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Scalar-tensor cosmological models converging to general relativity: potential dominated and matter dominated cases

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Abstract. We consider Friedmann-Lemaître-Robertson-Walker flat cosmological models in the framework of general Jordan frame scalar-tensor theories of gravity in two different cases: in the dust matter dominated era and in the potential dominated era. Motivated by the local weak field constraints and by cosmological observations, we develop and use an approximation scheme for the regime close to the so-called limit of general relativity. The ensuing nonlinear approximate equations for the scalar field and the Hubble parameter can be solved analytically in cosmological time in both cases. We find criteria for the functions ω and V characterizing a scalar-tensor theory, to determine whether the theory does or does not possess solutions converging to general relativity asymptotically in time. The converging solutions can be subsumed under two principal classes: exponential or polynomial convergence, and damped oscillations around general relativity. The classes of scalar-tensor theories of gravity which contain these types of solutions and satisfy observational constraints, are candidates to explain possible deviations from the standard Λ CDM model. Finally, the effective equation of state parameter w_{eff} is used to illustrate possible asymptotic cosmological dynamics.

1. Introduction

Various cosmological observations of our Universe can be fairly well accommodated within the ACDM concordance model [1, 2] based on the theory of general relativity (GR). However, there is a number of viable alternative theories of gravity which also manage to conform sufficiently well with observational data [3, 4, 5, 6]. One such family of theories is provided by scalartensor gravity (STG) which employ a scalar field Ψ besides the usual spacetime metric tensor $g_{\mu\nu}$ to describe the gravitational interaction. In the so-called Jordan frame and Brans-Dicke like parametrization STG form a collection of theories which contain two functional degrees of freedom, a coupling function $\omega(\Psi)$ and a scalar potential $V(\Psi)$. As has been discussed by many authors previously [7, 8, 9, 10, 11, 12, 13, 14, 15, 16], for a range of choices of ω and V the cosmological evolution of dust and potential dominated STG models naturally converges close to the one expected from GR. On the other hand, cosmological observations and data from the local weak field tests permit to suppose that only these STG models are viable which involve this property. Namely, limits of observed values of parametrized post-Newtonian (PPN) parameters $|\beta-1| < 10^{-4}, |\gamma-1| < 10^{-5}$ and the time variation of the gravitational constant $|\dot{G}/G| < 10^{-13}$ yr^{-1} strongly constrain the parameters of the theory at present [17, 18]. Constraints from the cosmological dynamics and from the Big-Bang nucleosynthesis (BBN) indicate that the Universe

was very close to GR already at the beginning of matter dominated era [19]. Yet, at the same time STG models may also offer a possibility to explain small observational differences from pure GR ACDM behaviour, e.g. recent data are not excluding the possibility that the effective equation of state (EoS) of dark energy is changing in the cosmological time [20, 21].

In this paper we consider the flat Friedmann-Lemaître-Robertson-Walker (FLRW) cosmological models in the framework of general STG, close to GR (referred to as 'the limit of GR' or 'GR point'). We apply the approximation scheme for the field equations to capture the dynamics near the GR limit. We follow our recent papers [22, 23, 24, 25] where we presented and justified an approximation scheme, as well as derived the non-linear equations for deviations of the scalar field and for the Hubble parameter. The resulting equations explicitly contain cosmological time and can be solved analytically in the potential dominated and in the matter dominated epoch. As an example, we use the effective barotropic index w_{eff} to illustrate the cosmological dynamics near GR point. For some related recent studies see Refs. [26, 27, 28, 29, 30, 31, 32, 33, 34, 35].

In section 2 we recall STG FLRW equations and outline the approximation method worked out in Ref. [22]. In section 3 we review the solutions of resulting nonlinear equations in the regime when the density of scalar potential $V(\Psi)$ dominates over the energy density of matter ρ at cosmological scales (i.e. $V \neq 0$, $\rho = 0$) as expected for the dark energy era [22, 23, 24]. Section 4 is devoted to the dust matter dominated era, introducing the results from a recent publication [25]. Some characteristic examples and implications for selecting a model of STG viable for realistic cosmological scenario is also briefly discussed in sections 3 and 4. Finally, section 5 provides a brief summary.

