Gravity-related wave function collapse: mass density resolution

To cite this article: Lajos Diósi 2013 *J. Phys.: Conf. Ser.* **442** 012001

View the article online for updates and enhancements.

Gravity-related wave function collapse: mass density resolution

Lajos Diósi

Wigner Research Center for Physics, H-1525 Budapest 114, P.O.Box 49, Hungary

E-mail: `diosi.lajos@wigner.mta.hu`

Abstract. Selected issues of the concept of spontaneous collapse are discussed, with the emphasis on the gravity-related model. We point out that without spontaneous collapses the Schrödinger cat states would macroscopically violate the standard conservation laws even in the presence of environmental noise. We prove that the collapse time of condensed matter c.o.m. superpositions is not sensitive to the natural uncertainty of the nuclear locations whereas we formulate the conjecture that superfluid He may show an anomalous low rate of spontaneous collapse compared to common condensed matter.

1. Introduction

If we apply quantum mechanics for macroscopic variables, e.g., to the c.o.m. motion of a given massive object, we face paradoxical as well as concrete physical issues. The hypothesis of spontaneous collapse (SC) has been thought a remedy. According to it, all macroscopic superpositions, i.e.: the Scrödinger cat (Cat) states, are quickly eliminated as if a quantum measurement were performed on them. This time, however, the measurement happens spontaneously and universally, without the presence of any measurement device. SC models contain a further innovation compared to standard wave function collapse. The latter is a sudden one-shot event whereas SCs take time, they have a certain stochastic dynamics. Formally, they are time-resolved standard one-shot collapses. The SC models take out the concept of standard one-shot collapse.

We discuss some old problems within the gravity-related model, proposed independently by Penrose and the author [1, 2, 3, 4, 5, 6, 7], of the yet hypothetical SC. Some problems are acute because of related experimental considerations and experiments [8, 9, 11, 12, 10, 13, 14]. Some are common for almost all SC models [15], some are specific for ours. For the sake of a minimum comparison, we introduce the elements of the gravity-related SC model together with the so-called continuous spontaneous localization (CSL) [16] model, using the notion of 'catness' (Sec. 2). Sec. 3 presents two novel results for the gravity-related model. First, we show that the equilibrium collapse rate of the Cat c.o.m. is a classical expression, independent of $\hbar$. Second, we prove that the calculated SC rate does not depend on the spread of nuclear locations even if the mass density spatial resolution assumes the nuclear size. Many question the physical relevance of SCs of the Cat states because their unavoidable entanglement with the environment prevents Cat states from being created. We point out in Sec. 4, that the immediate environmental decoherence may not help Cat states avoid their macroscopic violation of conservation laws.

2. Spontaneous collapse models

Most SC models are similar to each other. Let us introduce the notion of 'catness' for the superposition of two different macroscopic configurations, say, two mass densities $f(\mathbf{r})$ and $f'(\mathbf{r})$. Consider a typical Cat state:

$$|\text{Cat}\rangle = |f\rangle + |f'\rangle, \tag{1}$$

where f and f' are 'macroscopically' different. Macroscopicity, i.e.: catness, is measured by a distance $\ell(f, f')$. Let, for concreteness, ℓ^2 be of dimension of energy. SC models assume that Nature makes $|\text{Cat}\rangle$ decay (collapse) stochastically at mean life-time

$$\tau = \frac{\hbar}{\ell^2(f, f')}. \tag{2}$$

Each collapse model has its own choice of the measure ℓ of catness. The choice of the distance $\ell(f, f')$ must be careful: there should be practically no decay (extreme large life-time τ) for atomic cats, and immediate decay (very short life-time τ) for 'macroscopic' Cats. When specifying catness, the spatial mass density resolution is a crucial issue. The models set up a finite cutoff σ of spatial resolution.

