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Gravitational Waves in Viable Modified Gravity Theories

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Abstract. We review our recent work [1] on gravitational waves in viable f(R) models. We concentrate on the exponential gravity and Starobinsky models. We show that in both cases, the mass of the scalar mode is order of $10^{-33} eV$ when it propagates in vacuum. In the presence of matter density, such as galaxy, the scalar mode can be heavy. In particular, it becomes almost infinity so that the scalar mode of gravitational wave for the exponential model disappears like the Λ CDM, whereas it can be as low as $10^{-24} eV$ in the Starobinsky model, corresponding to the lowest frequency of 10^{-9} Hz, which may be detected by the current and future gravitational wave probes in space.

1. Introduction

It has been widely accepted that one simple way to modify general relativity is to promote the Ricci scalar R in the Einstein-Hilbert action into an f(R) function, which is the so-called f(R) theory [2, 3, 4]. A viable model of f(R) can generate a late-time accelerating expansion of our universe, have the radiation-dominated stage followed by the matter-dominated one, and be consistent with the solar-system constraint under chameleon mechanism. The conditions for such a viable f(R) model include (i) the positivity of the effective gravitational coupling; (ii) the stability of cosmological perturbations; (iii) the stability of the late-time de-Sitter point; (iv) the asymptotic behavior to Λ CDM at the high curvature regime; (v) the solar system constraint; and (vi) the constraint from the violation of the equivalence principle. The typical examples of the viable f(R) models are Hu-Sawicki [5], Starobinsky [6], Tsujikawa [7] and exponential gravity models [8] as shown in Table 1.

In Ref. [9], Chiba showed that an f(R) model will allow a new scalar degree of freedom. This corresponds to a new scalar mode of gravitational wave besides the ordinary tensor one of general relativity. This new scalar mode will be massive and propagate as a longitudinal polarization. Various discussions and predictions about this extra scalar mode of gravitational wave have been given in the literature [10, 11, 12, 13]. However, most of them were concentrated on either quadratic or inverse-curvature type of f(R) models, which is highly restricted by the observational results [3]. In the talk, we will review our recent work [1] on gravitational waves in viable f(R) models. To illustrate our results, we will concentrate on the exponential and Starobinsky models. Our study can be easily extended to other viable models.

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model	f(R)	Constant parameters			
(a) Hu-Sawicki	$R - \frac{c_1 R_{\rm HS} (R/R_{\rm HS})^p}{c_2 (R/R_{\rm HS})^p + 1}$	$c_1, c_2, p(>0), R_{\rm HS}(>0)$			
(b) Starobinsky	$R + \lambda R_{\rm c} \left[\left(1 + \frac{R^2}{R_{\rm c}^2} \right)^{-n} - 1 \right]$	$\lambda(>0), n(>0), R_{\rm c}$			
(c) Tsujikawa	$R - \mu R_{\rm T} \tanh\left(\frac{R}{R_{\rm T}}\right)$	$\mu(>0), R_{\rm T}(>0)$			
(d) Exponential	$R - \beta R_{\rm E} \left(1 - e^{-R/R_{\rm E}}\right)$	$\beta, R_{\rm E}$			

Table 1. Viable f(R) models

We use units of $k_B = c = \hbar = 1$ and the gravitational constant $G = M_{Pl}^{-2}$ with the Planck mass of $M_{Pl} = 1.22089 \times 10^{19} GeV$.

2. Gravitational Waves in Viable f(R) Gravity

We start by considering a general Einstein-Hilbert action

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} f(R) + S_m(g_{\mu\nu}, \Psi_{\mu\nu}),$$
(1)

where f(R) is an arbitrary function of the Ricci scalar R, S_m is the action of the matter part and $\kappa^2 \equiv 8\pi G$. In the metric formalism, we vary the action (1) with respect to $g_{\mu\nu}$, and the modified Einstein field equation can be obtained as

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_{\mu}\nabla_{\nu})f'(R) = \kappa^2 T_{\mu\nu},$$
(2)

where a prime denotes the derivative with respect to R, ∇_{μ} is the covariant derivative and $\Box = g^{\mu\nu}\nabla_{\mu}\nabla_{\nu}$ is the d'Alembert operator. The trace of the field equation (2) gives

$$f'(R)R - 2f(R) + 3\Box f'(R) = \kappa^2 T,$$
(3)

where $T = g^{\mu\nu}T_{\mu\nu} = -\rho + 3a^2P$ is the trace of the matter energy-momentum tensor, and a is the scale factor.

For f(R), the de Sitter stage is a vacuum solution with a positive constant background curvature R_d , which is assumed to be homogeneous and static. Consequently, one has

$$\nabla_{\mu} f'(R_d) = 0 \quad \text{and} \quad f'(R_d) R_d = 2f(R_d). \tag{4}$$

Moreover, from Eq. (2), the Ricci tensor satisfies $R_{\mu\nu}|_{R_d} = g_{\mu\nu}R_d/4$.

