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# Semi-active $H_{\infty}$ control of seat suspension with MR damper

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Abstract. The vibration control of a seat suspension system with magnetorheological (MR) damper is investigated in this study. Firstly, a dynamical model of the seat suspension system with parameter uncertainties (such as mass, stiffness, damping) and actuator saturation is established. Secondly, based on Lyapunov functional theory and considering constraint conditions for damping force, the semi-active controller is designed, and the controller parameters are derived in terms of linear matrix inequalities (LMIs), which guarantees  $H_{\infty}$  performance index. Finally, compared  $H_{\infty}$  control strategy and the passive, skyhook control

strategy, the simulation results in time and frequency domains demonstrate the proposed approach can achieve better vertical acceleration attenuation for the seat suspension system and improve ride comfort.

#### **1. Introduction**

The prolonged vibration could cause harm to human body organs, such as lumbar, spine, stomach and kidney, damaging the driver's physical and mental health [1]. If suffering from vibration or shock for a long time, the driver's fatigue and anxiety will increase, and their operating accuracy and work efficiency will also reduce. In serious cases, it even brings the hidden danger for driving safety, and increases the probability of traffic accident. The seat suspension system is a kind of important device in vibration attenuation, whose performance has a great influence on ride comfort. The study of the control of seat suspension has a great significance for it's vibration attenuation, improving the driver's working conditions and work efficiency and ensuring personnel health.

Three main types of seat suspension, i.e., conventional passive seat suspension, semi-active seat suspension and active seat suspension, have been presented so far. Normally, conventional passive seat suspensions are constituted of elastic elements of the fixed stiffness and damping components of the fixed damping force. Therefore, it can not adjust itself automatically with the change of the incentive. Active seat suspension is actually a power-driven system, and its suspension components relie on the external power, meanwhile, it can control the amount of energy. However, it is a complex and costly system with large energy consumption, which is the main drawback that prevents this technique from being used extensively in practice [2]. Semi-active seat suspension consists of spring and damper whose damping force can be adjusted in real time (ER damper or MR damper). MR damper fluid is an excellent semi-active control device, it possesses simple structure, large dynamic

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force, good durability, low energy consumption and strong reliability, which can be treated as a passive control devices even the control system failures. MR damper as a semi-active control device combines the reliability of passive control systems and the strong adaptability of active control system. It can achieve the similar control performance of the active control system by means of a certain control law. It has a good prospect of engineering application as semi-active damping device. Therefore, it has received more and more attention in recent years [3-6].

The performance of a semi-active control system is dependent on the choice of control strategy [7], such as skyhook control [8-10], groundhook control [11], neuro-fuzzy control [12], LQG control [13], adaptive control and nonlinear control [14] and  $H_{\infty}$  control [15-17]. The parameter uncertainties, caused by the structural vibration and load changes, affect the stability of the control system when modelling the seat suspension system. Therefore, the seat suspension control system requires a certain degree of robustness to deal with the uncertainty of the model and external incentives. However, previous studies often neglected the effect of the uncertainty on the control performance in the modelling process, whereas the  $H_{\infty}$  control can takes account of the effect of uncertainty and guarantee the robust stability of the system [18]. On the other hand, the damping force will increase with increasement of excitation. However, the damping force will not increase longer when the excitation current reaches a certain degree because of the internal structure of the damper, and the actuator saturation appears. Robust  $H_{\infty}$  control can effectively deal with the problem of the actuator saturation when exerting the control force over the seat suspension [18-19]. So, an appropriate robust  $H_{\infty}$  control for the semi-active seat suspension system is designed to provide a trade-off of the two main performance requirements (ride comfort and suspension deflection).

In this research, the dynamical model of a seat suspension system with parameter uncertainties and actuator saturation is established. Then, combined with the damping force characteristics of MR damper, a state feedback controller is designed which guarantees  $H_{\infty}$  performance index and the constraints of the actuator saturation and the suspension deflection. Finally, some necessary comparisons between the  $H_{\infty}$  control strategy and the passive and skyhook control strategy are given, and the simulations are used to validate the effectiveness of the proposed control strategy.

