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To cite this article: Sayan Kar 2012 *J. Phys.: Conf. Ser.* **405** 012002

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Bulk–brane models: an overview and some queries

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Abstract. In this article we first present a concise overview, highlighting the main features of the well-known bulk–brane models. We then discuss specific topics related to Lorentz violations, localisation of fields on the brane and braneworld gravity. Finally, we describe, in brief, a lower dimensional analog model for the warped two-brane system and conclude with some queries and remarks.

1. A very brief review

Almost a century ago, in 1914, Nordstrom [1] first introduced the notion of dimensions beyond the usual four. A few years later, pioneering work by Kaluza and Klein (KK) [2] laid the theoretical foundations of unification using compact extra dimensions. The KK idea, though pursued in terms of detail by many, remained largely dormant for quite some time till it reappeared and gained support mainly through the work of string theorists [3], during the second half of the twentieth century. In the late 1990s, another new, phenomenological idea arrived on the scene whose origins may be traced to early work by Akama, Rubakov–Shaposhnikov, Visser and Gogberashvili [4]. This new class of extra dimensional models which are significantly different from the Kaluza–Klein models are known today, collectively, as the *bulk-brane* scenario [5, 6].

1.1. Jargon

The term *bulk*, in these models, refers to the background higher dimensional spacetime whereas *brane* (derived from *membrane*) refers to an embedded $3 + 1$ dimensional, timelike hypersurface in the bulk. We assume we live on one such *brane*. The presence of the extra dimensions is manifest in phenomena on the brane. Advantages of the bulk-brane constructions include possibilities of (a) resolution of some well-known problems (b) verification of the existence of extra dimensions in collider and other terrestrial experiments (eg. non–Newtonian short distance forces) or astrophysical observations.

There are two types of bulk–brane models: the LED (large extra dimensional) model due to ADD [5] and the WED (warped extra dimensional) models due to RS [6]. We focus henceforth on the WED models, where we have two varieties: a *two-brane* model with an orbifolded extra dimension (topology S^1/Z_2 , finite and noncompact) and a *single brane* model where the extra dimension is of infinite extent. Fig. 1 illustrates the above notions, qualitatively.

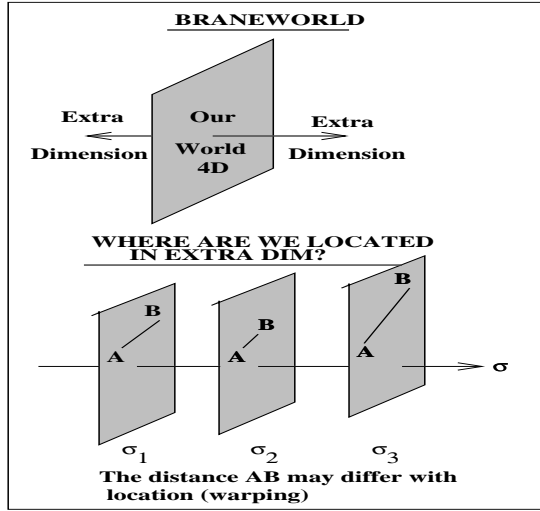


Figure 1. Bulk and brane, several branes, warping

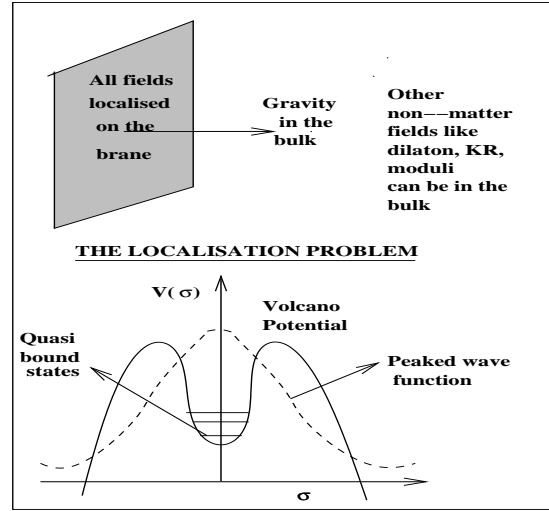


Figure 2. Localisation of fields on the brane, volcano potential.

