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The Weak and Strong Oblique Shock Waves Appeared on the Carbon Dioxide Two-phase Flow in the Ejector Refrigeration Cycle

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Abstract. After the Great East Japan Earthquake, the saving energy became the one of the most important issues. Especially, saving the electrical power for air-conditioning at summer time is a problem of great urgency. We have been developing the ejector refrigeration system which can improve coefficient of performance of refrigeration systems by converting exhausted expansion energy into useful pressure energy. The performance of the ejector is greatly affected by the pressure recovery at the mixing section in the ejector. It is elucidated by our recent research on the carbon dioxide refrigeration cycle that the pressure recovery at the ejector is performed by oblique shock waves appeared in the mixing section. The purpose of this research is to clarify the characteristics of the two-phase flow oblique shock waves in the supersonic carbon dioxide two-phase flow. It is shown by the theoretical analyses that the two types of oblique shock waves occur on the supersonic two-phase flow. One is the week shock wave behind of which the flow is still a subsonic state. The other is the strong oblique shock wave which has a large pressure recovery. And the experimental research also carried out by using the carbon dioxide two-phase flow channel. The theoretical results are compared with the experiment.

1. Introduction
The speed of sound in high-speed two-phase flow, which has been used extensively in areas such as total flow systems for geothermal power plants and in refrigeration cycles, is extremely low compared to that of single-phase flow [1]; moreover, a characteristic of the two-phase flow is its easy acceleration into a supersonic state. In general, both shock and expansion waves occur with rapid changes in the flow path of the supersonic state, and the effect of these waves on the flow field cannot be neglected. In addition, the shock and expansion waves that occur in the two-phase flow differ from those in a single-phase gas flow, and the pressure field tends to disperse. Therefore, we can expect a combination of more complex phenomena than those in a single-phase gas flow. We have studied the ejector-refrigeration cycle [2], which can improve the refrigeration cycle by converting the expansion energy that has so far been wastefully discarded into useful compression energy. The performance of the ejector [3] is considerably affected by pressure recovery in the mixing section inside the ejector. In past studies, because the speed of sound for the two-phase flow is low compared to that for the single-
phase flow, an oblique shock wave was generated at the nozzle exit of the ejector interior that affected the flow field of the supersonic two-phase flow inside the mixing section. Furthermore, it was found that most of the pressure recovery in the ejector depends on the kinetic energy of the two-phase flow in the mixing section. Theoretical studies have shown that in supersonic flow, there are two types of oblique shock waves with two-phase flow. One exists at the mach angle and is a weak oblique shock wave, which remains supersonic behind the shock wave. The other is a strong oblique shock wave, which is subsonic behind the shock wave, whose pressure is considerably recovered similar to that in case of a normal shock wave. The conditions under which the strong and weak oblique shock waves are generated were not discussed in these studies, thereby clarifying that the characteristics of these oblique shock waves is important for elucidating the pressure-recovery phenomenon of the ejector.

Accordingly, in this study, both strong and weak oblique shock waves were generated on an analytical semi-infinite plane to determine the conditions under which they arise and their characteristics, with the aim of comparing them to experimental results.

2. Basic equations for compressible two-phase flow

In our analysis, transport phenomena such as momentum, heat, and mass transports were considered between the gas and liquid phase. Two-phase flow is assumed to consist of a gas phase in the continuous phase and droplets in the dispersal phase by using the following assumptions:

- Phase changes occur in the flow field according to the heat-transfer limit.;
- The effect of friction other than interphase interactions is neglected.
- All droplets are spheres with diameter $d$, and any breakup or coalescence due to collisions among droplets is not considered.
- Both gas and liquid phases are assumed to refer to real gases.

Now, we provide the respective equations for the conservation of mass for gas and liquid phases:

$$\frac{\partial}{\partial t} (\alpha \rho g_{i}) + \frac{\partial}{\partial x_j} (\alpha \rho g_{i} v_{g_{i}}) = \dot{m},$$

$$\frac{\partial}{\partial t} [(1-\alpha) \rho l_{j}] + \frac{\partial}{\partial x_j} [(1-\alpha) \rho l_{j} v_{l_{j}}] = -\dot{m}. \tag{2}$$

Here, $\alpha$ expresses the void fraction represented by the volume fraction of the gas phase in the two-phase flow, $\rho$ is density, $\dot{m}$ is the rate of evaporation, and $v$ is flow velocity with the respective suffixes $g$ and $l$ denoting gas and liquid phases, and $i$ represents each of the directions in two dimensions.