2. Equations and Approximation Method

We consider a general scalar-tensor theory in the Jordan frame given by the action

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[\Psi R(g) - \frac{\omega(\Psi)}{\Psi} \nabla^{\rho} \Psi \nabla_{\rho} \Psi - 2\kappa^2 V(\Psi) \right] + S_m(g_{\mu\nu}, \chi_m) \,. \tag{1}$$

Here $\omega(\Psi)$ is a coupling function (we assume $2\omega(\Psi) + 3 \ge 0$ to avoid ghosts in the Einstein frame), $V(\Psi) \ge 0$ is a scalar potential, κ^2 is the non-variable part of the effective gravitational constant $\frac{\kappa^2}{\Psi}$, and S_m is the matter contribution to the action as all other fields are included in χ_m .

The field equations for the flat Friedmann-Lemaître-Robertson-Walker (FLRW) line element $ds^2 = -dt^2 + a(t)^2 \left(dr^2 + r^2 (d\theta^2 + \sin^2 \theta \, d\varphi^2) \right)$ and perfect barotropic fluid matter, $p = w\rho$, read

$$H^{2} = -H\frac{\dot{\Psi}}{\Psi} + \frac{1}{6}\frac{\dot{\Psi}^{2}}{\Psi^{2}} \omega(\Psi) + \frac{\kappa^{2}}{3}\frac{\rho}{\Psi} + \frac{\kappa^{2}}{3}\frac{V(\Psi)}{\Psi}, \qquad (2)$$

$$2\dot{H} + 3H^2 = -2H\frac{\dot{\Psi}}{\Psi} - \frac{1}{2}\frac{\dot{\Psi}^2}{\Psi^2}\,\omega(\Psi) - \frac{\ddot{\Psi}}{\Psi} - \frac{\kappa^2}{\Psi}w\rho + \frac{\kappa^2}{\Psi}V(\Psi)\,,\tag{3}$$

$$\ddot{\Psi} = -3H\dot{\Psi} - \frac{1}{2\omega(\Psi) + 3} \frac{d\omega(\Psi)}{d\Psi} \dot{\Psi}^2 + \frac{\kappa^2}{2\omega(\Psi) + 3} (1 - 3w) \rho + \frac{2\kappa^2}{2\omega(\Psi) - \Psi} \left[2V(\Psi) - \Psi \frac{dV(\Psi)}{d\Psi} \right], \qquad (4)$$

$$\dot{\rho} = -3H (w+1) \rho \qquad (4)$$

where $H \equiv \dot{a}/a$ and we will assume that $\rho \ge 0$.

Upon introducing the notation

$$A(\Psi) \equiv \frac{d}{d\Psi} \left(\frac{1}{2\omega(\Psi) + 3} \right), \qquad W(\Psi) \equiv 2\kappa^2 \left(2V(\Psi) - \frac{dV(\Psi)}{d\Psi} \Psi \right) \tag{6}$$

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and substituting H from Eq. (2) into Eq. (4), we get

$$\ddot{\Psi} = \left(\frac{3}{2\Psi} + \frac{1}{2}A(\Psi)(2\omega(\Psi) + 3)\right)\dot{\Psi}^2 + \frac{W(\Psi)}{2\omega(\Psi) + 3} + \frac{\kappa^2}{2\omega(\Psi) + 3}(1 - 3w)\rho \pm \frac{1}{2\Psi}\sqrt{3(2\omega(\Psi) + 3)\dot{\Psi}^2 + 12\kappa^2\Psi(\rho + V(\Psi))}\dot{\Psi}.$$
(7)

In the limit $\frac{1}{(2\omega(\Psi)+3)} \to 0$, $\dot{\Psi} \neq 0$ the system faces a spacetime curvature singularity, since H diverges, and likewise behaves $\ddot{\Psi}$. At first the limit (a) $\frac{1}{(2\omega(\Psi)+3)} \to 0$ and (b) $\dot{\Psi} \to 0$ seems only slightly less mathematically precarious for the equations are left just indeterminate (contain terms $\frac{0}{0}$). Yet the latter situation is of particular physical importance, as the experiments in the Solar System (where local matter density dominates over the scalar potential), i.e. the limits of observed values of PPN parameters and the time variation of the gravitational constant suggest the present cosmological background value of the scalar field to be very close to the limit (a)-(b) [18].