1.2. Hierarchy

The hierarchy problem, in a loose sense, asks why the electroweak (100 GeV) and Planck (10^{19} GeV) scales are so widely separated? In the LED models, one does answer this, though a new hierarchy is generated [5]. The RS model resolution is better and does not introduce a new hierarchy. Briefly, one begins with the following five dimensional *warped* metric ansatz

$$ds^2 = e^{-2k|\sigma|} (-dt^2 + |d\vec{r}|^2) + d\sigma^2 \quad (1)$$

where σ denotes the extra dimension and $e^{-2k|\sigma|}$ is known as the warp factor. A $\sigma = \text{constant}$ hypersurface is the *brane*. The bulk geometry here is AdS (constant negative scalar curvature), generated by a bulk, negative cosmological constant. Branes are placed at $\sigma = 0$ (+ve tension) and $\sigma = \pi r_c$ (-ve tension) – sourced by delta functions in the bulk energy momentum (the $|\sigma|$ in the warp factor ensures the delta function source terms). Thus, the bulk spacetime is a slice of AdS (between $\sigma = 0$ and $\sigma = \pi r_c$). Integrating the five dimensional Einstein–Hilbert action over the extra dimension one gets the relation between the four and five dimensional Planck scales (M_{Pl} and M , respectively),

$$M_{Pl}^2 = \frac{M^3}{k} (1 - e^{-2k\pi r_c}) \quad (2)$$

Therefore, for moderate $kr_c\pi$, the 4D Planck scale M_{Pl} is of the same order as the five dimensional Planck scale M , if $k \sim M_{Pl}$. On the other hand, writing down a massive wave equation in a scaled Minkowski metric (which is the metric on the brane hypersurface, see next section), one notices that physical masses (Higgs VEVs) on the visible brane at $\sigma = \pi r_c$ scale as

$$m = m_0 e^{-kr_c\pi} \quad (3)$$

where m_0 is the five dimensional mass-scale (say of the order of the Planck scale). m_0 is effectively warped by the factor $e^{-kr_c\pi}$ and the mass m seen on the brane can thus be at the TeV scale (assuming $kr_c \sim 12$). Hence, all fundamental scales (in five dimensions) are of the order of the Planck scale (no hierarchy) and the TeV scale on the brane arises because of extra dimensions and warping. This is the WED resolution of the hierarchy problem [6].

1.3. Localisation

In the WED models, there is no notion of compactification (like in Kaluza Klein theory). The concept of localisation of fields on the brane hypersurface [7] replaces compactification. One assumes that all Standard Model fields are localised on the brane whereas gravity (and some other fields like the dilaton, Kalb Ramond, moduli) can spread into the bulk. However, to check localisation one needs to give the wave function representing the field an extra dimensional dependence and then prove that normalisable wave functions peaked about the location of the brane exist. For decaying warp factors, like the one for the RS model, the localisation problem reduces to that of a quantum particle in a volcano potential (see Fig. 2). Later in this article we will look at the problems that arise while analysing the localisation of gauge fields.

1.4. Thick branes

Apart from the RS type warp factor ($e^{-2k|\sigma|}$) wherein the brane is *thin* (delta function sources), there are models where the warp factor is a smooth function with no derivative discontinuities. These models are usually sourced by a scalar field in a potential. The brane in such *thick* brane models is realised as a scalar field domain wall (see [8, 9] for examples).

1.5. Stability

It is obvious that branes in the two-brane RS-type WED models are not stable. Stability is ensured by introducing an additional bulk scalar— this is the Goldberger–Wise mechanism [10]. An effective potential $V(r_c)$ is derived by integrating the matter (scalar field) action over the extra dimension. The existence of minima in the effective potential ensures stability. The $V(r_c)$ for the RS model does not have a minimum, but with additional scalar fields in the bulk, this becomes possible [10]. A useful discussion on stability may be found in [11].