Similarly, the momentum-conservation equations for each of the gas and liquid phases are as follows:

$$\frac{\partial}{\partial t} (\alpha \rho_{g_{i}} v_{g_{i}}) + \frac{\partial}{\partial x_j} (\alpha \rho_{g_{i}} v_{g_{i}} v_{g_{i}}) = \dot{m} v_{g_{i}} - (1-\alpha) \rho_{l_{j}} v_{g_{i}} v_{l_{j}} \frac{\partial p}{\partial x_j} - \alpha \frac{\partial p}{\partial x_j}, \tag{3}$$

$$\frac{\partial}{\partial t} [(1-\alpha) \rho_{l_{j}} v_{l_{j}}] + \frac{\partial}{\partial x_j} [(1-\alpha) \rho_{l_{j}} v_{l_{j}} v_{l_{j}}] = -\dot{m} v_{l_{j}} + (1-\alpha) \rho_{l_{j}} v_{g_{i}} v_{l_{j}} \frac{\partial p}{\partial x_j}. \tag{4}$$

The velocity of evaporation is $v_{l_{j}} = (v_{g_{i}} : \dot{m} \geq 0, v_{g_{i}} : \dot{m} < 0)$, and the momentum relaxation time is $\tau_{v} = \rho d^{2}/(4 \eta_{g_{i}})$. The total-energy-conservation equation is:

$$\frac{\partial}{\partial t} \left[ \alpha \rho_{g_{i}} \left( e_{g_{i}} + \frac{v_{g_{i}}^{2}}{2} \right) + (1-\alpha) \rho_{l_{j}} \left( e_{l_{j}} + \frac{v_{l_{j}}^{2}}{2} \right) \right] + \frac{\partial}{\partial x_j} \left[ \alpha \rho_{g_{i}} v_{g_{i}} \left( h_{g_{i}} + \frac{v_{g_{i}}^{2}}{2} \right) + (1-\alpha) \rho_{l_{j}} v_{l_{j}} \left( h_{l_{j}} + \frac{v_{l_{j}}^{2}}{2} \right) \right] = 0. \tag{5}$$
Here, $\epsilon$ represents internal energy, and $h$ represents enthalpy. This equation does not contain a term for transport between phases.

From the heat-transfer equation for each phase, we can write the following equations considering entropy:

$$T_g \frac{DS}{Df} = C_{pg} \frac{1 - \alpha}{\tau_{rg}} (T_r - T_g)$$ \hspace{1cm} (6)

$$T_l \frac{DS}{Df} = C_{pl} \frac{T_r - T_l}{\tau_{rl}}$$ \hspace{1cm} (7)

Here, $C_{pg}$ and $C_{pl}$ represent the specific heat of gas and liquid phases, respectively, and then, the thermal-relaxation times are $\tau_{rg} = \rho_r C_{pg} d/(6 h_{rg})$ and $\tau_{rl} = \rho_l C_{pl} d/(6 h_{rl})$. Please refer to the reference section for details.

We used the NIST REFPROP v.8 software for calculating the thermophysical properties.

### 3. Relational expression for oblique shock waves in extreme states

Though momentum, heat, and mass transports are considered in this study, because phase changes occur according to the heat-transfer limit, only momentum and heat transports are important. Extreme states exist depending on the velocity at which these transport phenomena progress, and for extremely high transport velocity, they are referred to as equilibrium states. On the other hand, for low or negligible transport velocity they are referred to as frozen states. In this section, we illustrate the method to determine the relational expressions for the shock waves in each of these extreme states.