#### 2. Dynamic model of the seat suspension with MR damper

#### 2.1. The characteristic of MR damper

The MR damper is illustrated schematically in Figure 1 and the Bingham model is adopted in this study for the MR damper (shown in Figure 2). The expression of the damping force is shown as follow:

$$F = c_e \dot{x}_d + f_d \operatorname{sgn}(\dot{x}_d) \tag{1}$$



Where  $c_e$  is the viscous damping coefficient of the MR damper,  $\dot{x}_d$  is the relative velocity between the piston and the cylinder. Therefore, the MR damper can be seen as the sum of a passive viscous damper and a semi-active coulomb damping device. The passive viscous damping is not controllable

and coulomb damping force is controllable. By designing appropriate control algorithm, the current is changed, and the shear yield strength of the MR fluid is changed accordingly. Then, desired control force is achieved.



Figure 2. The Bingham model of MR damper.

RD 8040-1 MR damper from Lord Corporation is used in this study. By applying sinusoidal excitation with a fixed frequency of 1Hz and constant amplitude of 15mm, the response of force is changed with different input current from 0 to 2A with increments of 0.5A. Thus, the force versus displacement loops and the force versus velocity loops under various electric currents were obtained as shown in Figure 3 and Figure 4. It can be seen that the damper force will remarkably increase with the increasement of the input current.



under various currents .

**Figure 4.** Force versus velocity loops under various current.

By expression 1, we can put viscous damping force into the system damping matrix. So, the MR damper can be seen as a simple controlled friction damper.

#### 2.2. The dynamical model of a seat suspension system with MR damper

A physical model of the semi-active seat suspension system with a passive spring and a controllable MR damper is shown in Figure 5, whose dynamical model is abstracted in Figure 6.

Make  $c_e = c_s$ ,  $\dot{x}_d = \dot{z}_s - \dot{z}_r$ ,  $f_d = u$  and define  $w(t) = \dot{z}_r$ 

The dynamic equations of motion of the seat suspension system are given by:

$$m_{p}z_{p} + c_{p}(z_{p} - z_{s}) + k_{p}(z_{p} - z_{s}) = 0$$

$$m_{s}\ddot{z}_{s} + c_{s}(\dot{z}_{s} - \dot{z}_{r}) + k_{s}(z_{s} - z_{r}) - c_{p}(\dot{z}_{p} - \dot{z}_{s}) - k_{p}(z_{p} - z_{s}) = -u$$
(2)

Where  $m_p$ ,  $m_s$  are the mass of the driver and the seat, respectively;  $z_r$  is the road displacement input,  $z_p$ ,  $z_s$  are the displacements of the driver and the seat, respectively;  $k_p$ ,  $k_s$  are stiffnesses of the driver and the passive suspension system, respectively;  $c_p$ ,  $c_s$  represent the damping of the driver and the MR damper when the input current is zero, respectively; u is the active control input of the seat suspension.



**Figure 5.** Physical model of the semi-active seat suspension with MR damper.



**Figure 6.** A dynamical model of the seat suspension system with MR damper.

Define the set of states:

$$x = [x_1, x_2, x_3, x_4]^T$$

where the components of state variables are given as follows:

$$x_1 = z_p - z_s$$
,  $x_2 = \dot{z}_p$ ,  $x_3 = z_s - z_r$ ,  $x_4 = \dot{z}_s$ 