1.6. Queries

The above basic formalism invites quite a few queries, some of which we now list below.

- Is a *scaled Minkowski metric*, which is the metric on the brane, *compatible* with all known laws of physics?
- Why is a *special* warped five dimensional line element chosen? Isn't a more *general* one possible? Say, for example:

$$ds^2 = -e^{2f_1(\sigma)}dt^2 + e^{2f_2(\sigma)}dx^2 + e^{2f_3(\sigma)}dy^2 + e^{2f_4(\sigma)}dz^2 + r_c^2 d\sigma^2 \quad (4)$$

- In the two brane model, can the branes have a *slant* w.r.t each other?
- The usual choice of *embedding* is $\sigma = \sigma_0$ (constant) and bulk (t, x, y, z) *identical* to on-brane (t, x, y, z) . This is the *simplest* and is also *non-minimal*. Why this choice of embedding?

2. Lorentz violations

Let us now turn to the general issue in the first three queries mentioned above—that of Lorentz violations. We shall not discuss the fourth query in this article.

2.1. Scaled Minkowski

The line element on the visible brane at $\sigma = \pi r_c$ is given as

$$ds_{brane}^2 = e^{-2kr_c\pi} \left[-dt^2 + dx^2 + dy^2 + dz^2 \right]$$

This leads to a *violation of the principle of relativity*. Let us see why. Recall the *scaled* Lorentz transformations (see Pauli's book on Theory of Relativity [12]).

$$x' = \kappa\gamma(x - vt) \quad ; \quad y' = \kappa y \quad z' = \kappa z \quad ; \quad t' = \kappa\gamma\left(t - \frac{vx}{c^2}\right)$$

Thus, the distance function in the S' frame, written in the S frame coordinates is

$$ds'^2 = \kappa^2 \left[-dt^2 + dx^2 + dy^2 + dz^2 \right]$$

which is a scaled Minkowski line element. Note that the requirement $\kappa = 1$ follows from *reciprocity*, which, in turn, follows from the *principle of relativity*. Let us briefly recall what reciprocity is all about. Place a unit length rod in the S frame along y (the boost is along x , say). Its length in S' is κ . Now place the same rod in S' . Its length in S is $\frac{1}{\kappa}$. For reciprocity to hold, which is physically meaningful, we need $\kappa = 1$. Thus, with $\kappa \neq 1$ reciprocity and hence the principle of relativity are violated. We end up having a *preferred* frame. Thus, one needs to make a choice—*solve* heirarchy problem but *violate* the principle of relativity. Giving up the principle of relativity still *allows* some transformations between inertial frames, as demonstrated in a recent paper [13]. It may also be noted that von Ignatowsky in 1910 (see reference in [13]), first showed how the principle of relativity is *intimately tied* with the *group* structure of Lorentz transformations. With the *scaling* we have a larger group, the conformal group, and apart from Maxwell's equations in 4D other *massive* equations are *not* invariant, a fact which lies at the heart of the hierarchy resolution mentioned earlier. However, in the RS type WED models, the speed of light postulate of special relativity is not violated.

2.2. Asymmetric warping

We now move on towards more explicit Lorentz violations. This is achieved by the so-called asymmetrically warped line elements,

$$ds^2 = -e^{2f(\sigma)}dt^2 + e^{2g(\sigma)}|d\vec{r}|^2 + r_c^2 d\sigma^2$$

Thus, on the brane (at $\sigma = \sigma_0$), we get

$$ds^2 = -e^{2f(\sigma_0)}dt^2 + e^{2g(\sigma_0)}|d\vec{r}|^2$$

In such models [14] both the postulates of special relativity are violated. The speed of light can be greater than c , if $f(\sigma_0) \neq g(\sigma_0)$. This fact was exploited [15] to explain a recent claim on the existence of superluminal neutrinos, from a warped braneworld perspective. However, the experimental claim has now been found to be erroneous, but the theoretical construction in [15] remains an intriguing result.