Let us define Ψ_{\star} by $\frac{1}{2\omega(\Psi_{\star})+3} = 0$ and focus upon the solutions near this point,

$$\Psi = \Psi_{\star} + x(t), \qquad \dot{\Psi} = \dot{\Psi}_{\star} + \dot{x}(t) = \dot{x}(t), \qquad (8)$$

where x and \dot{x} span the neighbourhood of first order small distance from $(\Psi_{\star}, \dot{\Psi}_{\star})$. As phase space variables x and \dot{x} are independent from each other, their ratio \dot{x}/x is indeterminate at $(x = 0, \dot{x} = 0)$.

Under additional mathematical assumptions, (c) $A_{\star} \equiv A(\Psi_{\star}) \neq 0$ and (d) $\frac{1}{2\omega+3}$ is differentiable at Ψ_{\star} , we can expand in series

$$\frac{1}{2\omega(\Psi)+3} = \frac{1}{2\omega(\Psi_{\star})+3} + A_{\star}x + \dots \approx A_{\star}x, \qquad (9)$$

$$(2\omega(\Psi) + 3)\dot{\Psi}^2 = \frac{\dot{x}^2}{0 + A_{\star}x + \dots} = \frac{\dot{x}^2}{A_{\star}x}(1 + O(x)) \approx \frac{\dot{x}^2}{A_{\star}x}.$$
 (10)

The latter result actually informs us that in order to avoid a spacetime singularity $\frac{\dot{x}^2}{x}$ must not diverge, hence we should treat x(t) and $\dot{x}(t)$ as the same order (small) quantities. In passing let us remark that in our previous papers [22, 23, 24, 25] we have tentatively called (a)-(d) 'the limit of general relativity' since under these conditions the set of STG cosmological equations (2)-(5) reduces to those of pure GR (with a cosmological constant if $V(\Psi_{\star}) \neq 0$).

3. Potential Dominated Regime

In this section we focus upon the case when matter density can be neglected in favor of the potential (i.e. take $V \neq 0$, $\rho = 0$) and consider the limit (a)-(d). In approximate equations we must recognize the term $\frac{\dot{x}^2}{x}$ as being of the same order as x and \dot{x} . Thus, keeping the term $\frac{\dot{x}^2}{x}$ in the approximation of Eq. (7) we obtain a second order nonlinear differential equation [22]

$$\ddot{x} + C_1 \, \dot{x} - C_2 \, x = \frac{\dot{x}^2}{2x}.$$
(11)

Here we have defined the values of some parameters at $(\Psi_{\star}, \dot{\Psi}_{\star})$ as

$$C_1 \equiv \pm \sqrt{\frac{3\kappa^2 V(\Psi_\star)}{\Psi_\star}}, \qquad C_2 \equiv A_\star W_\star, \qquad (12)$$

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where $W_{\star} \equiv W(\Psi_{\star})$ and $V(\Psi_{\star}) \geq 0$. The three parameters A_{\star} , W_{\star} , C_1 determine the leading terms in expansions of the two functions $\omega(\Psi)$, $V(\Psi)$ which specify a STG. The phase trajectories for the nonlinear approximate system corresponding to equation (11) near GR point are classified and analysed in Ref. [22]. The fixed points and the corresponding eigenvalues in the case of linear system (neglecting the term $\dot{x}^2/2x$) are found in Refs. [36, 37].

The general solution of Eq. (11) reads [24]

$$\pm x(t) = \begin{cases} e^{-C_1 t} \left[M_1 e^{\frac{1}{2} t \sqrt{C}} - M_2 e^{-\frac{1}{2} t \sqrt{C}} \right]^2, & C > 0, \\ e^{-C_1 t} \left[\tilde{M}_1 t - \tilde{M}_2 \right]^2, & C = 0, \\ e^{-C_1 t} \left[N_1 \sin(\frac{1}{2} t \sqrt{|C|}) - N_2 \cos(\frac{1}{2} t \sqrt{|C|}) \right]^2, & C < 0, \end{cases}$$
(13)

where $C = \sqrt{C_1^2 + 2C_2}$ and M_1 , M_2 , $\tilde{M}_1 \tilde{M}_2$, N_1 , N_2 are constants of integration determined by initial conditions. In order to successfully meet the observational constraints, let us now focus upon solutions which approach the GR limit asymptotically in time (PPN parameters approach the GR values $\beta = 1$ and $\gamma = 1$ as $t \to \infty$). Our results [22, 24] allow one to immediately decide whether any STG with particular $\omega(\Psi)$ and $V(\Psi)$ is viable or not. Furthermore, the behavior of solutions which approach GR can be classified under two characteristic types: (i) exponential or linear exponential convergence (C > 0 or C = 0), (ii) damped oscillations around GR (C < 0).