Therefore, the dynamic equations in (2) can be written into a state space equation form as:

$$\dot{x}(t) = Ax(t) + Bu(t) + B_w w(t) \tag{3}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_p}{m_p} & -\frac{c_p}{m_p} & 0 & \frac{c_p}{m_p} \\ 0 & 0 & 0 & 1 \\ \frac{k_p}{m_s} & \frac{c_p}{m_s} & -\frac{k_s}{m_s} & -\frac{c_p + c_s}{m_s} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 & -\frac{1}{m_s} \end{bmatrix}^T, B_w = \begin{bmatrix} 0 & 0 & -1 & \frac{c_s}{m_s} \end{bmatrix}^T$$

taking the parameter uncertainties and actor saturation into account, the seat suspension model can be expressed as:

$$\dot{x}(t) = A_1 x(t) + B_1 u(t) + B_w w(t)$$
(4)

The parameter uncertainties considered here are norm-bounded of the form:

$$A_{1} = A + \Delta A, B_{1} = B + \Delta B$$
$$[\Delta A \quad \Delta B] = L\varepsilon(t) [E_{A} \quad E_{B}]$$

where  $\Delta A$ ,  $\Delta B$  are unknown real matrices, reflecting the parameter uncertainties in the system model. L,  $E_A$ ,  $E_B$  are known constant real matrices of appropriate dimensions.  $\varepsilon(t)$  is an unknown unreal matrix function of appropriate dimensions and satisfies  $\varepsilon^{T}(t)\varepsilon(t) \leq I$ , I represents the unit matrix. And the actuator saturation nonlinearity is described by:

$$|u(t)| \le u_{\max} \tag{5}$$

where  $u_{\text{max}}$  is the maximum controlled damping force.

Ride comfort is closely related to the vertical acceleration of the body. Denote body acceleration as control output:

$$z_1(t) = \ddot{z}_p \tag{6}$$

Meanwhile, in order to ensure the ride comfort and prevent the seat mechanical structure from damaging, the controller should be able to inhibit suspension disturbance from exceeding the limits of its vibration. In order to satisfy the performance constraint, denote the

$$z_2(t) = \left| \frac{z_s - z_r}{z_{\text{max}}} \right| < 1 \tag{7}$$

where  $Z_{max}$  is the maximum limit of suspension deflection.

Therefore, the seat suspension control system can be described by equation 4 and the following equations:

$$z_{1}(t) = C_{1}x(t) + D_{1}u(t)$$
(8)

$$z_2(t) = C_2 x(t) \tag{9}$$

where

 $C_1 = \begin{bmatrix} -\frac{k_p}{m_p} & -\frac{c_p}{m_p} & 0 & \frac{c_p}{m_p} \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 & 0 & \frac{1}{z_{max}} & 0 \end{bmatrix}, \quad D_1 = 0, \quad C_1, \quad C_2 \text{ are known constant}$ 

real matrices of appropriate dimensions for describing the system model.

According to the performance requirements, the design goal can be summed up as follows:

Define  $T_{w}(s)$  as the transfer function from the disturbance inputs w(t) to the controlled (1)output  $\ddot{z}_p(t)$ . The  $H_{\infty}$  controller is designed for improving the ride comfort such that the closedloop system guarantees:

$$\left\|T_{zw}(s)\right\|_{\infty} < \gamma \tag{10}$$

where  $\gamma$  is a prescribed scalar.

Taking the actuator saturation into account, equation 5 should be satisfied. (2)

(3)Taking the safety of mechanical structure into account, equation 7 should be satisfied.

# 3. Robust $H_{\infty}$ controller design for the seat suspension system with MR damper

In order to get the controller parameters, the following lemmas are necessary.

**Lemma 1** [20]. Given appropriately dimensioned matrices D, F(t) and E. For all F(t) satisfying  $F^{T}(t)F(t) \leq I$ , we have

 $DF(t)E + E^T F^T(t)D^T \leq \varepsilon^{-1}DD^T + \varepsilon E^T E$ ,

where  $\varepsilon$  is any scalar.

**Lemma 2** [20]. For any matrices (or vectors) U and V with appropriate dimensions, we have  $U^T V + V^T U \le \alpha U^T U + \alpha^{-1} V^T V$ 

where  $\alpha$  is any scalar.