2.3. Slanted branes

Another Lorentz violating scenario is the recently proposed slanted braneworld model [16]. The Hernandez-Sher tilted line element is given as

$$ds^2 = e^{-2C|\sigma|+ax|\sigma|}\eta_{ij}dx^i dx^j + \left(R^2 + Dxa\right)d\sigma^2$$

where a is the tilt parameter— a small quantity. The off-diagonal component of the Einstein tensor

$$G_{14} = -3\left(1 + \frac{CD}{2R^2}\right)a$$

which implies, for $G_{14} = 0$, $2R^2 = -CD$. The two branes are not parallel, but slightly tilted. Interestingly, to order a , almost all results of the RS two-brane model are retained. However, the tilt does affect the fermion masses through a spatially varying Higgs VEV. The electron–proton mass ratio is modified. Upper bounds on the tilt parameter a can be calculated using data from quasar spectra. These yield a value: $a < 7.5 \times 10^{-7} GPC^{-1}$. The tilt aspect is of course, meaningful in a two-brane model. It will be worth exploring this model in further detail—for example, its phenomenological and cosmological consequences.

3. Localisation again

Even though one can show that most of the physical fields are localisable on the brane in the RS type WED models, a problem arises with gauge fields. We shall now highlight this problem.

3.1. Gauge fields in RSI

In the two-brane model, the extra dimensional coordinate is bounded. However, it was shown in [17], for a $U(1)$ gauge theory, that the Kaluza Klein excitations of the gauge field are *strongly coupled* to fermions. Precision electroweak data constrain the *lowest* KK states to lie *above* 25 TeV. Taking the weak scale to be 1 TeV, the resulting implications on the model parameters force the bulk curvature to be larger than the higher dimensional Planck scale, M , a fact which violates theoretical consistency. To preserve $|R| < M^2$, the weak scale must be pushed to > 100 TeV.

A possible way out of the above problem is to introduce *brane-localised kinetic terms* [18] in the action, as follows,

$$S_g = -\frac{1}{4} \int d^5x \left\{ \sqrt{-g_5} F_5^2 + [c_0 \delta(y) + c_\pi \delta(y - \pi)] F_4^2 \right\} \quad (5)$$

This modification leads to, for natural choices of parameters, a substantial *suppression* of the KK couplings, compared to the original model. Details may be found in [18]. However, though things work, note that such terms do not have a clear motivation in a curved spacetime setting and are introduced mainly as an extrapolation of flat spacetime results [18].

3.2. Gauge fields in RSII

In the single brane RS model, it is easy to see the localisation problem of gauge fields. Consider, the $U(1)$ theory with equation of motion

$$\partial_i \left(\sqrt{-g} g^{ij} g^{kl} F_{jl} \right) = 0 \quad ; \quad \partial_i A^i = 0, A_5 = 0. \quad (6)$$

The equation for zero mode $A'_i = 0$ yields a constant wave function along the fifth dimension $A_i = A_0 a_i(x)$. The constant wave function is normalisable for scalars but for Maxwell fields we have

$$\int d^5x \sqrt{-g} g^{ij} g^{kl} F_{ik} F_{jl} = \int dy A_0^2 \int d^4x F_{ij}^{(4)} F^{(4)ij} \quad (7)$$

and the y integration diverges. Therefore, free gauge fields cannot be localised in single brane models. A possible resolution is to modify the action as follows:

$$S_{gauge} = -\frac{1}{4} \int d^5x A(\phi) F_5^2$$

The $A(\phi)$ can be such that the divergent integral is rendered finite. Such modifications can arise due to various reasons: eg. dilaton, moduli couplings to gauge fields. Several other proposals also exist—however, the problem is far from resolved completely.

4. Brane gravity

There are three ways of studying brane gravity. We list them below and then discuss each briefly.