The gravity-related (G-related, or DP) model defines

$$\ell_G^2(f, f') = G \int \int [f(\mathbf{r}) - f'(\mathbf{r})][f(\mathbf{s}) - f'(\mathbf{s})] \frac{d\mathbf{r}d\mathbf{s}}{|\mathbf{r} - \mathbf{s}|}, \tag{3}$$

where f, f' are coarse-grained with the spatial resolution σ. The model is further discussed in Sec. 3. The closest alternative to the DP model is the mass-proportional CSL model [16]. The CSL choice of catness reads

$$\ell_{CSL}^2(f, f') = \frac{\hbar \lambda \sigma^3}{m_0^2} \int [f(\mathbf{r}) - f'(\mathbf{r})]^2 d\mathbf{r}, \tag{4}$$

where f, f' are Gaussian coarse-grained with the spatial resolution $\sigma \sim 10^{-5}$cm. The CSL model uses the nucleon mass m_0 as a reference mass and the further fenomenological parameter $\lambda \sim 10^{-17}$/s.

The predictions of the DP and CSL models may differ by orders of magnitudes, but they fulfill the main request. Practically infinite life-times are preserved for superpositions of microscopic systems whereas the Cat states (those apparently missing from Nature) will decay immediately. According to both DP and CSL models, the competition of SCs with the standard unitary dynamics may reach a regime of equilibrium. The DP model is unique in that the equilibrium collapse rate is classical, $\hbar$ cancels from it (Sec. 3.1).

3. G-related spontaneous collapse

We can re-express the (squared) catness (3) as the following combination of three Newton interaction energies:

$$\ell_G^2(f, f') = 2U(f, f') - U(f, f) - U(f', f'), \tag{5}$$

where

$$U(f, f') = -G \int \int \frac{f(\mathbf{r})f'(\mathbf{s})}{|\mathbf{r} - \mathbf{s}|} d\mathbf{r}d\mathbf{s}. \tag{6}$$

Suppose our system is a single rigid homogeneous ball of mass M and radius R and we consider its c.o.m. wave function. The popular 'mechanical' Cat state [1, 2, 3, 4, 5, 6, 7, 9] is the superposition of two macroscopically different c.o.m. locations $\mathbf{x}$ and $\mathbf{x}'$:

$$|\text{Cat}\rangle = |\mathbf{x}\rangle + |\mathbf{x}'\rangle. \tag{7}$$

The corresponding two mass densities of the Cat state (1) read

$$
\begin{aligned}
f(\mathbf{r}) &= \rho\theta(|\mathbf{r} - \mathbf{x}| \le R), \\
f'(\mathbf{r}) &= \rho\theta(|\mathbf{r} - \mathbf{x}'| \le R),
\end{aligned}
\tag{8}
$$

where $\rho = M/(4\pi R^3/3)$ is the constant mass density, θ is the step-function. The central quantity is the mean collapse time τ_G, cf. (2), of the c.o.m. wave function. It becomes the function of the c.o.m. displacement $\Delta\mathbf{x} = \mathbf{x} - \mathbf{x}'$; we calculate the mean collapse rate in the first non-vanishing order [1, 2, 3]:

$$
\frac{1}{\tau_G} = \frac{1}{2}\frac{M\omega_G^2}{\hbar}(\Delta\mathbf{x})^2,
\tag{9}
$$

valid if $|\Delta\mathbf{x}| \ll R$. The R-dependence has been absorbed into the parameter

$$
\omega_G = \sqrt{4\pi G\rho/3}
\tag{10}
$$

which we call the frequency of the Newton oscillator (cf. Fig. 1). This frequency is purely classical.
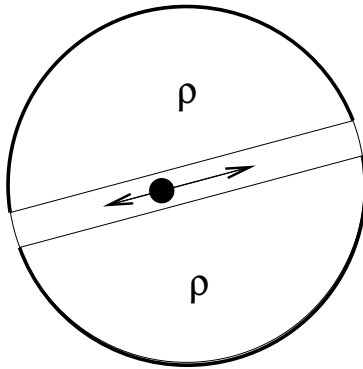

Figure 1. Schematic view of the Newton Oscillator: a homogeneous ball with an infinite narrow diagonal hole where the probe is oscillating under the linear directional force of the Newton field of the ball. The frequency of the oscillator $\omega_G = \sqrt{4\pi G\rho/3}$ does not depend on the size but on the density ρ of the ball.

