Optimal placement of sensors and actuators for active vibration reduction of a flexible structure using a genetic algorithm based on modified $H_{\infty}$

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Optimal placement of sensors and actuators for active vibration reduction of a flexible structure using a genetic algorithm based on modified $H_{\infty}$

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Abstract. This paper is concerned with active vibration reduction of a square isotropic plate, mounted rigidly along one edge to form a cantilever. Optimal placement of ten piezoelectric sensor/actuator pairs is investigated using a genetic algorithm to suppress the first six modes of vibration. A new objective function is developed based on modified $H_{\infty}$ to locate the sensor/actuator pairs. The plate, with piezoelectric sensor/actuator pairs bonded to its surfaces, is modelled using the finite element method and Hamilton's principle based on first order shear deformation theory including bending, membrane, and shear deformation effects. The effects of piezoelectric mass, stiffness and electromechanical coupling are taken into account. The first six natural frequencies are validated by comparison with the finite element ANSYS package using two dimensional SHELL63 and three dimensional SOLID45 elements and also experimentally. Vibration reduction for the cantilever plate with piezoelectric patches bonded in the optimal location was investigated to attenuate the first six modes of vibration using a linear optimal control scheme. The new fitness function has reduced the computational cost and given greater vibration reduction than other previously published results.

1. Introduction
Vibration reduction of a flexible structure can be optimised by locating piezoelectric sensors and actuators in efficient locations using optimisation methods such as genetic algorithm. The complexity of optimisation process is directly proportional to the fitness function type and search space number of possible solutions which equal to the statistical combination of finite element discretisation and sensors or actuators number to be optimised. The reduction of optimisation problems for large scale structure has attracted researchers to reduce search space, computer time calculation, exploring global optimal sensors and actuators location to achieve controller optimality.

The genetic algorithm has been utilized for the optimal location of piezoelectric sensors and actuators by many researchers for flat plates and shells. The Rayleigh-Ritz method was implemented by Sadri et al to model a thin plate, with modal and grammian controllability taken as an objective function to locate two actuators [1]. Two actuators and piezofilm sensors were optimized using controllability, observability and spillover as an objective function to suppress the first three modes of vibration by Han and Lee [2]. Quek, et al optimized two piezoelectric sensor/actuator pairs based on modal controllability to suppress the first two modes of vibration [3]. Optimal placement of ten sensor/actuator pairs studied by Kumar and Narayanan using minimization of linear quadratic index as an objective function to suppress the first six modes of vibration [4]. The half and quarter chromosome technique has been developed by Daraji and Hale to reduce the genetic algorithm search space by 99.99% to locate ten and eight sensors/actuators pairs for a flat plate using linear quadratic index minimization as an objective function [5]. A new placement strategy including conditional filter is
proposed by Daraji and Hale to reduce genetic algorithm search space and explore the global optimal configuration of ten and four piezoelectric pairs and to attenuate the first six modes of vibration [6].

The special configuration for placement of finite numbers of sensors and actuators on a continuous flexible structure is investigated by Chemishkian and Arabyan to reduce structural deformation to a minimum in $H_\infty$ sense. The computational overhead is reduced by evaluating the minimum $H_\infty$ norm by each genetic algorithm step and controller $H_\infty$ is designed only when the lower limit is below a predefined value [7, 8]. Optimal placement of two sensor/actuator pairs on a beam is investigated by Hiramoto et al using modified $H_\infty$ based on simple addition and multiplication matrices to reduce the computational time compared with standard norm [9]. Optimal placement of sensors and actuators is investigated by Gawronski for a fixed ended beam based on standard $H_2$ norm to suppress the first mode individually and collectively. The placement indices for sensors and actuators are explained thoroughly with modelling based on the $H_\infty$, $H_2$ and $H_s$ norm. The proposed approach consists of determination of the norm of each sensor or actuator for selected modes and then grading them according to their participation in the system norm to reduce the computation time [10]. The spatial $H_2$ norm is developed by Liu et al to optimise sensor and actuator locations in controlled flexible structures based on dominant modes which have most effect on the structure. They showed that the spatial $H_2$ norm gives better performance index for structural vibration control [11].