**Lemma 3** [21]. For given symmetric matrix  $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$ , the following three conditions are

equivalent to each other, where  $S_{11}$  has the dimension of  $r \times r$  .

(i) 
$$S < 0$$
; (ii)  $S_{11} < 0, S_{22} - S_{12}^{T}S_{11}^{-1}S_{12} < 0$ ; (iii)  $S_{22} < 0, S_{11} - S_{12}S_{22}^{-1}S_{12}^{T} < 0$ .

In order to achieve the effect of the semi-active control, assume the MR damper as an active control device firstly, and the ideal force is derived from the active control algorithm, obtained control force is constrained by the semi-active control law, and then the force of MR damper is approximate to the ideal force to perform the semi-active control.

In this study, it is assumed that all the state variables can be measured, and we are interested in designing a state feedback controller:

$$u(t) = Kx(t) \tag{11}$$

where K is the state feedback gain matrix to be designed, such that the  $H_{\infty}$  norm of the closed-loop system is minimized, while satisfying the constraints in (5) and (7) for all nonzero  $w(t) \in L_2[0, \infty)$  with the initial condition.

**Theorem 1:** Let positive scalars  $\gamma$ ,  $\eta$  and  $\rho$  be given. If there exist matrices X > 0, Z, satisfying inequalities (12), (13), (14), such that the closed-loop system (4) is asymptotically stable for the state feedback controller (11),  $||T_{zw}(s)||_{\infty} < \gamma$  is satisfied and the constraints in (5) and (7) are guaranteed for all nonzero  $w(t) \in L_2[0,\infty)$ . Moreover, the state feedback gain matrix is obtained as  $K = ZX^{-1}$ .

$$\begin{bmatrix} XA^{T} + Z^{T}B^{T} + AX + BZ & L & XE_{A}^{T} + Z^{T}E_{B}^{T} & B_{w} & XC_{1}^{T} \\ L^{T} & -\eta^{-1}I & 0 & 0 & 0 \\ E_{A}X + E_{B}Z & 0 & -\eta I & 0 & 0 \\ B_{w}^{T} & 0 & 0 & -\gamma^{2}I & 0 \\ C_{1}X & 0 & 0 & 0 & -I \end{bmatrix} < 0$$
(12)

$$\begin{bmatrix} -I & \sqrt{\rho}Z \\ \sqrt{\rho}Z^T & -u^2_{\max}X \end{bmatrix} < 0$$
<sup>(13)</sup>

$$\begin{bmatrix} -I & \sqrt{\rho}C_2 X\\ \sqrt{\rho}XC_2^T & -X \end{bmatrix} < 0$$
(14)

**Proof:** Define a Lyapunov function for system as

$$V(x(t),t) = x^{T}(t)Px(t)$$
(15)

where P is a positive definite matrix.

By differentiating (15), we obtain

$$\dot{V}(x(t),t) = \dot{x}^{T}(t)Px(t) + x^{T}(t)P\dot{x}(t)$$

$$= x^{T}(t)\Big[(A+BK)^{T}P + P(A+BK)\Big]x(t)$$

$$+x^{T}(t)\Big[(E_{A}+E_{B}K)^{T}\varepsilon^{T}(t)(PL)^{T} + PL\varepsilon(t)(E_{A}+E_{B}K)\Big]x(t)$$

$$+\Big[B_{w}w(t)\Big]^{T}Px(t) + x^{T}(t)P\Big[B_{w}w(t)\Big]$$
(16)

By using Lemma 1, we obtain

$$\dot{V}(x(t),t) \le x^{T}(t)\Gamma x(t) + \left[B_{w}w(t)\right]^{T}Px(t) + x^{T}(t)P\left[B_{w}w(t)\right]$$
(17)

where  $\Gamma = (A + BK)^T P + P(A + BK) + \eta^{-1} PL(PL)^T + \eta (E_A + E_BK)^T (E_A + E_BK)$ , and  $\eta$  is any positive scalar.