#### 3.1. Jump equation for the normal shock wave in momentum and thermal-equilibrium states

Interphase velocity is equal to temperature in the momentum and thermal-equilibrium states. If we denote the forward state for the normal shock wave with suffix 1 and the rear state with suffix 2, the conservation equations for mass, momentum, and energy for a uniform steady flow are given by integrating Eqs. (1) + (2), (3) + (4), and (5):

- **mass-conservation equation**
  \[ \alpha_1 \rho_{g1} v_{w1} + (1 - \alpha_1) \rho_{l1} v_{w1} = \alpha_2 \rho_{g2} v_{w2} + (1 - \alpha_2) \rho_{l2} v_{w2} \] \hspace{1cm} (8)

- **momentum-conservation equation**
  \[ \alpha_1 \rho_{g1} v_{w1} \left( \frac{w_{g1}^2 + h_{g1}}{2} \right) + (1 - \alpha_1) \rho_{l1} w_{w1} \left( \frac{w_{g1}^2 + h_{g1}}{2} \right) = \alpha_2 \rho_{g2} v_{w2} \left( \frac{w_{g2}^2 + h_{g2}}{2} \right) + (1 - \alpha_2) \rho_{l2} w_{w2} \left( \frac{w_{g2}^2 + h_{g2}}{2} \right) \] \hspace{1cm} (9)

- **energy-conservation equation**
  \[ \alpha_1 \rho_{g1} v_{w1} \left( \frac{w_{g1}^2 + h_{g1}}{2} \right) + (1 - \alpha_1) \rho_{l1} w_{w1} \left( \frac{w_{g1}^2 + h_{g1}}{2} \right) = \alpha_2 \rho_{g2} v_{w2} \left( \frac{w_{g2}^2 + h_{g2}}{2} \right) + (1 - \alpha_2) \rho_{l2} w_{w2} \left( \frac{w_{g2}^2 + h_{g2}}{2} \right) \] \hspace{1cm} (10)

Because the physical quantities for the forward shock wave are known, the unknowns terms in Eqs. (8), (9), and (10) are $\alpha_2$, $\rho_{g2}$, $\rho_{l2}$, $w_{w2}$, $p_2$, $h_{g2}$, and $h_{l2}$. However, $h_{g2}$, $h_{l2}$, $\rho_{g2}$, and $\rho_{l2}$ can be calculated from the pressure corresponding to the saturation state of thermal equilibrium in the two-phase flow. Therefore, the three unknown terms $\alpha_2$, $w_{w2}$, and $p_2$ can be obtained using Eqs. (8), (9), and (10).

#### 3.2. Jump equation for the normal shock wave in momentum equilibrium and thermal frozen states

For the thermal frozen state in which only a small amount of heat is transferred between gas and liquid phases, because the thermodynamic variable is a function of pressure and entropy, when adiabatic conditions are applied to Eqs. (8) and (9), we get Eq. (11) in addition to Eqs. (8), (9), and (10).

$$s_{w1} = s_{w2}, \quad s_{l1} = s_{l2}$$ \hspace{1cm} (11)
From the tabulated thermodynamic properties, $\rho_{g2}, \rho_{l2}, h_{g2},$ and $h_{l2}$ can be estimated using the pressure, $p_2$, and entropy, $s_2$, for mutual phases. Therefore, the five unknown terms $\alpha_2, w_{m2}, \rho_2, s_2, \rho_{g2}$, and $\rho_{l2}$ can be found by solving Eqs. (8), (9), (10), and (11).

### 3.3. Jump equation for the normal shock wave in the momentum frozen and thermal equilibrium states

If the mutual forces acting on the phases are reduced to approximately zero, the mutual phases flow independently. Therefore, the momentum equation is required for both gas and the liquid phases.

- **mass-conservation equation**
  \[
  \alpha_2 \rho_{g1} w_{m1} + (1 - \alpha_2) \rho_{l1} w_{m1} = \alpha_2 \rho_{g2} w_{m2} + (1 - \alpha_2) \rho_{l2} w_{m2},
  \]  
  \[
  (1 - \alpha_2) \rho_{l1} w_{m1} = (1 - \alpha_2) \rho_{l2} w_{m2}.
  \]  

- **total momentum-conservation equation**
  \[
  \alpha_2 \rho_{g1} w_{m1}^2 + (1 - \alpha_2) \rho_{l1} w_{m1}^2 + p_1 = \alpha_2 \rho_{g2} w_{m2}^2 + (1 - \alpha_2) \rho_{l2} w_{m2}^2 + p_2.
  \]  

