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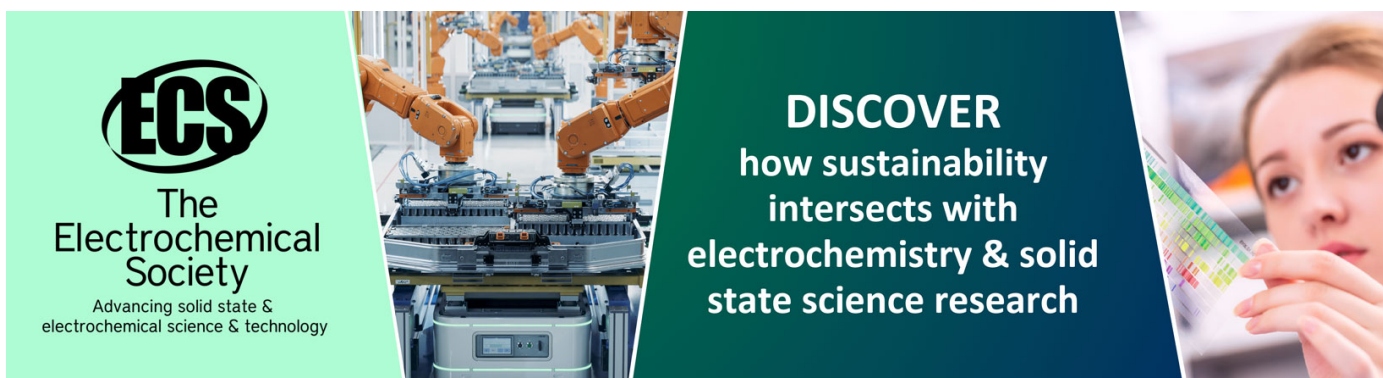
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Mean-field and RPA approaches to stable and unstable nuclei with semi-realistic NN interactions

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Abstract. Important roles of the NN interaction in the shell evolution, which have been pointed out by Otsuka and his collaborators, can be investigated by the self-consistent mean-field approaches with the semi-realistic interactions. We show that the shell evolution in the $Z = 50$ and $Z = 20$ nuclei is appropriately described by the semi-realistic interactions that include the realistic tensor force. The $M1$ strengths in ^{208}Pb are also reproduced by the semi-realistic interactions in the HF+RPA, which are another manifestation of the tensor force in nuclear structure.

1. Introduction

Experimental data on unstable nuclei have disclosed that the nuclear shell structure has significant Z - and N -dependence, even altering magic numbers [1]. This Z - and N -dependence of the shell structure is often called *shell evolution* [2]. Although the ℓ -dependence of the single-particle (s.p.) energies in the loosely bound orbitals could influence the shell evolution [3], there has been no clear evidence that the ℓ -dependence of the s.p. energies gives rise to appearance or disappearance of magic numbers. On the other hand, significant roles of several parts of the effective nucleonic interaction have been pointed out. The central spin-isospin channel of the NN interaction was first considered [4], although it seems only subsidiary in many cases. In contrast, it has been established that the tensor channels are important in the shell evolution [5]. As discarded in the conventional mean-field approaches, tensor-force effects are one of the current hot topics in nuclear structure physics. In addition to the NN interaction, roles of the NNN interaction have also been investigated [6]. Because these mechanisms of the shell evolution according to the NN and NNN interactions were first proposed by Otsuka, together with his collaborators, I would call them *Otsuka scheme* in this paper.