For the evolution of the universe in scalar-tensor cosmology we may envisage a realistic scenario where during the matter domination era the scalar field has already dynamically relaxed sufficiently close to the GR limit. Later when the cosmological energy density of the potential becomes more significant, the solutions given here can be taken to provide a rough description. As an illustration for dynamics given by the solution (13), we use the effective barotropic index which determines the behaviour of dark energy. The same approximation scheme applied to the scalar field equation (7) allows to expand the effective barotropic index as (the expansion of H, \dot{H} and PPN parameters, see Ref. [24])

$$w_{\text{eff}} \equiv -1 - \frac{2\dot{H}}{3H^2} \approx -1 + \frac{1}{C_1^2 \Psi_{\star}} \left[\frac{3}{2} \left(1 + \frac{1}{\Psi_{\star} A_{\star}} \right) \frac{\dot{x}^2}{x} - 4C_1 \dot{x} + 3C_2 x \right] + \dots$$
(14)

Among the models for which the 'GR point' works as an attractor, we can determine whether a model in the theory characterized by distinct parameters (C_1, C_2, A_\star) approaches the de Sitter spacetime from the quintessence side ($w_{\text{eff}} > -1$) or from the phantom side ($w_{\text{eff}} < -1$). Note that a necessary condition for crossing the so-called phantom divide, $w_{\text{eff}} = -1$, is vanishing of the second term in Eq. (14). Depending on the model, exponential solutions may cross the phantom divide line at most twice before approaching $w_{\text{eff}} = -1$ from either above or below. In the oscillating type of solutions the dark energy effective barotropic index oscillates either in the quintessence regime ($w_{\text{eff}} > -1$), phantom regime ($w_{\text{eff}} < -1$), or crossing the phantom divide line at most twice before.

As an illustration, Figure 1 depicts the dynamics of w_{eff} for three sample solutions in different STG models:

$$\omega(\Psi) = \frac{3\Psi}{2(1-\Psi)}, \qquad \kappa^2 V(\Psi) = \frac{2}{3} \left[1 + (1-\Psi)^2 \right], \tag{15}$$

$$\omega(\Psi) = \frac{5\Psi}{7(1-\Psi)}, \qquad \kappa^2 V(\Psi) = 3e^{3(1-\Psi)}, \tag{16}$$

$$\omega(\Psi) = \frac{\Psi}{2(1-\Psi)}, \qquad \kappa^2 V(\Psi) = 3e^{3(1-\Psi)}.$$
(17)

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Figure 1. Examples of the time evolution of w_{eff} for different STG models: (15) left, (16) middle, (17) right. The evolution is measured in the units of the analogue of Hubble time, $T = H_{\star} t = \frac{C_1}{3} t$, and present moment is chosen to be at T = 0.

The first model belongs to the class with C > 0, $C_1 > 0$, $C_2 < 0$ and the sample solution shows a monotonic quintessence type convergence towards de Sitter. The second model belongs to the class with C < 0, $C_1 > 0$, $C_2 < 0$ and is characterized by damped oscillations in the quintessence regime. The third model also belongs to the same class as previous example but exhibits oscillations through the phantom divide line. The initial conditions of these solutions have been chosen such that the corresponding PPN parameters are within observationally allowed limits. We may notice that it is possible to have the period of oscillations to be about the same order of magnitude as the age of the Universe.