Define  $H_{\infty}$  performance index  $J = \int_{0}^{\infty} \left[ z_{1}^{T}(t) z_{1}(t) - \gamma^{2} w(t)^{T} w(t) \right] dt$ . By using inequality (17) and J

$$\dot{V}(x(t),t) + z_1^T(t)z_1(t) - \gamma^2 w(t)^T w(t) \leq \left[ x^T(t) \quad w^T(t) \right] \Pi \left[ x(t) \quad w(t) \right]$$
(18)

where,

$$\Pi = \begin{bmatrix} \Gamma + C_1^T C_1 & PB_w \\ B^T_w P & -\gamma^2 I \end{bmatrix}$$
(19)

By using lemma 3, if  $\Pi < 0$ , we obtain

$$\begin{bmatrix} (A+BK)^{T} P + P(A+BK) & PL & (E_{A}+E_{B}K)^{T} & PB_{w} & C_{1}^{T} \\ L^{T}P & -\eta^{-1}I & 0 & 0 & 0 \\ E_{A}+E_{B}K & 0 & -\eta I & 0 & 0 \\ B_{w}^{T}P & 0 & 0 & -\gamma^{2}I & 0 \\ C_{1} & 0 & 0 & 0 & -I \end{bmatrix} < 0$$
(20)

It is now deduced from the above that if  $\Pi < 0$ , then  $\dot{V}(x(t),t) < 0$ . Therefore, the closed-loop system (4) with the controller (11) is asymptotically stable for the parameter uncertainties and actuator saturation of the seat suspension system. And if  $\Pi < 0$ ,  $\dot{V}(x(t),t) + z_1^T(t)z_1(t) - \gamma^2 w(t)^T w(t) < 0$ . Therefore, the system satisfies the  $H_{\infty}$  performance index for all nonzero  $w(t) \in L_2[0,\infty)$ .

By using equation (16) and lemmas 2, we have

$$\dot{V}(x(t),t) \le \gamma^2 w(t)^T w(t) \tag{21}$$

and by integrating both sides of inequality(21), we obtain

$$V(x(t),t) - V(x(0),0) \le \int_0^t \gamma^2 w(t)^T w(t) dt \le \gamma^2 \left\| w(t) \right\|_2^2 = \gamma^2 w_{\max}$$
(22)

where  $w_{\text{max}}$  is the disturbance energy.

This shows that,

$$x^{T}(t)Px(t) \le V(x(0), 0) + \gamma^{2} w_{\max} = \rho$$
(23)

Define  $\overline{x}(t) = P^{\frac{1}{2}}x(t)$ , from inequality (23), it follows that  $\overline{x}^{T}(t)\overline{x}(t) \le \rho$ . Hence

$$\max_{t\geq 0} |u(t)|^{2} = \max_{t\geq 0} ||Kx(t)||_{2}^{2} = \max_{t\geq 0} ||x^{T}(t)K^{T}Kx(t)||_{2}$$

$$= \max_{t\geq 0} ||\overline{x}^{T}(t)P^{-\frac{1}{2}}K^{T}KP^{-\frac{1}{2}}\overline{x}(t)||_{2} \leq \rho\lambda_{\max} \left(P^{-\frac{1}{2}}K^{T}KP^{-\frac{1}{2}}\right)$$

$$\max_{t\geq 0} |z_{2}(t)|^{2} = \max_{t\geq 0} ||x^{T}(t)C_{2}^{T}C_{2}x(t)||_{2}$$

$$= \max_{t\geq 0} ||\overline{x}^{T}(t)P^{-\frac{1}{2}}C_{2}^{T}C_{2}P^{-\frac{1}{2}}\overline{x}(t)||_{2} \leq \rho\lambda_{\max} \left(P^{-\frac{1}{2}}C_{2}^{T}C_{2}P^{-\frac{1}{2}}\right)$$
(24)

where  $\lambda_{\max}(\cdot)$  represents the maximum eigenvalue. Then, the constraints in (5) and (7) hold if

$$\rho P^{-\frac{1}{2}} K^T K P^{-\frac{1}{2}} < u^2_{\max} I$$
(25)

$$oP^{-\frac{1}{2}}C_2^{T}C_2P^{-\frac{1}{2}} < I$$
(26)