The mean-field (MF) theories, the Hartree-Fock (HF) and the Hartree-Fock-Bogolyubov (HFB) theories in particular, give us a framework to describe nuclear shell structure from the nucleonic degrees of freedom in a self-consistent manner. On top of the MF theories, the random-phase approximation (RPA) is useful in describing properties of excited states at relatively low energy. The only input to these theories is effective interactions (or energy density functionals), in principle. In the non-relativistic MF approaches, the Skyrme interaction [7] has most widely been employed, in which all terms of the interactions are restricted to zero range, though certain part of finite-range effects is pretended by the derivative terms. The Gogny interaction [8] has also been applied systematically, in which the central channels have finite range. The author



has developed effective interactions [9, 10, 11] based on the Michigan 3-range Yukawa (M3Y) interaction [12]. The M3Y interaction was derived by fitting the Yukawa functions to the G -matrix. Whereas the original M3Y interaction cannot reproduce the saturation and the spin-orbit splitting within the MF regime, this is remedied by adding a density-dependent contact term and multiplying the LS channels by an enhancement factor. This type of interactions has been called *semi-realistic* interactions. They are contrasted to the Skyrme and the Gogny interactions whose parameters have usually been determined in a phenomenological manner.

In harmony with the Otsuka scheme, the MF approaches with semi-realistic interactions are expected to be useful in investigating the shell evolution. The M3Y-type semi-realistic interactions contain the central channels of the one-pion exchange potential (OPEP), and thereby have reasonable spin-isospin channels. They also have the realistic tensor force originating from the G -matrix. Though with restricted form and fully phenomenological treatment at present, the density-dependent terms may simulate effects of the NNN interaction. Using the recently developed algorithm [13, 14, 15, 16], semi-realistic interactions have been applied to the MF and RPA calculations of medium- to heavy-mass spherical nuclei. In this paper we shall show some of the results, primarily focusing on the tensor-force effects on nuclear structure. It is commented that, with attempts of application to nuclear reactions [17] and neutron-star problems [18] as well as nuclear structure studies, the semi-realistic interactions may give a step toward unified description of all these problems.

2. M3Y-type semi-realistic interactions

The M3Y-type semi-realistic interactions have been developed [9, 10, 11], with aiming at high predictive power and wide applicability. The M3Y- Pn parameter-sets ($n = 1, 2, \dots$, etc.) have been obtained from the M3Y-Paris interaction [19], by replacing a part of short-range repulsion by density-dependent contact term so as to reproduce the saturation, and by tuning some of the parameters. In the M3Y- Pn interactions with $n \geq 5$, the tensor channels ($v^{(TN)}$) of the M3Y-Paris interaction have been kept unchanged, as well as the longest-range central channels that are fixed to be those of the OPEP ($v_{\text{OPEP}}^{(C)}$). While these channels are closely connected to the chiral symmetry breaking, they have not explicitly been taken into account in the conventional MF approaches. It is remarked that these channels in the M3Y-type interactions provide realistic basis corresponding to roles of the spin-isospin channels and the tensor channels in the Otsuka scheme, and that, owing to explicit and realistic $v_{\text{OPEP}}^{(C)}$ and $v^{(TN)}$, we obtain reasonable spin properties with the M3Y-type interactions. For instance, M3Y- Pn gives the Landau-Migdal parameter g_0' consistent with the data [20], to which contribution of $v_{\text{OPEP}}^{(C)}$ is significant. Thus the M3Y-type semi-realistic interactions are suitable to investigating their effects on the shell evolution within a self-consistent MF regime. For the NNN interaction, the density-dependent terms may carry certain part of its effects, although these terms may also represent medium effects. Since the density-dependent terms are currently determined in a phenomenological manner, its connection to the microscopic NNN interaction is obscure and we shall not argue their effects in this paper.

Density profile of the symmetry energy $a_t(\rho)$ may be important in nuclear reaction problems and in neutron stars. Although the M3Y-P5 interaction [10] has been applied to these problems [17, 18], $a_t(\rho)$ is not well constrained in M3Y-P5, typified by instability of the symmetric nuclear matter at $\rho \approx 4\rho_0$ (ρ_0 is the saturation density). In the newly explored M3Y-P6 and P7 interactions [21], several parameters have been fitted to the neutron matter energy obtained from microscopic calculations of Refs. [22] (M3Y-P6) and [23] (M3Y-P7). This improves $a_t(\rho)$ to substantial extent, and the spin-saturated symmetric matter is found stable in wide range of ρ , in contrast to many phenomenological interactions including the tensor force [24]. However, in the subsequent sections we mainly focus on the shell evolution and on the spin excitation, by employing the M3Y-P5 or P5' interaction.