In order to investigate gravitational wave in f(R) theories, we need to study the linearized theory of f(R) gravity. Consider a small perturbation from the FRW metric:

$$g_{\mu\nu} = \overline{g}_{\mu\nu} + h_{\mu\nu},\tag{5}$$

where $|h_{\mu\nu}| \ll 1$ is the perturbation and $\overline{g}_{\mu\nu} = diag(-1, a^2, a^2, a^2)$ is the FRW background metric. If the evolution of the system is much shorter than Hubble time, we can approximate the background spacetime to be nearly the Minkowski one with $\overline{g}_{\mu\nu} \approx \eta_{\mu\nu} = diag(-1, 1, 1, 1)$. We keep the theory to be the first order in $h_{\mu\nu}$ and neglect terms higher than $\mathcal{O}(h^2)$.

The different between gravitational waves in f(R) and general relativity is that it contains an extra scalar degree of freedom in f(R). This comes from the non-vanishing trace of the

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field equation. Eq. (3) can be viewed as equation of motion for a scalar field Φ . By the identifications [14, 15]

$$\Phi \to f'(R) \quad \text{and} \quad \frac{dV_{eff}}{d\Phi} \to \frac{2f(R) - f'(R)R - \kappa^2 \rho}{3},$$
(6)

we obtain the Klein-Gordon equation for the scalar field Φ :

$$\Box \Phi = \frac{dV_{eff}}{d\Phi}.$$
(7)

In order to have a stable perturbation of spacetime, we must require the background scalar Φ_0 to stay at the stable minimum of the effective potential V_{eff} , i.e., $dV_{eff}/d\Phi = 0$ and $d^2V_{eff}/d\Phi^2 > 0$, corresponding to the conditions for the de-Sitter point curvature (4) and the positivity of the scalar mass, respectively, in vacuum. Perturbing the trace of the field equation (3) with a nonzero constant background curvature R_0 yields

$$3\Box \delta f' + R_0 \delta f' + f'(R_0) \delta R - 2\delta f = 0.$$
(8)

Using the relations $\delta f = f'(R_0)\delta R$ and $\delta f' = f''(R_0)\delta R$, we obtain the massive wave equation for the scalar mode [11]

$$\Box h_f = m_s^2 h_f,\tag{9}$$

where $h_f \equiv \delta f'/f'(R_0)$ is the field of the scalar mode and

$$m_s^2 = \frac{1}{3} \left(\frac{f'(R_0)}{f''(R_0)} - R_0 \right) \tag{10}$$

is the mass squared of it. Note that $m_s^2 = V_{eff}''(\Phi)$ [5]. For any viable f(R) model, the condition $m_s^2 > 0$ is needed for the stability of the cosmological perturbation and to prevent the field from being a tachyon [14].

For the FRW metric, Eq. (9) should be expressed as

$$\left(-\partial_0^2 + \frac{\partial_i^2}{a^2} - 3H\partial_0\right)h_f = m_s^2 h_f,\tag{11}$$

where the term $-3H\partial_0$ gives a damping factor caused by the expansion of the universe. To illustrate the solution of Eq. (11), we take the de Sitter universe with a constant H. In this case, the solution is a damped plane wave

$$h_f = A(\vec{k}) e^{-\frac{3}{2}Ht} exp\left(iq^{\mu}x_{\mu}\right),$$
(12)

where $q^{\mu} \equiv (\omega_m, \vec{k}), \ \omega_m = \sqrt{\vec{k}^2/a^2 + m_s^2 - \frac{9}{4}H^2}$ is the angular frequency and $A(\vec{k})$ is the amplitude. For simplicity and without loss of generality, we take a = 1 and neglect the damping effect as $\vec{k}^2/a^2 \gg H^2$. As a result, Eq. (9) leads to a simple plane wave solution

$$h_f = A(\vec{p}) exp\left(iq^{\mu}x_{\mu}\right),\tag{13}$$

with $\omega_m = \sqrt{\vec{k}^2 + m_s^2}$. We can see that m_s is the cutoff frequency of the scalar mode of gravitational wave. For $\omega_m < m_s$, the wave vector becomes imaginary. The waveform is an exponential decay in distance, i.e., $h_f \propto \exp(-\vec{k} \cdot \vec{x})$. Thus, the scalar will not propagate in space

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below the cutoff frequency. The massive scalar mode will not propagate at the speed of light with the group-velocity

$$v_g = \frac{\left|\vec{k}\right|}{\omega_m} = \frac{\sqrt{\omega_m^2 - m_s^2}}{\omega_m}.$$
(14)

Note that the tensor mode in f(R) is exactly the same as that in GR when a traceless gravitational wave propagates in a non-zero de-Sitter curvature R_d background.

