir operator of the global SU(2) algebra. If the background electron neutrino density is negligible (a good approximation near the proto-neutron star where neutrino-neutrino interactions are dominant) and a single angle dominates the second term, the Hamiltonian of Eq. (2) takes the form

$$H = \sum_p \frac{\delta m^2}{2p} \hat{B} \cdot \vec{J}_p + \frac{\sqrt{2} G_F}{V} \vec{J} \cdot \vec{J}, \tag{4}$$

where $\vec{J} = \sum_p \vec{J}_p$. The Hamiltonian in Eq. (4) is mathematically very similar to the BCS Hamiltonian in the quasi-spin basis[2]

$$\hat{H}_{\mathrm{BCS}} = \sum_k 2\epsilon_k \hat{t}_k^0 - |G| \hat{T}^+ \hat{T}^-, \tag{5}$$

where the quasi-spin operators also form an SU(2) algebra:

$$[\hat{t}_k^+, \hat{t}_l^-] = 2\delta_{kl} \hat{t}_k^0 \ , \qquad [\hat{t}_k^0, \hat{t}_l^\pm] = \pm \delta_{kl} \hat{t}_k^\pm \tag{6}$$

with $T^+ = \sum_i \hat{t}_i^+$ and so on. An exact solution of this problem was given by Richardson some time ago [14]. This solution was later generalized by Gaudin [15]. A similar solution also exists for the neutrino Hamiltonian of Eq. (4) [12]. It is manifest that the Hamiltonian of Eq. (4) possesses an $SU(N)_f$ rotation symmetry in the neutrino flavor space with N flavors [11, 17, 18]. Since the BCS Hamiltonian is shown to be integrable, there must be constants of motion associated with it [16]. One can write down the constants of motion of the collective neutrino Hamiltonian in Eq. (4) as [12]

$$\hat{h}_p = \hat{B} \cdot \vec{J}_p + 2 \sum_{q(\neq p)} \frac{\vec{J}_p \cdot \vec{J}_q}{\omega_p - \omega_q}. \tag{7}$$

where we defined $\omega_p = \frac{1}{\mu} \frac{\delta m^2}{2p}$ with $\mu = \frac{\sqrt{2} G_F}{V}$. The Hamiltonian of Eq. (4) itself can be written to include a linear combination of these invariants:

$$H = \sum_p w_p \hat{h}_p + \sum_p \vec{J}_p \cdot \vec{J}_p \tag{8}$$

It was shown that existence of such constants of motion could lead to collective neutrino oscillations [19].

To solve the full Hamiltonian of Eq. (2) for the large number of neutrinos present in the supernovae is a numerically very challenging, if not almost impossible, task. The full Hamiltonian is usually simplified using an RPA-like linearization procedure by writing the two-body terms

[2] Note that neutrino Hamiltonian in the single-angle approximation is *not* identical to the BCS Hamiltonian as the sign of the two-body term is different. Also the Hamiltonian in Eq. (4) does not include the antineutrino terms.

as a one-body operator times its expectation value in a suitably chosen basis. For example applying this procedure to the Hamiltonian of Ref. (4) one gets

$$H \to \hat{H}^{\mathrm{RPA}} = \sum_p \omega_p \hat{B} \cdot \vec{J}_p + \vec{P} \cdot \vec{J} \tag{9}$$

where the polarization vector $\vec{P}$ ensues from the linearization of the flavor isospin operators:

$$\vec{P}_{\mathbf{p},s} = 2\langle \vec{J}_{\mathbf{p},s} \rangle. \tag{10}$$

Using the SU(2) coherent states associated with the flavor isospin to calculate the operator averages in the above equations yields a reduced collective neutrino Hamiltonian, widely used in the literature [11]. This introduces an approximation to the description of the neutrino gas, however abandoning this approximation seems to be unlikely in the near future for practical reasons.