Table 1. Binding energies $-E$ (MeV) and rms matter radii $\sqrt{\langle r^2 \rangle}$ (fm) of several doubly magic nuclei.

		SLy5	D1S	M3Y-P5'	M3Y-P7	Exp.
^{16}O	$-E$	128.6	129.5	124.1	125.9	127.6
	$\sqrt{\langle r^2 \rangle}$	2.59	2.61	2.60	2.57	2.61
^{40}Ca	$-E$	344.3	344.6	331.7	334.3	342.1
	$\sqrt{\langle r^2 \rangle}$	3.29	3.37	3.37	3.35	3.47
^{48}Ca	$-E$	416.0	416.8	411.5	414.9	416.0
	$\sqrt{\langle r^2 \rangle}$	3.44	3.51	3.51	3.49	3.57
^{90}Zr	$-E$	782.4	785.9	775.7	780.8	783.9
	$\sqrt{\langle r^2 \rangle}$	4.22	4.24	4.23	4.22	4.32
^{208}Pb	$-E$	1635.2	1639.0	1635.7	1635.5	1636.4
	$\sqrt{\langle r^2 \rangle}$	5.52	5.51	5.51	5.51	5.49

All the numerical calculations in this paper are implemented by applying the Gaussian expansion method [13, 14, 15, 16]. The Hamiltonian is $H = H_N + V_C - H_{\text{c.m.}}$, where $H_N = \sum_i \mathbf{p}_i^2/2M + \sum_{i<j} v_{ij}$ with the effective NN interaction v_{ij} , V_C represents the Coulomb interaction among protons, and $H_{\text{c.m.}} = \mathbf{P}^2/2AM$. Note that finite-range nature of V_C and $H_{\text{c.m.}}$ is handled exactly, up to the pairing channel in the HFB case.

The spherical HF results for the binding energies and the rms matter radii of the doubly-magic nuclei with the M3Y-P5' and P7 interactions are compared to those with Skyrme SLy5 [25] and Gogny D1S [26] as well as to the measured values [27, 28, 29], in Table 1. We view that the M3Y-type semi-realistic interactions are adjusted to the properties of the doubly magic nuclei comparably well with the conventional MF interactions.

3. Shell evolution in medium-mass nuclei

Since the shell structure is a fundamental concept in nuclear structure physics, the shell evolution is a highly important subject. A significant role of the tensor force, which is a central issue in the Otsuka scheme, was first pointed out for the N -dependence of $\varepsilon_p(0h_{11/2}) - \varepsilon_p(0g_{7/2})$ just above $Z = 50$ [5]. We here view the s.p. energies of these orbits relative to $p1d_{5/2}$, $\Delta\varepsilon_p(j) = \varepsilon_p(j) - \varepsilon_p(1d_{5/2})$. In Fig. 1, $\delta\Delta\varepsilon_p(0h_{11/2})$ and $\delta\Delta\varepsilon_p(0g_{7/2})$, where $\delta\Delta\varepsilon_p(j)$ indicates the difference between $\Delta\varepsilon_p(j)$ at a given N and at $N = 64$, are plotted. The HF results with M3Y-P5' are compared to those with D1S and to the experimental data [30], for which we use the energies of the lowest states in Sb having appropriate J^π . It is noted that the N -dependence of $\varepsilon_p(0h_{11/2}) - \varepsilon_p(0g_{7/2})$ evaluated from the lowest states is similar to that obtained after averaging the known states with the weight of the spectroscopic factors [31]. Unlike D1S that does not explicitly include tensor channels, the M3Y-type interactions including the realistic tensor force well describe the N -dependence of $\delta\Delta\varepsilon_p(j)$, particularly $\delta\Delta\varepsilon_p(0h_{11/2})$, as pointed out in Ref. [10]. This result confirms the Otsuka scheme both qualitatively and quantitatively. It is remarked that the realistic tensor force gives reasonable slope of $\delta\Delta\varepsilon_p(j)$.