4. Matter Dominated Regime

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In what follows we focus to the era when the dust matter density dominates over potential (i.e. take $\rho \neq 0$, w = 0, V = 0) and consider the limit (a)-(d). The system (2)-(5) contains three variables { Ψ, H, ρ }, but one of them is related to the others via the Friedmann constraint (2). Upon eliminating ρ we have two equations [25]

$$\ddot{\Psi} = -3H\dot{\Psi} + \frac{1}{2}(2\omega+3)A(\Psi)\dot{\Psi}^2 + \frac{1}{(2\omega+3)}\left(3\Psi H^2 + 3H\dot{\Psi} - \frac{\dot{\Psi}^2}{2\Psi}\omega\right), \quad (18)$$

$$\dot{H} = -\frac{3}{2}H^2 + H\frac{\dot{\Psi}}{2\Psi} - \frac{\dot{\Psi}^2}{4\Psi^2}\omega - \frac{1}{4}(2\omega+3)A(\Psi)\frac{\dot{\Psi}^2}{\Psi} - \frac{1}{2(2\omega+3)}\left(3H^2 + 3H\frac{\dot{\Psi}}{\Psi} - \frac{\dot{\Psi}^2}{2\Psi^2}\omega\right),$$
(19)

where we have introduced notation (6).

Let us Taylor expand the functions in Eqs. (18), (19) using approximations (8), (9) and an additional approximation for the Hubble parameter

$$H(t) = H_{\star}(t) + h(t)$$
. (20)

We assume that $H_{\star}(t)$ satisfies the equation $\dot{H}_{\star} = -\frac{3}{2}H_{\star}^2$ as in the Friedmann solution of the dust dominated pure GR model. It determines the time evolution of H_{\star} to be

$$H_{\star} = \frac{2}{3(t - t_s)} \,. \tag{21}$$

Here t_s is a constant of integration which fixes the beginning of time scale; in what follows we choose $t_s = 0, t > 0$.

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We take $x, \dot{x}, h, \dot{x}^2/x$ to be of the same order small quantities. Approximate first order equations now read with H_{\star} given by Eq. (21) as

$$\ddot{x} = \frac{\dot{x}^2}{2x} - 3H_\star \dot{x} + 3A_\star \Psi_\star H_\star^2 x \,, \tag{22}$$

$$\dot{h} + 3H_{\star}h = -\frac{1}{4\Psi_{\star}} \left(1 + \frac{1}{2A_{\star}\Psi_{\star}} \right) \frac{\dot{x}^2}{x} + \frac{1}{2\Psi_{\star}}H_{\star}\dot{x} - \frac{3}{2}A_{\star}H_{\star}^2x.$$
(23)

The effective barotropic index is

$$w_{\rm eff} \equiv -1 - \frac{2H}{3H^2} \approx -\frac{2}{3H_{\star}^2} \left(\dot{h} + 3H_{\star}h \right) \,. \tag{24}$$

Notice that Eqs. (22), (23) contain time t explicitly due to H_{\star} (21). This means that the corresponding system of first order equations is not autonomous and the standard phase space analysis is not applicable. However, we can straightforwardly integrate Eqs. (22), (23) in cosmological time and analyse the behaviour of solutions in the neighbourhood of the limit of general relativity. It turns out that the type of solution x(t) of Eq. (22) depends on the constant

$$D \equiv 1 + \frac{8}{3} A_{\star} \Psi_{\star} \tag{25}$$

which characterizes the underlying STG. Knowing solutions x(t) we can also find the solutions h(t) of Eq. (23) and determine the effective barotropic index w_{eff} from Eq. (24).

The general solution of Eq. (22) reads [25]

$$\pm x(t) = \begin{cases} \frac{1}{t} \left[M_1 t^{\frac{\sqrt{D}}{2}} - M_2 t^{-\frac{\sqrt{D}}{2}} \right]^2, & D > 0, \\ \frac{1}{t} \left[\tilde{M}_1 \ln t - \tilde{M}_2 \right]^2, & D = 0, \\ \frac{1}{t} \left[N_1 \sin(\frac{1}{2}\sqrt{|D|}\ln t) - N_2 \cos(\frac{1}{2}\sqrt{|D|}\ln t) \right]^2, & D < 0. \end{cases}$$
(26)

Here M_1 , M_2 , \tilde{M}_1 , \tilde{M}_2 , N_1 , N_2 are constants of integration and are related to the initial data $x_* = x(t_*)$, $\dot{x}_* = \dot{x}(t_*)$ at some arbitrary time t_* . The \pm follows from an invariance property of Eq. (22) under reflection $x \to -x$, i.e. it allows solutions which can lie in the region $\Psi > \Psi_*$ (x > 0) or $\Psi < \Psi_*$ (x < 0).