3.1. Equilibrium collapse rate

The existence of equilibrium is a generic feature of the SC models. We are going to use the heuristic arguments of [2] which anticipated the exact derivations [3].

Let us start from the collapse rate (9) of a mechanical Cat, previously determined in the leading order of the coherent displacement $\Delta\mathbf{x} = \mathbf{x} - \mathbf{x}'$. We can heuristically extend this expression for a c.o.m. wave packet of coherent dispersion $\Delta\mathrm{x}$: the SCs at about the rate (9) tend to shrink the width $\Delta\mathrm{x}$. Now we take the unitary evolution of the wave packet into the account: it tends to spread $\Delta\mathrm{x}$ at the rate about $\hbar/M(\Delta\mathrm{x})^2$. The two contrary tendencies get balanced when the two rates coincide:

$$
\frac{\hbar}{M(\Delta\mathrm{x})^2} \sim \frac{M\omega_G^2(\Delta\mathrm{x})^2}{\hbar}.
\tag{11}
$$

This is the condition of equilibrium between SC and unitary dynamics, resulting in a sort of natural quantum state of the massive object. The corresponding natural coherent size of the c.o.m. wave packet yields

$$
\Delta\mathrm{x}^{\mathrm{eq}} \sim \sqrt{\hbar/M\omega_G},
\tag{12}
$$

which is a very tiny size for massive objects due to the smallness of $\hbar$. Similar strong localization of the macro-objects follows from the CSL model, too. Nevertheless, we find a unique feature of the DP model if we calculate the equilibrium collapse rate. Let's just take the geometric mean of both sides of the balance condition above, it yields the equilibrium collapse rate directly:

$$\frac{1}{\tau_G^{\text{eq}}} \sim \omega_G. \tag{13}$$

The equilibrium collapse rate coincides approximately with the frequency of the Newton oscillator, does not depend on the mass M and size R, moreover, it is fully classical, independent of $\hbar$.

3.2. Mass resolution issue, He superfluid

Catness (3) and collapse rates would diverge for point-like objects, that's why we need the finite spatial resolution σ of $f(\mathbf{r})$. It's crucial to decide that we resolve the microscopic structure or we don't. Adopting the resolution $\sigma \sim 10^{-5}$cm of the CSL model, the mass density in condensed matter becomes the smooth macroscopic density ρ equal to a few times $1\text{g}/\text{cm}^3$. We can estimate the parameter ω_G which governs the collapse rate (9) and yields the equilibrium collapse rate (13):

$$\frac{1}{\tau_G^{\text{eq}}} = \omega_G^{\text{eq}} \sim \frac{1}{\text{h}}. \tag{14}$$

This extreme low rate questions the significance of DP collapses in common objects. The microscopic structure of the mass density may be a loophole.

We can take a much finer resolution with the 'nuclear' cutoff $\sigma \sim 10^{-12}$cm. Then $f(\mathbf{r})$ shows the granular structure of localized nuclear contributions. For simplicity, we consider that the mass M is carried by the nuclei which are homogeneous balls of radius σ and density $\rho^{\text{nucl}} = (R/\sigma)^3 \rho$. For small displacements $|\Delta \mathbf{x}| \ll \sigma$, each nucleus will contribute separately to the (squared) catness, like a homogeneous ball of radius σ and density ρ^{nucl}. At the end of the day we obtain the collapse rate similar to (9):

$$\frac{1}{\tau_G} = \frac{1}{2} \times \frac{M(\omega_G^{\text{nucl}})^2}{\hbar}(\Delta \mathbf{x})^2, \tag{15}$$

just with ω_G^{nucl} instead of ω_G [18]. The expression is valid if $|\Delta \mathbf{x}| \ll \sigma$. Here

$$\omega_G^{\text{nucl}} = \sqrt{4\pi G \rho^{\text{nucl}}/3} \tag{16}$$

is the frequency of the Newton oscillator in nuclear matter. Its order of magnitude is $\omega_G^{\text{nucl}} \sim 10^3/s$.