Fragmented over certain energy range, the s.p. energies obtained in the MF approaches should better correspond to the averaged energy of the states weighted by the spectroscopic factors, rather than energies of individual observed levels. For the proton $1s_{1/2}$ and $0d_{3/2}$ hole states in ^{40}Ca and ^{48}Ca , sum of the measured spectroscopic factor exceeds 90% [32, 33], by which inversion of the $p1s_{1/2}$ and $p0d_{3/2}$ levels is indicated as N increases from ^{40}Ca to ^{48}Ca . This

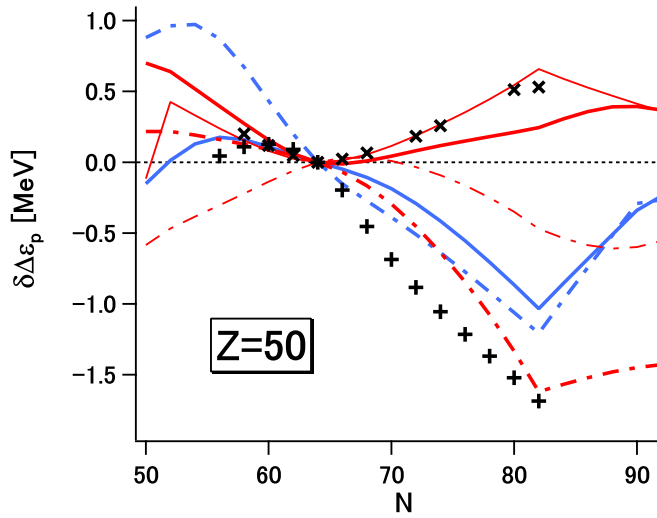


Figure 1. $\delta\Delta\varepsilon_p(0g_{7/2})$ (dot-dashed lines) and $\delta\Delta\varepsilon_p(0h_{11/2})$ (solid lines) in the $Z = 50$ nuclei obtained from the spherical HF calculations with D1S (blue lines) and M3Y-P5' (thick red lines), in comparison with the observed levels of the Sb nuclei (+ for $0g_{7/2}$ and x for $0h_{11/2}$) [30]. Thin red lines are the $v^{(TN)}$ contribution in the M3Y-P5' results.

$p1s_{1/2}$ - $p0d_{3/2}$ difference and their inversion can be a good test of the MF effective interactions. We have found that M3Y-P5' reproduces the N -dependence of $\Delta\varepsilon_p = \varepsilon_p(1s_{1/2}) - \varepsilon_p(0d_{3/2})$ from ^{40}Ca to ^{48}Ca remarkably well (Fig. 2). A crucial role of the realistic tensor force in the slope of $\Delta\varepsilon_p$ is obvious, as $\Delta\varepsilon_p$ becomes almost parallel to that of D1S when the tensor force is excluded. This consequence is qualitatively consistent with those from phenomenological tensor force [34, 35], though $\Delta\varepsilon_p$ has not been reproduced quantitatively in those studies. Further details including possibility of the bubble structure are discussed in Ref. [36].

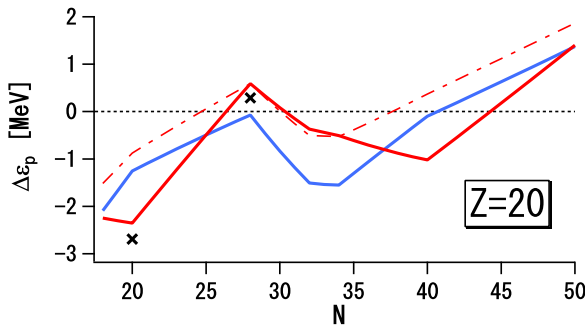


Figure 2. $\Delta\varepsilon_p$ in the $Z = 20$ nuclei obtained from the spherical HF calculations with D1S (blue lines) and M3Y-P5' (thick red lines), in comparison with the experimental values in $^{40,48}\text{Ca}$ (x) [32, 33]. Thin red dot-dashed line shows the values by excluding the $v^{(TN)}$ contribution from the M3Y-P5' results, with shifting so as to coincide at ^{48}Ca .