- Investigate the effects of the excited KK gravitons on particle phenomenology and find bounds on the masses of these states. This approach is ofcourse entirely perturbative.
- Look at the effective theory on the brane—the Shiromizu-Maeda-Sasaki equations [20] and focus on departures from standard Einstein gravity. A major problem here is the absence of a prescription to go back uniquely to the bulk.
- Investigate bulk higher dimensional gravity and its classical effects with reference to branes. This is a formal GR-motivated approach and needs to connect up with the other two.

4.1. Newtonian potential

The usual Newtonian gravitational potential is known to have corrections due to the presence of extra dimensions. The corrections due to the excited KK gravitons, have been calculated for both the LED and WED models. For instance, in the WED models, the corrected potential is of the form

$$U(r) = -\frac{Gm_1m_2}{r} \left(1 + \frac{C}{(kr)^2}\right)$$

One might be able to test this correction in the ongoing Newtonian gravity experiments at EOTWASH and STANFORD. However, no such attempt has been made yet. The groups are primarily focused on Yukawa-type corrections which arise in LED models. Interestingly, a possible experiment involving the WED corrections has been suggested in [19] though it is yet to be realised.

4.2. Effective Einstein

What are the Einstein equations induced on the four dimensional brane from the five dimensional Einstein equations in the bulk? From the work of [20], we know that these equations are

$$\begin{aligned} G_{ij} &= -\Lambda h_{ij} + \kappa^2 T_{ij} + 6\frac{\kappa^2}{\lambda} Q_{ij} - \mathcal{E}_{ij} \quad ; \quad \kappa^2 = \frac{1}{6}\lambda\kappa_5^4, \Lambda = \frac{1}{2} [\Lambda_5 + \kappa^2\lambda] \\ Q_{ij} &= \frac{1}{12} T T_{ij} - \frac{1}{4} T_{ik} T_j^k + \frac{1}{24} h_{ij} [T_{kl} T^{kl} - T^2] \quad ; \quad \mathcal{E}_{ij} = {}^{(5)}C_{abcd} n^a n^c h_i^b h_j^d \end{aligned} \quad (8)$$

The significant new terms here are a quadratic stress energy Q_{ij} , and the nonlocal \mathcal{E}_{ij} which is traceless and has no off-brane component. The usual Einstein equations are recovered for situations where (i) the bulk Weyl is zero and (ii) the quadratic contributions are small. However, we can also show that the quadratic contributions are important in the early universe. Recently, similar quadratic contributions have been shown to arise in the Eddington-inspired Born-Infeld (EiBI) theory of gravity [22]. Though the quadratic stress energy terms in the EiBI theory are qualitatively similar there are quantitative differences. It would be interesting to see if there are any connections between the two.

The effects of \mathcal{E}_{ij} have been explored in the context of black holes[21] and cosmology [21]. Further, a nonzero \mathcal{E}_{ij} can be used to model dark matter[23]. But any nonzero \mathcal{E}_{ij} would mean a non-zero bulk Weyl and therefore a bulk which is not of RS type. So, all conclusions using the RS metric will have to be revisited.

4.3. Bulk gravity

The study of bulk gravity is largely that of higher dimensional GR. One can begin with a generalised line element

$$ds^2 = e^{2f(\sigma)} \left(-dt^2 + a^2(t) |d\vec{x}|^2 \right) + b^2(t) d\sigma^2$$