By using lemma 3, inequalities (25) and (26) are equivalent to

$$\begin{bmatrix} -I & \sqrt{\rho}K \\ \sqrt{\rho}K^{T} & -u^{2}_{\max}P \end{bmatrix} < 0$$
<sup>(27)</sup>

$$\begin{bmatrix} -I & \sqrt{\rho}C_2 \\ \sqrt{\rho}C_2^T & -P \end{bmatrix} < 0$$
(28)

Since expressions like (20) cannot be handled directly by LMI optimization. In order to solve the nonlinear problem, pre- and post-multiply by  $J_1 = diag(P^{-1}, I, I, I, I)$  and its transpose to (20) and pre-multiply and post-multiply by  $J_2 = diag(I, P^{-1})$  and its transpose to (27) and (28). Define  $X = P^{-1}$  and  $Z = KP^{-1}$ , inequalities (12), (13) and (14) are derived.

**Remark 1**. It is noticed that inequality (12), (13) and (14) are LMIs to  $\gamma^2$ . In order to obtain the lower limit of  $H_{\infty}$  performance index, we can minimize the variable  $\gamma$ . Therefore, the controller design problem can be changed to a problem of finding a solution to:

min 
$$\gamma^2$$
 s.t (12), (13) and (14) (29)

This minimization problem can be solved by using the Matlab LMI Toolbox. And the solution to (29) will be dependent on the values of  $\eta$  and  $\rho$ . The choice of values for  $\eta$  and  $\rho$  is a trial and error process. In general, the use of small values of  $\eta$  and  $\rho$  may mean that a high gain controller design could be obtained.

The controller gain matrix, K, obtained by solving the problem of (29), will be used to generate a desired control force. Then desired damping force of MR damper could be obtained by the semi-active control law, and the expression is as follows

$$f_{d} = \begin{cases} u(t)_{\max} & u(t)(\dot{z}_{s} - \dot{z}_{r}) \ge 0, \ |u(t)| \ge u(t)_{\max} \\ u(t) & u(t)(\dot{z}_{s} - \dot{z}_{r}) \ge 0, \ |u(t)| < u(t)_{\max} \\ 0 & u(t)(\dot{z}_{s} - \dot{z}_{r}) \le 0 \end{cases}$$
(30)

The actual damping force can be obtained by applying the corresponding current to the MR damper.

#### 4. Simulations

In order to validate the effectiveness of the control method proposed in the above section, simulations are conducted in this section. The parameters that describe seat suspension system with MR damper are listed in table 1 for reference. And the maximum suspension deflection is defined as  $z_{max} = 0.1m$  [19], the maximum control force is assumed as  $u_{max} = 3000N$ . The norm-bounded parameter uncertainties are expressed as:

$$L = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^{T}, E_{A} = \begin{bmatrix} 0 & 0 & -\frac{0.1k_{s}}{m_{s}} & \frac{0.1c_{s}}{m_{s}} \end{bmatrix}$$

According to the Table 1, constant matrices for describing the seat suspension system model are as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{44100}{50} & -\frac{1420}{50} & 0 & \frac{1420}{50} \\ 0 & 0 & 0 & 1 \\ \frac{44100}{26} & \frac{1420}{26} & -\frac{17085}{26} & -\frac{2420}{26} \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 & -\frac{1}{26} \end{bmatrix}^T$$

$$B_{w} = \begin{bmatrix} 0 & 0 & -1 & \frac{1000}{26} \end{bmatrix}^{T}, C_{1} = \begin{bmatrix} -\frac{44100}{50} & -\frac{1420}{50} & 0 & \frac{1420}{50} \end{bmatrix}, C_{2} = \begin{bmatrix} 0 & 0 & 10 & 0 \end{bmatrix}$$