Collective neutrino oscillations produce an interesting effect, called spectral swappings or splits, on the final neutrino energy spectra: at a particular energy these spectra are almost completely divided into parts of different flavors [20, 21]. The nature of the such swaps can be elucidated by referring to the Hamiltonian in Eq. (9). As in the BCS theory, this Hamiltonian does not conserve particle number. Particle number conservation can be enforced by introducing a Lagrange multiplier:

$$\hat{H}^{\mathrm{RPA}} \to \hat{H}^{\mathrm{RPA}} + \omega_c \hat{J}^0. \tag{11}$$

Diagonalization of this Hamiltonian using an appropriately chosen Bogoliubov transformation gives rise to spectral splits or swaps in the neutrino energy spectra with the Lagrange multiplier playing the role of the swap frequency [12].

It should be emphasized that the collective neutrino oscillations could significantly impact r-process nucleosynthesis yields [22, 23]. To understand the r-process is clearly one of the drivers of the experiments with rare ion beams. This physics program requires measuring the beta-decay rates of nuclei both at and far from stability, in particular half-lifes at the r-process ladders as well as the initial and final state energies. A complete knowledge of the spin-isospin response of a broad range of nuclei to a various probes is crucial for not only for the r-process nucleosynthesis, but also for many other applications. Recently much progress was made in understanding the nuclear spin-isospin response, both experimentally and theoretically. On the experimental side the matrix elements of the Gamow-Teller operator $\vec{\sigma}\,\vec{\tau}$ between the initial and final states were successfully measured using inverse kinematics [24]. A major theoretical development is the proper inclusion of the tensor force in the shell model Hamiltonians. Monopole component of the nucleon-nucleon force is the same in nuclear medium and free space, however the monopole effect of the tensor force alters the shell structure in a significant way [25]. This is because the monopole component of the tensor interaction changes depending on whether the nucleon spin is parallel or antiparallel to its orbital angular momentum. In most cases the monopole component is an average over all possible spin orientations, so the tensor component does not contribute for the filled orbits. However, near the Fermi surfaces where the spin-orbit force splits the orbits, the $j = \ell + s$ orbit fills first altering the mean field. Indeed residual effective force between the valence nucleons, beyond that represented by the mean field, is very well described by the tensor force [26]. A new p-sd shell model Hamiltonian including up to 2-3 $\hbar\Omega$ excitations can describe the magnetic moments and Gamow-Teller (GT) transitions in p-shell nuclei well with a small quenching for the spin g-factor and the axial-vector coupling constant [27]. These new developments lead to a description of the spin properties of such nuclei better than the conventional shell model Hamiltonians resulting in a better description of the weak interactions for astrophysical applications. For example, this new Hamiltonian significantly improves the description of the cross sections for the reactions ν_e+^{12}C [28] and ν_e+^{13}C [29], potentially important for scintillator-based neutrino experiments. These new Hamiltonians can successfully describe not only the Gamow-Teller, but also the first-forbidden transitions. Inclusion of the latter terms can significantly change the lifetimes [30]. These recent advances in the shell model studies of stable and exotic rare nuclei as well as their use in the description of the spin-dependent nuclear weak processes are reviewed in references [32] and [31].

Neutrinos form the bridge between many astrophysical phenomena and laboratory nuclear physics, investigating stable as well as unstable, exotic, and rare elements. Exploiting this connection in an intellectually beneficial way necessitates a multitude of experimental and theoretical efforts. On the theoretical side, these include a better understanding of neutrino properties and improving our knowledge of nuclear structure to calculate neutrino interactions with nuclei more accurately. On the experimental side, these include measurements of the spin-isospin response of both stable and exotic nuclei to various probes as well as measuring salient properties of such nuclei. On the observational side, these include better determinations of cosmic abundances. One of the prizes of this quest is understanding the origin of the elements. At least to some of us, another prize will be the full appreciation of the considerable role neutrinos play in the cosmos.

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