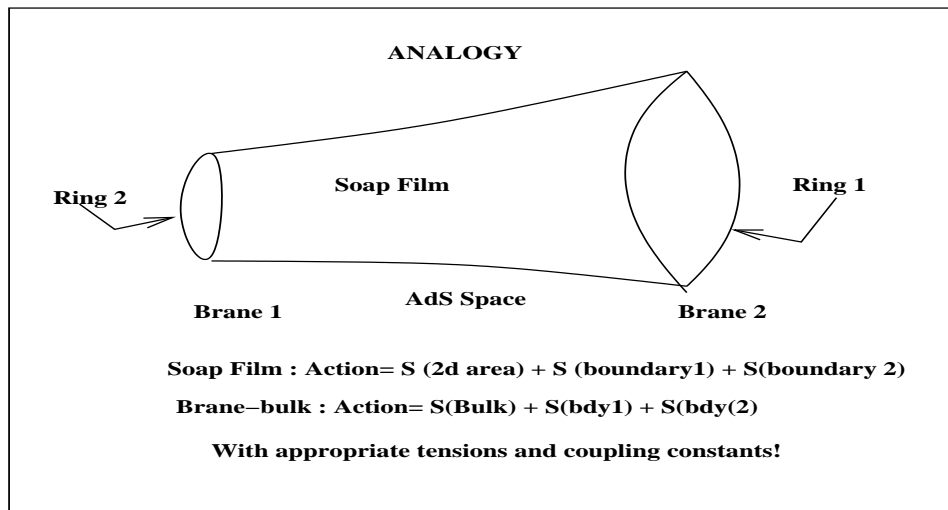


Figure 3. Lower dimensional analog model: soap film between rings.

and ask various questions. How does a variation of the warp factor reflect on geodesic motion? Is confined motion possible? How does a variation of the on-brane cosmological metric $a(t)$ and a time varying extra dimension $b(t)$, reflect on geodesic motion? What happens to geodesic flows (congruences) in the full five dimensional bulk? Do geodesic flows focus? If so, when and how? Under what conditions? Can we see/visualise the effect? What happens if there are more than one brane? Some of these questions are answered in [24].

Besides the above, there is significant research on *Brane Cosmology*. We refrain from discussing it here and refer the reader to the excellent review in [21].

5. Analog model

A useful lower dimensional analog model for the warped two-brane system is that of a soap film suspended between two co-axial rings. Fig. 3 depicts the analogy and correspondence in detail. The geometry on the 2D surface is that of a negative curvature catenoid (minimal surface). One can study fluctuations about the minimal surface and it turns out that the effective potential for fluctuations is a volcano potential—similar to potentials that arise in the study of localisation of fields on the brane. The interested reader may look up [25] for further details on the soap film problem.

6. Bounds from LHC

Finally, let us turn to experiments and the latest bounds from LHC. Using diphoton events in 7 TeV proton-proton collisions with the ATLAS detector [26] the 95 % CL lower limits on the lightest RS graviton mass is between 0.79 and 1.85 TeV, for values of the dimensionless coupling k/\bar{M}_{Pl} varying from 0.01 to 0.1. Combining with ATLAS results from the dielectron and dimuon final states, the 95% CL lower limit on the RS graviton mass for $k/\bar{M}_{Pl} = 0.01(0.1)$ is 0.80(1.95) TeV. Note, this is a lower bound and therefore not very conclusive.

7. Queries again and remarks

To end we provide a list of queries once again.

- What is there in the bulk? How do we decide on the content of bulk matter?
- How many extra dimensions are there? Is there a way to decide theoretically?

- What about models which consistently include the Lovelock, Gauss–Bonnet terms in higher dimensional gravity?
- Is it feasible to violate the principle of relativity in a classical world? How do we eliminate/retain asymmetrically warped or the slanted brane models?
- Is it possible to identify all SM particles using the localisation mechanism? Do we need a hybrid of compactification and localisation?
- How do we understand the quadratic stress energy and the Weyl–dependent terms better? What is their link with KK gravitons?
- How does one continue a four dimensional solution of the effective Einstein equations into the bulk? Taylor series type approaches give only local bulk geometry.
- Even if we have warped RS type extra dimensions, why will nature choose an AdS bulk? Unless experiments/observations decide on the existence of extra dimensions, one cannot answer many of the questions above. Even if experiments/observations prove the existence of extra dimensions, one would have to derive these phenomenological models from a fundamental theory, in a concrete way. Research in extra dimensions began in 1914 with Nordstrom’s work. Maybe we will have good reasons to celebrate the centenary of extra dimensions, in 2014!

Acknowledgments

The author thanks the organisers of COSGRAV 2012 for their warm hospitality.

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