- **droplet momentum-conservation equation**
  \[
  \frac{1}{2} w_{m2}^2 - \frac{1}{2} w_{m1}^2 + \int_1^{\rho_2} \frac{1}{\rho_2} \frac{dp}{dT} = 0.
  \]  

- **energy-conservation equation**
  \[
  \alpha_2 \rho_{g1} w_{m1}^2 \left(\frac{w_{m1}^2}{2} + h_{g1}\right) + (1 - \alpha_2) \rho_{l1} w_{m1}^2 \left(\frac{w_{m1}^2}{2} + h_{l1}\right) = \alpha_2 \rho_{g2} w_{m2}^2 \left(\frac{w_{m2}^2}{2} + h_{g2}\right) + (1 - \alpha_2) \rho_{l2} w_{m2}^2 \left(\frac{w_{m2}^2}{2} + h_{l2}\right).
  \]

Similar to the momentum and thermal-equilibrium states, $\rho_{g2}, \rho_{l2}, h_{g2},$ and $h_{l2}$ can be calculated from the pressure corresponding to the saturation state of themal equilibrium in the two-phase flow. Therefore, the four unknown terms $\alpha_2, w_{m2}, \rho_{g2},$ and $\rho_{l2}$ can then be found from Eqs. (12), (13), (14), and (15).

### 3.4. Jump equation for the normal shock wave in the momentum and thermal frozen states

By assuming that all transport phenomena can be neglected, mass, momentum, and energy for each phase are conserved.

- **mass-conservation equation**
  \[
  \alpha_2 \rho_{g1} w_{m1} = \alpha_2 \rho_{g2} w_{m2},
  \]  
  \[
  (1 - \alpha_2) \rho_{l1} w_{m1} = (1 - \alpha_2) \rho_{l2} w_{m2}.
  \]  

- **momentum-conservation equation**
  \[
  \alpha_2 \rho_{g1} w_{m1}^2 + p_1 = \alpha_2 \rho_{g2} w_{m2}^2 + p_2,
  \]  
  \[
  (1 - \alpha_2) \rho_{l1} w_{m1}^2 + p_1 = (1 - \alpha_2) \rho_{l2} w_{m2}^2 + p_2.
  \]  

- **energy-conservation equation**
  \[
  \alpha_2 \rho_{g1} w_{m1}^2 \left(\frac{w_{m1}^2}{2} + h_{g1}\right) = \alpha_2 \rho_{g2} w_{m2}^2 \left(\frac{w_{m2}^2}{2} + h_{g2}\right),
  \]  
  \[
  (1 - \alpha_2) \rho_{l1} w_{m1}^2 \left(\frac{w_{m1}^2}{2} + h_{l1}\right) = (1 - \alpha_2) \rho_{l2} w_{m2}^2 \left(\frac{w_{m2}^2}{2} + h_{l2}\right).
  \]

The terms $h_{g2}$ and $h_{l2}$ can be estimated using the mutual phase pressure $p_2$ and the densities $\rho_{g2}$ and $\rho_{l2}$. Therefore, the six unknown terms $\alpha_2, w_{m2}, \rho_{g2}, p_2, \rho_{l2},$ and $p_2$ can be estimated using Eqs. (16)–(21).
3.5. Relationship between normal and oblique shock waves

The equation for the oblique shock wave can be calculated from its relationship with the normal shock wave. As shown in figure 1, because the velocity in the tangential direction does not change for the oblique shock wave, Eq. (22) is obtained:

\[
\tan \frac{\phi - \theta}{\tan \theta} = \frac{w_{n2}}{w_{n1}} \quad \text{i.e.} \quad \theta = \phi - \tan^{-1}\left(\frac{w_{n2}}{w_{n1}} \tan \theta\right).
\]

(22)

Here, \(\theta\) is the turning angle of the inclined wall, \(\phi\) is the shock-wave angle for the oblique shock wave, and \(w_{c}\) is the normal velocity. The shock-wave angle \(\phi\) can be calculated from Eq. (22) using the turning angle of the inclined wall, \(\theta\), and the ratio of normal velocities, \(w_{n2}/w_{n1}\). The ratio of the normal velocities is similar to that for the normal shock wave. Therefore, if the turning angle of the inclined wall is given, the oblique shock-wave angle is determined using the relational expression for the normal shock wave.