With the 'nuclear' resolution, the Cat life-time τ_G becomes cca 10^{12} times shorter! Without this amplification, first shown in [18], the experimental significance of the SCs may remain illusory for another while, as recognized in [11, 12, 13, 14]. It seems therefore a sort of necessity that we define the DP model with the 'nuclear' cutoff to take granular structure of the mass density $f(\mathbf{r})$ into the account.

We must note that the exact interpretation of the nuclear mass distribution does not exist. We don't know the principles either. In the best case we believe that high energy gravitons would copy how high energy photons explore the electronic charge density. But the available non-relativistic interpretation of nuclear mass density is fenomenological, we have to rely upon the simple fenomenology.

Finally, we ought to discuss whether or not the thermal and quantum blurredness of the nuclear locations matters when we calculate the DP collapse rate of the Cat state. In solids,

the blurredness of nuclear location is about one tenth of the interatomic distance. If this broadening of mass distribution did matter then the above said factor 10^{12} of amplification would be lost and replaced by some 10^3. Fortunately, this is not the case: the DP collapse of the c.o.m. superposition is always independent of the quantum spread of microscopic constituents. The general proof is trivial in the density matrix formalism. Let us introduce the coordinates $\mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_N$ of the N nuclei w.r.t. the c.o.m. $\mathbf{x}$. We choose the following set of independent coordinates: $\mathbf{x}, \mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_{N-1}$. The corresponding density matrix reads

$$\rho(\mathbf{x}, \mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_{N-1} | \mathbf{x}', \mathbf{q}_1', \mathbf{q}_2', \ldots, \mathbf{q}_{N-1}'). \tag{17}$$

The full DP collapse rate of this state of the Cat is described by the following term [1, 2, 3]:

$$\frac{\ell_G^2(f, f')}{\hbar} \rho(\mathbf{x}, \mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_{N-1} | \mathbf{x}', \mathbf{q}_1', \mathbf{q}_2', \ldots, \mathbf{q}_{N-1}'), \tag{18}$$

where f is the (σ-coarse-grained) mass density of the configuration $(\mathbf{x}, \mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_{N-1})$ and f' is the (σ-coarse-grained) mass density of the configuration $(\mathbf{x}', \mathbf{q}_1', \mathbf{q}_2', \ldots, \mathbf{q}_{N-1}')$. We are not interested in the collapse rate of the integral degrees of freedom, therefore we trace over them, yielding

$$\int \frac{\ell_G^2(f, f')}{\hbar} \rho(\mathbf{x}, \mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_{N-1} | \mathbf{x}', \mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_{N-1}) d\mathbf{q}_1 d\mathbf{q}_2 \ldots d\mathbf{q}_{N-1}, \tag{19}$$

where f is the (σ-coarse-grained) mass density of the configuration $(\mathbf{x}, \mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_{N-1})$ and f' is the (σ-coarse-grained) mass density of the configuration $(\mathbf{x}', \mathbf{q}_1, \mathbf{q}_2, \ldots, \mathbf{q}_{N-1})$. The expression (19) is the exact c.o.m. collapse rate. This shows that the coherence of the spread of the nuclear locations is completely irrelevant. And there is an even more important result. We must not blur the nuclear locations first. Rather we must calculate the c.o.m. squared catness and the corresponding collapse rate with the sharp locations $\mathbf{q}_1, \mathbf{q}_2, \ldots$ of the nuclei. Second, the effect of the spread of nuclear locations will be taken into the account by blurring the above squared catness (i.e., the collapse rate) over the spread of the nuclear locations. In our special case of very small displacements $D\mathbf{x} \ll \sigma$, the squared catness $\ell_G^2(f, f')$ is, as we said before, well approximated by the sum of separate contributions of the nuclei. The contribution of a single nucleus, say the 'first' one, depends on the displacement between the two superposed locations $\mathbf{x} + \mathbf{q}_1$ and $\mathbf{x}' + \mathbf{q}_1$, which is just $|\Delta \mathbf{x}| = |\mathbf{x} - \mathbf{x}'|$. Neither the precise nuclear locations nor their spreads will influence the c.o.m. SC of the Cat.