Let us make some general comments about the solutions, while the detailed analysis as well as solutions for h(t) is presented in the recent paper [25]. For the first class of solutions, D > 0, we can conclude that asymptotically at $t \to \infty$ there are two distinctive behaviors. For STGs with $\sqrt{D} < 1$ (i.e. $A_{\star}\Psi_{\star} < 0$) all cosmological solutions irrespective of their initial conditions monotonically approach the general relativistic dust matter FLRW cosmology, $\Psi(t) \to \Psi_{\star} = \text{const.}, H(t) \to H_{\star}(t) = 2/(3t), \text{ w}_{\text{eff}}(t) \to 0, \text{ since all first order corrections}$ vanish at this limit. On the other hand STGs with $\sqrt{D} > 1$ (i.e. $A_{\star}\Psi_{\star} > 0$) allow only solutions that will diverge, $x(t) \to \infty$, $h(t) \to \infty$, $w_{\text{eff}}(t) \to \infty$, meaning that solutions in these theories can linger near general relativity only for a certain period, while as time evolves they will leave and the approximation scheme will break down eventually. The second class of solutions, D = 0, are of marginal interest due to the fine-tuning of the STG by the condition $A_{\star}\Psi_{\star} = -\frac{3}{8}$. For the third class of solutions, D < 0, the first order corrections to the GR FLRW dust cosmological model are proportional to negative powers of cosmological time t, so they vanish at $t \to \infty$ and solutions approach the general relativistic dust matter cosmology in the manner of damped oscillations. The amplitude of the deviations monotonically decreases while the period monotonically increases in time.



Figure 2. Examples of the time evolution of w_{eff} for different STG models: (27) left and (28) right. The evolution is measured in the units of the analogue of Hubble time, $T = H_* t$ and the initial conditions are fixed at $T_* = 1$.

As an illustration, Figure 2 depicts the dynamics of w_{eff} for two sample solutions in different STG models:

$$\omega(\Psi) = \frac{5\Psi}{3(1-\Psi)}, \qquad (27)$$

$$\omega(\Psi) = \frac{\Psi}{9(1-\Psi)}.$$
(28)

The first model belongs to the class with D > 0 and the sample solution shows a monotonic convergence towards $w_{\text{eff}} = 0$. The second model belongs to the class with D < 0 and is characterized by damped oscillations around $w_{\text{eff}} = 0$.

Finally, let us once again summarize the conditions of the models with the aim to view different epochs as parts of a single cosmological scenario. In both cases there are general conditions for solutions to converge towards the GR value Ψ_{\star} asymptotically in time: in the dust dominated model it reads

$$A_{\star}\Psi_{\star} \equiv \left[\frac{d}{d\Psi}\left(\frac{1}{2\omega(\Psi)+3}\right)\Psi\right]_{\star} < 0 \tag{29}$$

and in the potential dominated model [22]

$$V(\Psi_{\star}) > 0, \qquad \left[\frac{\Psi}{2V}\frac{dV}{d\Psi}\right]_{\star} < 1.$$
(30)

A realistic STG cosmological scenario compatible with observations would better need to have GR as an attractor in both dust dominated and matter dominated regimes. Therefore for a credible STG both conditions (29) and (30) must be satisfied, thus constraining the set of functions $\omega(\Psi)$ and $V(\Psi)$ one can consider for constructing a viable model.

5. Summary

We have derived and solved nonlinear approximate equations for small deviations of the scalar field in the framework of general STG FLRW flat cosmological models in two eras near the limit of general relativity as favored by various observational constraints. First we look the era when the energy density of the scalar potential dominates over the energy density of the ordinary matter and the Universe has evolved close to the limit of GR (which acts as an attractor for certain classes of STG). Secondly we consider the era of dust dominated matter close to the limit of GR. The behaviour of solutions can be used to analyse the cosmological expansion near the 'GR point' where the weak field constraints are satisfied. The solutions which approach GR can be classified under two characteristic types: (a) exponential or polynomial convergence, and (b) damped oscillations around general relativity. The classes of STGs which contain these solutions are of particular interest since there is a dynamical mechanism naturally driving the solutions to satisfy observational constraints. On the other hand, they are good candidates to explain possible deviations from the Λ CDM. Combining the results of the dust dominated epoch with results of the potential dominated regime, provides a reasonable viability filter for STG models in terms of the conditions (29),(30).

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