\[\text{Oblique shock wave}\]

\[\text{Inclined wall}\]

\[\text{Inlet (c_p = c_v)}\]

\[\text{Outlet}\]

\[\text{Wall}\]

\[\text{Oblique shock wave}\]

\[\text{(Slip for gas, outlet for liquid)}\]

\[\text{Wall}\]

\[\theta\]

\[\phi\]

\[L\]

\[w_1\]

\[w_2\]

\[w_{n1}\]

\[w_{n2}\]

\[\phi - \theta\]

\[\theta\]

\[\phi\]

\[\text{Figure 1. Velocity vector diagram for the oblique shock wave.}\]

\[\text{Figure 2. Calculation domain for the weak oblique shock wave.}\]

4. Relationship between strong and weak oblique shock waves

4.1. Weak oblique shock wave

4.1.1. Analysis method for the weak oblique shock wave. Weak oblique shock waves are generated by mach waves, and in supersonic flow, entropy production is small. In addition, with only slight disturbance to the flow field of weak oblique shock waves, the back of the shock wave is supersonic at the inlet.

In past studies [4][5], weak oblique shock waves were simulated using numerical analysis. In numerical analysis, Constrained Interpolation Profile Scheme method is applied to the basic equations of the compressible two-phase flow, which considers momentum, heat, and mass transports. Non-steady calculations are performed, and a steady-state solution is obtained at which a constant value is reached. The calculation domain in figure 2 has the same shape as that of the inclined wall so that oblique shock waves are generated over a semi-infinite plane. The inclined wall has an angle of \(\theta\). For the domain, we make 70 divisions in the direction horizontal to the inclined wall and 100 divisions in the direction perpendicular to the wall. In the calculation domain, we define a typical length of \(L\) for
the inlet-section height and a typical velocity of $u$ for the gas-flow rate in the inlet section. The two-phase flow enters into the domain from the left side and flows out from the top and right sides. On the bottom side of the wall surface, the component of the gas velocity in the direction perpendicular to the wall is 0, and the horizontal component is symmetrical with the wall. It has been assumed that, after collision with the wall, the liquid flows as a liquid film with no reflection. Because the volume of such a liquid film is small, the volume of the liquid-film flow and the boundary layer were neglected in the analysis. Therefore, as for the outlet, the boundary conditions of the free outflow for the liquid are given. The inflow conditions were that constant pressure and velocity were maintained, and for an equilibrium state, the velocities of both gas and liquid phases at the inlet were assumed equal.

This analysis was performed by simulating the four extreme states of the transport phenomena for the two-phase flow, total-equilibrium state, thermal frozen state, momentum frozen state, and total frozen state. The momentum relaxation time, $\tau_v$, and the thermal relaxation times, $\tau_g$ and $\tau_l$, are important in simulating these extreme states. The relaxation times are functions of the droplet size. Therefore, the analysis of weak oblique shock waves proceeds by assuming each non-dimensional relaxation time as shown in table 1.

<table>
<thead>
<tr>
<th>Extremal state</th>
<th>Momentum relaxation time</th>
<th>Thermal relaxation time</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Droplet size $d$</td>
<td>$\tau_v$</td>
</tr>
<tr>
<td>$^a\text{m: eq, t: eq}$</td>
<td>0.1 $\mu$m</td>
<td>$1.42 \times 10^{-4}$</td>
</tr>
<tr>
<td>$^b\text{m: eq, t: fr}$</td>
<td>0.1 $\mu$m</td>
<td>$1.42 \times 10^{-4}$</td>
</tr>
<tr>
<td>$^c\text{m: fr, t: eq}$</td>
<td>1000 $\mu$m</td>
<td>$1.42 \times 10^{4}$</td>
</tr>
<tr>
<td>$^d\text{m: fr, t: fr}$</td>
<td>1000 $\mu$m</td>
<td>$1.42 \times 10^{4}$</td>
</tr>
</tbody>
</table>

$^a$ Momentum and thermal equilibrium state.
$^b$ Momentum equilibrium and thermal frozen state.
$^c$ Momentum frozen and thermal equilibrium state.
$^d$ Momentum and thermal frozen state.