Liquid He is an exception. In superfluid He the quantum dispersion of the nuclei is of the order of the interatomic distance, the nuclei have no identity and the above proof does not apply. Distinction between internal and c.o.m. collapses, respectively, is more involved. In liquid He the c.o.m. collapse rate may be much smaller than the value (15) valid in common condensed matter. Superfluid He may have an anomalous low rate of SCs in it. Of particular interest is the temperature dependence of the collapse rate. Too high temperatures would push back the coherent dispersion of the nuclei below the interatomic scale, resulting in increased c.o.m. collapse rate. It is a further question yet to be answered whether cold liquid He is sufficient for the anomalous low collapse rates or superfluidity is essential for it.

4. Environment does not mask spontaneous collapse

We know that typical environmental noise and entanglement used to decohere any Cat state much earlier than SCs would kill it. Environmental noise in Nature used to be much stronger than SCs. The experimental test of SCs appears very hard. Since SCs are masked by environmental noise, one might think that SCs are irrelevant, the corresponding models are superfluous. This conclusion is not correct.

Let us concentrate on the mechanical Cat. In the naive Cat state (7) we considered the c.o.m. but ignored all internal and environmental degrees of freedom. Now we refine the naive Cat state and include the environmental degrees of freedom:

$$|\text{Cat}\rangle = |\mathbf{x}\rangle \otimes |E\rangle + |\mathbf{x}'\rangle \otimes |E'\rangle \,, \tag{20}$$

where $|E\rangle$ and $|E'\rangle$ stand for two different states of the environment. To distill the central issue, we assume that the only massive object is the Cat itself (a rigid homogeneous ball with c.o.m. $\mathbf{x}, \mathbf{x}'$) and the environment is massless compared to the Cat. The above state is a macroscopic superposition of different mass configurations (8), so it is a genuine Cat. The two states $|\mathbf{x}\rangle$ and $|\mathbf{x}'\rangle$ are entangled with the environmental states $|E\rangle$ and $|E'\rangle$, respectively. Since in practice the environmental states $|E\rangle$ and $|E'\rangle$ are orthogonal, the coherence of the macroscopic superposition is not observable locally, our Cat is not observable locally. To see the Cat, we need to observe the remote parts of the entangled environment that we are not able to do. How can the Cats keep annoying us? Why should we deploy SCs or any mechanism against them at all?

Suppose the Cat state in the following form:

$$|\text{Cat}\rangle = |\mathbf{x}\rangle \otimes |E\rangle \otimes |1\rangle + |\mathbf{x}'\rangle \otimes |E'\rangle \otimes |2\rangle \,, \tag{21}$$

where $|1\rangle, |2\rangle$ are orthogonal single photon states. Say, the photon (like billions of other environmental photons) bounced back from the Cat some time ago and went to the states $|1\rangle, |2\rangle$ entangled with the Cat c.o.m. states $|\mathbf{x}\rangle, |\mathbf{x}'\rangle$, respectively. Suppose that we measure the photon so that, according to the standard quantum measurement theory, the Cat collapses into $|\mathbf{x}\rangle$ or into $|\mathbf{x}'\rangle$ at random. The c.o.m. of the Cat (and of the whole system) is $(\mathbf{x} + \mathbf{x}')/2$ before the measurement. After the measurement it becomes either $\mathbf{x}$ or $\mathbf{x}'$. We got a macroscopic shift of the c.o.m. of a closed macroscopic system whose only interaction with the outer world was that we measured a single photon that belonged to the system. This is not merely paradoxical. Actually all (non-superselected) conservation laws are macroscopically violated by the mere existence of certain Cats whether they are naive isolated ones or real ones entangled with the environment. Violation of energy-momentum conservation in standard collapse has been discussed earlier [17]. Note that our present concern is the possible macroscopicity of the violation, not the violation itself. A microscopic defect of conservation can always be attributed to (and balanced by) the measuring device. But macroscopic defects are annoying.