Since the ACDM model can be viewed as a special case of f(R) with

$$f(R) = R - 2\Lambda,\tag{15}$$

where Λ is the cosmological constant, the mass of the scalar mode is infinite, i.e., $m_s^2 = \infty$, which requires infinite large energy to excite the scalar mode. Clearly, there is no scalar mode in the Λ CDM model. Although the de Sitter curvature R_d is not zero, i.e.,

$$R_d = 4\Lambda \quad (\Lambda CDM) \,, \tag{16}$$

the contribution from R_d is negligible because $\Lambda \approx H_0^2 \approx (10^{-33} eV)^2$, where H_0 is the present Hubble parameter.

In the exponential gravity model, the viable conditions are satisfied when $\beta > 1$ and $R_S > 0$ [8, 16]. The feature is that it is free from the fine tuning problem and it has only one parameter more than the Λ CDM model. According to the condition for the de-Sitter curvature in Eq. (4), R_d satisfies

$$\left(1 - \beta e^{-R_d/R_S}\right) R_d = 2R_d - 2\beta R_S \left(1 - e^{-R_d/R_S}\right).$$
(17)

Defining $x \equiv R_d/R_S$, Eq. (17) becomes

$$x = 2\beta - \beta e^{-x} \left(x + 2\right). \tag{18}$$

The factor $e^{-x}(x+2)$ decreases very fast when $\beta > 1$, which is generally required by the viable condition for the exponential gravity. Therefore, we can obtain the asymptotic solution of x for a large β :

$$x = 2\beta \quad \text{for} \quad \beta \gg 1.$$
 (19)

From Eq. (10), we derive the mass squared of the scalar mode in the exponential gravity as

$$m_s^2 = \frac{1}{3} R_S \left(\frac{e^{R_d/R_S} - \beta}{\beta} - \frac{R_d}{R_S} \right) = \frac{1}{3} R_S \left(\frac{1}{\beta} e^x - 1 - x \right).$$
(20)

Since in the large curvature regime $R/R_S \gg 1$, the theory will recover the cosmological constant model, R_S is roughly inverse proportional to β in the way that

$$\beta R_S \cong 2\Lambda = 9.94 \times 10^{-66} eV^2 \tag{21}$$

with the value of Λ obtained from WMAP 7 [17], SDSS 7 [18] and SCP Union2 observations [19]. Eq. (20) then can be approximated as

$$m_s^2 \cong \frac{2\Lambda}{3\beta} \left(\frac{1}{\beta} e^x - x - 1 \right).$$
(22)

In Table 2, we show the exact m_s without any approximation for $\beta = 1.27, 2, 3$ and 4, respectively,

β	h	y_H^{ini}	Ω_m^0	$R_S (10^{-66} eV^2)$	$m_s \left(10^{-33} eV \right)$
4	0.7050	2.618	0.2761	2.452	24.36
3	0.7059	2.609	0.2758	3.263	11.39
2	0.7103	2.558	0.2738	4.824	5.069
1.27	0.7194	2.45	0.2701	7.39	1.86

Table 2. Numerical results of the scalar mode mass m_s in vacuum with respect to different β in the exponential gravity model.

where we have used the values of R_S obtained from in Ref. [20] under the constraints of WMAP 7, SDSS 7 and SCP Union 2 measurements.

When β is $\mathcal{O}(1)$, m_s is around $10^{-33} eV$. However, the cosmological observations do not give any significant upper bound on β . Thus, m_s could be arbitrary large in this case. As $\beta \to \infty$, corresponding to the Λ CDM model with $m_s \to \infty$, the scalar mode of gravitational wave vanishes.

In the Starobinsky Model, R_c is roughly the present cosmological density and λ and n are positive model parameters. From the solar system constraint and the bound on the violation of the equivalence principle, one gets n > 0.9 [21]. Since for $R \gg R_c$, the model will restore the Λ CDM model, we have $\lambda R_c \simeq 2\Lambda$, $R_d/R_c \simeq 2\lambda$ and $R_d \simeq 4\Lambda = 1.99 \times 10^{-65} eV^2$ when $\lambda \gg 1$.

We now consider gravitational waves in inner galaxy. In the presence of matter density, the scalar mode might not be able to exist in the viable f(R) models. Consider a scalar mode of gravitational wave propagating within our Galaxy halo. The local homogeneous density of dark matter and baryonic matter is roughly $\rho \approx 10^{-24} g/cm^3$. If we take this matter density into our analysis, it will give a large contribution to the background curvature compared to the vacuum de Sitter curvature. (The ratio of the matter density ρ to de Sitter curvature R_d is about $\kappa^2 \rho/R_d \simeq \kappa^2 \rho/4\Lambda \approx 10^5$.) In this case, the condition for the background curvature R_0 in Eq. (4) should be modified as

$$f'(R_0)R_0 = 2f(R_0) - \kappa^2 \rho, \tag{23}$$

where R_0 is the background curvature with matter. Note that for viable f(R) models, the solutions to Eq. (23) can be approximated as $R_0 \simeq \kappa^2 \rho$ at the high curvature regime.