		1 5	
Mass	Stiffness	Damping coefficient	
(kg)	(N/m)	(Ns/m)	
$m_p$ 50	k <sub>p</sub> 44100	<i>c</i> <sub><i>p</i></sub> 1420	
$m_s$ 26	k <sub>s</sub> 17085	$c_{s}$ 1000	

**Table 1.** Parameter values of the seat suspension system model.

By choosing  $\rho = 0.0009$ ,  $\eta = 1$  and solving the minimization problem of (29), we obtain the controller gain matrix by using the theorem 1 with a guaranteed  $H_{\infty}$  performance index and constraints in time domain,  $K = 10^6 \times [-1.0690 - 0.0333 - 0.0160 0.0361]$ 

Substitude K into (30), we can get the semi-active control force.

To show the advance of the proposed method, some necessary comparisons with the passive and skyhook control strategy are given.

#### (1) Passive control

There is no controllable control force for passive control system, it can only play certain a vibration reduction on the system, relying on elastic elements of the fixed stiffness and damping components of the fixed damping force. That is

$$u = 0 \tag{31}$$

## (2) Skyhook control

Skyhook control strategy assumes that the imaginary damper is mounted between the sprung mass  $m_s$  and virtual inertial space (Sky), and the skyhook control model shown in Figure 7.



Figure 7. The skyhook control model

The control strategy for the skyhook is given as:

$$u = -C_{sky}\dot{z}_s \tag{32}$$

$$f_{d} = \begin{cases} u(t)_{\max} & \dot{z}_{s}(\dot{z}_{s} - \dot{z}_{r}) \ge 0, \ |u(t)| \ge u(t)_{\max} \\ u(t) & \dot{z}_{s}(\dot{z}_{s} - \dot{z}_{r}) \ge 0, \ |u(t)| < u(t)_{\max} \\ 0 & \dot{z}_{s}(\dot{z}_{s} - \dot{z}_{r}) \le 0 \end{cases}$$
(33)

(34)

where  $C_{sky}$  is the skyhook damping coefficient. It can be determined according to the suspension system parameter optimization [22], and here  $C_{sky} = 1000 Nm/s$ .

To validate the seat suspension performance in the time domain, two typical types of road disturbance, i.e., bump road disturbance and random road disturbance, will be considered in the simulation.

#### 4.1. Simulation of bump road disturbance input

Road disturbances can be generally assumed as shocks. Shocks are discrete events of relatively short duration and high intensity, such as a convex bag or pothole on a smooth road surface. It is assumed that the bump road disturbance input has the following form: (the step input in time domain is shown in Figure 8)



Figure 8. The step input in time domain

Table 2. The comparison of the body acceleration under the step input

Control method	Max peak-to-peak value	Attenuation time
	$(ms^{-2})$	(s)
Passive	17.6486	0.50
Skyhook	15.9290	0.28
$H_{\infty}$	14.4778	0.04

11

Control method	Max suspension deflection	Attenuation time
	(m)	(s)
Passive	0.0155	0.78
Skyhook	0.0150	0.37
$H_{\infty}$	0.0146	0.38

Table 3. The comparison of the suspension deflection under the step input

Tables 2 and Table 3 are the comparison results of the body acceleration and the suspension deflection under the step input. As can be seen from the tables, the acceleration under the  $H_{\infty}$  control attenuates faster than the other two control methods (passive and skyhook control) and the settling time is reduced more quickly. The max peak-to-peak value of the body vertical acceleration under the skyhook control and the  $H_{\infty}$  control reduces by 9.74% and 17.97%, respectively. The time domain responses of body vertical acceleration under the step input is shown in Figure 9. The time domain responses of seat suspension deflection under the step input is shown in Figure 10.