4.1.2. Characteristics of weak oblique shock waves. Figure 3 shows the results of the numerical analysis for weak oblique shock waves and the differences in the oblique shock waves by assuming an inflow equilibrium Mach number $M_{eq} = 2.17$ for the three extreme states: (a) the total equilibrium state, (b) the thermal frozen state, and (c) the momentum frozen state. In figure 3, according to the differences between these extreme states, differences appear in the generated shock-wave angle of the oblique shock wave. To compare the results of the numerical analysis with the theoretical values for the oblique shock waves in the extreme states, the relationship between the shock-wave angle $\phi$ and the inflow equilibrium Mach number $M_{eq}$ is shown in figure 4, and the relationship between the pressure ratio $p_2/p_1 = p_{21}$ for the front and back of the shock wave and the inflow Mach number, $M_{eq}$, is shown in figure 5. Figures 4 and 5 are the results for a constant turning angle of the inclined wall, $\theta = 10^\circ$, and for the calculations in the previous section, which uses the relational expression for the oblique shock waves in the extreme states. The curves in the diagram are shown as a red solid line for the momentum and thermal equilibrium state, blue dashed line for the momentum equilibrium and thermal frozen state, green dashed–dotted line for the momentum frozen and thermal equilibrium state, and purple dashed-two dotted line for the momentum and thermal frozen state. The domain for the weak oblique shock wave is the region ranging from the point of inflection in figure 4 with a smaller shock-wave angle $\phi$, but in figure 5, it is the region ranging from the point of inflection for which the pressure ratio, $p_{21}$, tends to increase by a smaller amount with increasing inflow equilibrium Mach number $M_{eq}$. In addition, the domain for the strong oblique shock wave, which will be discussed later, is the region in figure 4 ranging from the point of inflection with the larger shock-wave angle $\phi$, but in
figure 5 is the region ranging from the point of inflection for which the pressure ratio $p_{21}$ tends to increase by a larger amount with increasing inflow equilibrium Mach number, $M_{e1}$.

The angles of the oblique shock waves obtained from the numerical analysis are shown in figure 4 as circles. From figure 4, because the numerical-analysis results match approximately with the theoretical curves for the oblique shock waves in the extreme states, the weak oblique shock waves may be predicted using this type of analysis.

![Figure 4. Shock wave angles versus front Mach number for extreme states.](image)

![Figure 5. Strength of oblique shock wave versus front Mach number for extreme states.](image)

4.2. Strong oblique shock waves

4.2.1. Analysis method for strong oblique shock waves. Another solution for oblique shock waves are strong oblique shock waves, as is the case for normal shock waves, with distinctive large changes in the physical quantities in the front and back of the shock wave and within its interior. Phenomena such as viscous stress and heat transfer occur in a non-equilibrium state. In addition, an increase in the
changes in the physical quantities means that the flow changes from supersonic to subsonic. The calculation domain created to predict the strong oblique shock waves on a semi-infinite plane is shown in figure 6. For the calculation domain, we defined the number of divisions in the direction horizontal to the inclined wall as 120 and the divisions in the direction perpendicular to the inclined wall as 100. Moreover, in this domain, we defined a typical length, \( L \), for the height of the inlet section and a typical velocity, \( u \), for the gas-flow rate in the inlet section. The boundary and inlet conditions are defined similarly for the weak oblique shock waves.

The flow at the back of the shock wave, in case of strong oblique shock waves, is subsonic. In general, within the supersonic flow, because the flow exceeds maximum propagation velocity, the effects on the downstream cannot be propagated forward. However, within the subsonic flow, the effects on the downstream can be propagated forward. Therefore, not only the inflow conditions were given but also the initial distribution as well as the physical quantities derived from the relational expression for the oblique shock wave in extreme states are given for the back of the shock wave shown in the gray region in figure 6. The non-dimensional relaxation times used in the analysis of the strong oblique shock wave in the extreme states were assumed to be the same as those shown in table 1.