Influence of $v_{\text{OPEP}}^{(C)}$ on the shell evolution, which forms another issue in the Otsuka scheme, has not been so clear. However, in certain cases it can give additional effects to those of $v^{(TN)}$, as illustrated by the $N = 32$ nuclei in Refs. [10, 37].

Although our MF approaches with the semi-realistic interactions have realized the Otsuka scheme on the shell evolution, the tensor-force effects in particular, they have given different consequence on the magicity from those predicted by Otsuka and his collaborators for several cases. Even though contribution of the tensor force is similar in qualitative respect, interplay of the tensor force and the other channels of the effective NN interaction can make sizable difference. Such an example is found in $N = 34$ magicity around ^{54}Ca , which has been claimed in Ref. [38] but not supported by the self-consistent MF calculations with the semi-realistic interactions [10, 11]. Another is erosion of the $Z = 28$ magicity at ^{78}Ni , which is not predicted in the MF approaches with the semi-realistic interactions [37] as with the conventional interactions. Experiments on these points are awaited.

Table 2. Comparison of $M1$ strength in ^{208}Pb , quoted from Ref. [39].

			M3Y-P5		Exp.
			$\hat{v} - \hat{v}^{(\text{TN})}$	$\hat{v}$	
“IS”	E_x	(MeV)	6.87	5.85	5.85
	$B(M1)\uparrow$	(μ_N^2)	4.7	2.4	2.0
“IV”	E_x	(MeV)	9.2 – 10.9	9.2 – 10.9	7.1 – 8.7
	$\sum B(M1)\uparrow$	(μ_N^2)	16.3	19.4	16.3 or 18.2

4. $M1$ excitations of ^{208}Pb

The tensor-force effects are expected to appear in excitations involving spin degrees of freedom. However, self-consistent calculations within the HF+RPA including the tensor force have not been performed until recently. An earliest calculation in this line was implemented for the $M1$ excitation of ^{208}Pb with the semi-realistic interaction [39], and the results were compared to those of the high-precision measurements. By adopting the $M1$ operator including the two-particle-two-hole ($2p$ - $2h$) as well as the meson-exchange current corrections [40], we have successfully reproduced the separation of the isoscalar (IS) dominant $M1$ strength at low energy and the isovector (IV) dominant ones at higher energies.

In ^{208}Pb , the $M1$ excitation strengths are basically carried by the $(p0h_{11/2})^{-1}(p0h_{9/2})$ and the $(n0i_{13/2})^{-1}(n0i_{11/2})$ configurations. The residual interaction mixes these two configurations. The excitation energy and the $M1$ strength of the IS-dominant state are sensitive to the residual interaction (see arguments in Ref. [41]). We have found that the repulsive IV spin-isospin interaction in the central channel is not necessarily sufficient to give rise to the separation as large as viewed in the measurement, and that the additive effect of the tensor force well accounts for the excitation energy and the $M1$ strength of the IS-dominant state, as presented in Table 2.

5. Summary

The M3Y-type semi-realistic NN interactions have been developed and applied to describing nuclear structure in the mean-field and RPA framework. They are useful in investigating roles of the NN interaction in the shell evolution within the self-consistent mean-field approaches, in harmony with the mechanisms proposed by Otsuka and his collaborators (Otsuka scheme). We have shown that the shell evolution in the $Z = 50$ and $Z = 20$ nuclei predicted by the semi-realistic interactions is in good agreement with the experimental data, owing primarily to the realistic tensor force. The realistic tensor force gives important and appropriate contribution also to the $M1$ strengths in ^{208}Pb , as shown within the HF+RPA. These results exemplify that the semi-realistic effective NN interactions provide us with a promising tool to understand nuclear properties on a microscopic basis.

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