0.25



Figure 9. The time domain responses of the body vertical acceleration under the step input

#### 4.2. Simulation of the random road input



time-domain response

Figure 10. The time domain responses of the suspension deflection under the step input

Random disturbance of the road is a continuous vibration, it refers to the continuous excitation along the road length direction, such as asphalt pavement and washboard road. For the continuous and random road surface, it is generally described by spatial frequency power spectral density function and the corresponding time domain representation:

$$G_q(n) = G_q(n_0) (\frac{n}{n_0})^{-w}$$
(35)

where *n* is the spatial frequency and  $n_o$  is the reference spatial frequency of  $n_o = 0.1(1/m)$ ;  $G_q(n_o)$  stands for the road roughness coefficient; w = 2 is the road roughness constant.

In order to facilitate the analysis, the spatial frequency functions need to be translated into time frequency function, set the vehicle forward velocity as v, then we have :

$$T = nv$$
 (36)

where, f is the time frequency, and its unit is Hz.

Therefore, the ground displacement power spectral density (PSD) is

$$G_q(f) = G_q(n_0) n_0^2 \frac{v}{f^2}$$
(37)

which is only related with the vehicle forward velocity. When the vehicle forward velocity is fixed, the ground displacement can be viewed as a white-noise signal. Select the road roughness as  $G_q(n_0) = 64 \times 10^{-6} m^3$  [23], which is corresponded to B Grade. Set the vehicle forward velocity as 30 km / h to generate the random road profile as Figure 11.

Table 4 is the comparison results of RMS value of the body acceleration and the suspension deflection under the random road input. As can be seen from the Table 4, the RMS value of body vertical acceleration under the skyhook control and the  $H_{\infty}$  control reduces by 10.13% and 39.25%, respectively. It shows that the  $H_{\infty}$  control outperforms the passive control and the skyhook control and realizes a better ride comfort.

**Remark 2.** Reducing the body acceleration and reducing the suspension deflection are a pair of contradiction, so that the improvement of ride comfort is at the expense of sacrificing the operational stability. Therefore, the suspension deflection has an increase for the  $H_{\infty}$  control. However, the time domain constraint is still guaranteed for the seat suspension system.



Figure 11. The random road input in time domain.

Table 4. RN	MS of	driver	body	acceleration	and	suspension	deflection	for a	B
grade road (	30Kı	m/h ) .							

Control method	RMS (A)	RMS (D)	
	$(m s^{-2})$	(m)	
Passive	0.4283	8.5803e-004	
Skyhook	0.3849	7.6642e-004	
$H_{\infty}$	0.2602	9.9959e-004	



**Figure 12.** The time domain responses of body vertical acceleration under the random road input.

**Figure 13.** The time domain responses of seat suspension deflection under the random road input.

The time domain responses of body vertical acceleration under the random road input is shown in Figure 12. The time domain responses of the seat suspension deflection under the random road input is shown in Figure 13. The frequency domain responses of body vertical acceleration under the random road input is shown in Figure 14. There are two distinct peaks, the first one is the corresponding resonance frequency for the body (6Hz), and the second one is the corresponding resonance frequency for the seat (12Hz). It can be seen that, compared with the passive control and the skyhook control, the  $H_{\infty}$  control greatly reduces the second-order resonance peak of the seat suspension system. The power spectral density of body acceleration under the the random road input is shown in Figure 15. It is observed from Figure 15 that the system for  $H_{\infty}$  control has lower PSD of body acceleration and smaller PSD of body acceleration value results in better ride comfort.



**Figure 14.** The frequency domain responses of body vertical acceleration with the random input.



Figure 15. The power spectral density of body acceleration with the random input.

# 5. Conclusions

This study presents the robust  $H_{\infty}$  control method for the semi-active seat suspension system with MR damper with parameter uncertainties and actuator saturation. The solution of the state feedback controller is given in terms of LMIs, which guarantees  $H_{\infty}$  performance index and the constraints. The simulation results in time and frequency domain demonstrate that it is closer to the actual system by considering the model parameter uncertainties and actuator saturation. And the proposed approach can achieve better vertical vibration attenuation and improve ride comfort for seat suspension.

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