### 4.2.2. Establishing the initial distribution for the calculation domain

The physical quantities that are important in the initial distribution for the calculation domain are the pressure ratio, \( p_{11} \), for the front and back of the shock wave and the shock-wave angle, \( \phi \). Both values are computed from the previously mentioned shock-wave relational expression. However, if the analysis is performed using these two values, an extremely unstable solution is obtained for given arbitrary values. The analysis results are given below. First, figure 7 shows the results for the case in which the pressure ratio is large for initial conditions, rather than just the required pressure ratio, to sustain an oblique shock wave at a given shock-wave angle. In figure 7, we can observe that the fundamental oblique shock wave moves in front of the inclined wall and is generated with an angle approximately equal to the normal shock wave. On the other hand, figure 8, which is the opposite case of figure 7, shows the results for a small pressure ratio. In figure 8, from the turning starting point of the inclined wall, a weak oblique shock wave is generated in the backward direction with a small shock-wave angle. Then, at some arbitrary position, a shock wave that is close to a normal shock wave is generated from the inclined wall, and the oblique shock waves that are weak in the forward direction overlap to generate a comparatively strong oblique shock wave. This configuration conforms to the \( \lambda \)-shaped shock wave in gas dynamics.

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**Figure 6.** Calculation domain for strong oblique shock wave.
If the time-step progresses further, the results in figure 7 proceed forward, and those in figure 8 proceed backward. Because the domain for the strong oblique shock wave is an extremely unstable region, these waves are sensitive to very slight differences in the initial values and tend to migrate toward a stable solution.

4.2.3. Characteristics of strong oblique shock waves. The numerical-analysis results for strong oblique shock waves are shown in figure 9. In figure 9, extreme states are shown for the total equilibrium state with \( M_{\text{eq}} = 1.5 \) in (a), the thermal frozen state with \( M_{\text{eq}} = 1.7 \) in (b), and the total frozen state with \( M_{\text{eq}} = 2.6 \) in (c). From figure 9, it can be observed that, even if the inflow Mach number for each of the extreme states is different, there is no considerable difference in the shock-wave angle for the generated oblique shock wave. By plotting the shock-wave angle obtained by this analysis as triangles on the theoretical curves, as shown in figure 4, we get figure 10. From figure 10, given that the numerical-analysis results are an approximate match with the theoretical curves for the oblique shock waves in extreme states, this demonstrates the possibility of predicting strong oblique shock waves by using this type of analysis.

![Figure 7. Case of large pressure ratio.](image1)

![Figure 8. Case of small pressure ratio.](image2)

**Figure 9.** Strong oblique shock waves from numerical analysis.
4.3. Comparison with the oblique shock waves obtained by experiments

In past studies [5], the authors have experimentally measured the oblique shock waves generated in a convergent-divergent (Laval) nozzle using carbon dioxide gas as a refrigerant. The static-pressure distribution inside the nozzle in the experiments is shown in figure 11. For the measured oblique shock waves, when a third-order approximation was made for the pressure distribution around the points at which the pressure started rising and a cross-correlation was performed, the average value was found to be 67°. This result demonstrates that for this shock wave angle, which was shown theoretically for carbon dioxide gas, the experimentally generated oblique shock waves exhibit a relatively strong pressure ratio of the front and back of the shock wave. Accordingly, if we plot the angles for the strong oblique shock waves obtained by numerical analysis and the shock wave obtained from experiments on the shock-wave-angle-relationship diagram, we get figure 12. We can observe from figure 12 that the shock-wave angle for the oblique shock wave obtained from experiments is approximately equal to the strong oblique shock-wave region.

5. Conclusion

We analytically investigated the weak and strong oblique shock waves that are generated in the supersonic two-phase flow of carbon-dioxide gas and found the following wave characteristics:

- For each respective extreme state, a stable solution for weak oblique shock waves was found with outflow boundary conditions.
• The angles for the weak oblique shock waves varied considerably with differences in the transport processes, but the solution matches with the theoretical solution and demonstrates that this analysis was correct.
• Because the back of the strong oblique shock waves is subsonic, it is necessary to define the pressure at the back of the wave. For a small change in the pressure at the back of the wave, a solution was obtained in which either the oblique shock wave becomes a normal shock wave that progresses forward or becomes a wave associated with a $\lambda$-shaped weak oblique shock wave that retreats backward.
• Strong oblique shock waves are moderately unstable, but steady-state solutions were obtained with this analysis method for each respective extreme state.
• We revealed that the oblique shock waves obtained in the experiments within the two-phase flow nozzle for carbon dioxide are strong waves.

6. Reference