The SC models, like DP and CSL, offer a simple treatment of the Cat problem. They collapse the Cats (20), kill them before they can come up, much before we could measure them and provoque the macroscopic violation of conservation laws. The collapse models themselves impose violation of the conservation laws at a tinier scale, but they prevent macroscopic violations at least.

5. Closing remarks
We have added some new facts, thoughts and conjectures to the gravity-related SC model. On one end, sound analysis has clarified how the thermal and quantum motion of the nuclei constituting a massive body should enter the calculation of the c.o.m. collapse rate. On the other extreme, we have conjectured that in superfluid He the c.o.m. SC rate may be much lower than in common condensed matter. Ref. [19] by the present author discusses similar topics and a related possible extension of the DP model.

References

[1] Diósi L 1986 *A quantum-stochastic gravitation model and the reduction of the wavefunction* (Thesis in Hungarian)
    http://www.rmki.kfki.hu/~diosi/thesis1986.pdf
[2] Diósi L 1987 A universal master equation for the gravitational violation of the quantum mechanics *Phys. Lett.* A **120** 377
[3] Diósi L 1989 Models for universal reduction of macroscopic quantum fluctuations *Phys. Rev.* A **40** 1165
[4] Penrose R 1994 *Shadows of the Mind* (Oxford: Oxford University Press)
[5] Penrose R 1996 On gravity's role in quantum state reduction it Gen. Rel. Gra. **28** 581
[6] Penrose R 1998 Quantum computation, entanglement and state reduction *Phil. Trans. R. Soc. Lond.* A **356** 1927
[7] Penrose R 2004 *The Road to Reality* (London: Jonathan Cape Publishers)
[8] Marshall W, Simon C, Penrose R and Bouwmeester D 2003 Towards quantum superpositions of a mirror *Phys. Rev. Lett.* **91** 130401
[9] Adler SL 2007 Comments on proposed gravitational modifications of Schrödinger dynamics and their experimental implications *J. Phys.* A **40** 755
[10] Vanner MR, Pikovski I, Cole GD, Kim MS, Brukner Č, Hammerer K, Milburn GJ and Aspelmeyer M 2011 Pulsed quantum optomechanics *PNAS* **108** 16182
[11] Romero-Isart O 2011 Quantum superposition of massive objects and collapse models *Phys. Rev.* A **84** 052121
[12] Li T, Kheifets S and Raizen MG 2011 Millikelvin cooling of an optically trapped microsphere in vacuum *Nat. Phys.* **7** 527
[13] Pepper B, Ghobadi R, Jeffrey E, Simon C and Bouwmeester D 2012 Optomechanical superpositions via nested interferometry *Phys. Rev. Lett.* **109** 023601
[14] Yang H, Miao H, Lee D, Helou B and Chen Y 2012 Macroscopic quantum mechanics in a classical spacetime *Preprint* arXiv:1210.0457
[15] Bassi A and Ghirardi GC 2003 Dynamical reduction models *Phys. Rep.* **379** 257
[16] Ghirardi GC, Grassi R and Benatti F 1995 Describing the macroscopic world: Closing the circle within the dynamical reduction program *Found. Phys.* **25** 5
[17] Pearl P 2000 Wavefunction collapse and conservation laws *Found. Phys.* **30** 1145
[18] Diósi L 2007 Notes on certain Newton gravity mechanisms of wave function localisation and decoherence *J. Phys.* A **40** 2989
[19] Diósi L 2013 Gravity-related wave function collapse: Is superfluid He exceptional? *Preprint* arXiv:1302.5364