In the case of the exponential gravity, Eq. (23) gives

$$x = 2\beta + r - \beta e^{-x} (x+2), \qquad (24)$$

where $x \equiv R_0/R_S$ and $r \equiv \kappa^2 \rho/R_S$ are the ratios of the background curvature and matter density to R_S , respectively. Since $\beta R_S \cong 2\Lambda$ from (21), we find that the solution of Eq. (24) is extremely large,

$$x \simeq r \simeq \frac{\kappa^2 \rho}{R_d/2\beta} \simeq 2 \times 10^5 \beta, \tag{25}$$

which just leads to $R_0 \simeq \kappa^2 \rho$. Thus, in the exponential gravity, the mass of the scalar mode will become an extreme in the galaxy region:

$$m_s \approx \sqrt{\frac{2\Lambda}{3\beta^2}} e^{2 \times 10^5 \beta} \approx \infty.$$
 (26)

The corresponding cutoff frequency ω_m is also infinite. As a result, it is almost impossible to detect this scalar mode within our Galaxy under the exponential gravity scenario. Moreover, for

any source that is massive enough to generate gravitational waves, we expect them to lay in the region with density higher than the baryonic/dark matter density $10^{-24}g/cm^3$. Therefore, the scalar mode will not have the chance to propagate from the source in the exponential gravity.

In the case of the Starobinsky model, the situation is quite different. The scalar mode of gravitational wave can have a light mass in the galaxy region. The minimum bound of the scalar mode mass is $m_s \gtrsim 10^{-24} eV$ when $\rho = 10^{-24} g/cm^3$. The corresponding cutoff frequency is quite small $f_m \gtrsim 10^{-9}$ Hz. This feature will allow the propagation of the scalar mode inside the galaxy. Hence, detecting the scalar mode in the Starobinsky model will be possible. In Fig. 1,



Figure 1. m_s^2 versus *n* in the Starobinsky model with matter density $\rho = 10^{-24} g/cm^3$ and $\lambda R_c \cong 2\Lambda$.

we depict the mass squared versus the model parameter n with different fixed values of λ , where we have used $\lambda R_c \cong 2\Lambda$. The scalar mode can still be very heavy when the index n goes to a large value, but the mass dependence on the parameter λ is not quite significant.

3. Conclusions

We have discussed gravitational waves in viable f(R) theories. Using the weak field approximation on the field equation, we have confirmed that f(R) will give an extra massive scalar mode besides the ordinary tensor mode in the standard GR. We have explicitly investigated the situations of the extra scalar mode of gravitational wave in the exponential gravity and Starobinsky models of the viable f(R) gravity theories. In vacuum, we have shown that the typical mass squared of the scalar mode is in the order of the de-Sitter curvature $m_s^2 \sim R_d \approx$ $10^{-66} eV^2$ in both models. In the galaxy region, the galactic matter density of $\rho = 10^{-24} g/cm^3$ is about 10^5 larger than the de-Sitter curvature R_d in both models. In the exponential gravity, the mass of the scalar mode in galaxy is undetectable large. However, in the Starobinsky model, the mass can be much smaller with its lower bound in galaxy being about $10^{-24} eV$ (or 10^{-9} Hz). Therefore, it is possible to observe the scalar mode of gravitational wave in the Starobinsky scenario if there is an astrophysical source which generates this scalar mode.

Recently, there is an underway space-based gravitational wave probing experiment, the Laser Interferometer Space Antenna (LISA) [22], which is a proposed joint mission of the European Space Agency (ESA) and NASA. It will measure the low-frequency band $(10^{-5} \text{ to } 1 \text{ Hz})$ of gravitational waves with high signal-to-noise ratio. Since LISA will be located far from the Earth and other gravitational sources, the background curvature of it is very low compared to the ground-based experiments, which allows the propagation of the scalar mode in some viable f(R). As a result, LISA has a great chance to direct detect not only the ordinary gravitational wave but also the clues of the deviation from Einstein's GR by analyzing the scalar mode behavior of gravitational wave. We note that other gravitational wave probes, such as ASTROD-GW [23] with the sensitivity in the $10^{-7} - 10^{-1}$ Hz band, may also detect the extra scalar mode. Finally, we remark that the scalar mode in a viable f(R) cannot be observed by the ground gravitational searches due to the large background curvature.

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