In this paper, the placement of ten sensor/actuator pairs, used to attenuate the first six modes of vibration, is optimised for a cantilever plate using a genetic algorithm. A new objective function, based on modified $H_\infty$, is proposed to minimize an average summation of absolute dB gain for the open loop transfer function for all sensors and modes as a result of applying a unity amplitude disturbance voltage at a predefined single actuator location through all genetic algorithm generations. An isotropic plate with the piezoelectric sensor/actuator pair bonded to its surfaces is modelled according to first order shear deformation theory to include bending, membrane and shear deformation effects. An isoparametric two dimensional element is chosen with four nodes and five degrees of freedom per node. The full derivation and parameters are explained by Daraji and Hale, the modal dynamic equation and state space matrices are[12];

\[
\begin{align*}
\{\ddot{\eta}\} + 2\xi\omega\{\dot{\eta}\} + \Omega(\eta) &= [\varphi]^T[F_u] - [\varphi]^T[K_{u\varphi}]^a\{\varphi_a\} \\
\{\varphi_a\} &= -[\varphi]^T[K_{\varphi\varphi}]^{-1}[K_{u\varphi}]^a\{\eta\} \\
\{\dot{X}\} &= \begin{bmatrix} 0 & \omega \\ -\omega & -2\xi\omega \end{bmatrix}\{X\} + \begin{bmatrix} 0 \\ -[\varphi]^T[K_{u\varphi}] \end{bmatrix}\{\varphi_a\} \\
\{\ddot{X}\} &= [A_i]\{X\} + [B_i]\{u\}, \quad \{C_i\} = [C]\{X\}
\end{align*}
\]

\[
[A_i] = \begin{bmatrix} 0 & \omega_i \\ -\omega_i & -2\xi_i\omega_i \end{bmatrix}, \quad [B_i] = \begin{bmatrix} 0 \\ -[\varphi_i]^T[K_{u\varphi}]^a \end{bmatrix}, \quad [C_i] = \begin{bmatrix} 0 & -[\varphi_i]^T[K_{\varphi\varphi}]^{-1}[K_{u\varphi}]^s\omega_i^{-1} \end{bmatrix}
\]

Where $[A_i]$, $[C_i]$ and $[B_i]$ are the individual modal state, sensor and actuator matrices in which subscript $(i)$ refers to the mode number. The state and actuator matrices for the number of modes $n_m$ and the number of actuators $r_a$ are:

\[
\{X\}_{(2n_m \times 1)} = \{\omega_1 \eta_1 \quad \dot{\eta}_1 \quad \omega_m \eta_m \quad \dot{\eta}_m\}^T
\]
\[ A_{(2n_m \times 2n_m)} = \begin{bmatrix} A_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & A_{n_m} \end{bmatrix}, \quad B_{(2n_m \times r_a)} = \begin{bmatrix} (B_1)_1 & \cdots & (B_1)_{r_a} \\ \vdots & \ddots & \vdots \\ (B_{n_m})_1 & \cdots & (B_{n_m})_{r_a} \end{bmatrix} \] (7)

Linear quadratic optimal controller design is based on minimization of a performance index \( J \). Values of positive-definite weighted matrices \( Q \), of dimension \((2n_m \times 2n_m)\), and \( R \) of dimension \((r_a \times r_a)\) are controlled the value of the performance index, where \( n_m \), \( r_a \) represent the number of modes and actuators, Ogata has shown it is possible to follow this derivation to design a linear quadratic controller [13], which leads to the Riccati equation:


\[ [K] = [R]^{-1}[B]^T[P], \quad \{u\} = -[K]\{X\} \] (9)

Solution of the Reduced Riccati equation (8) gives the value of matrix \( [P] \); if matrix \( [P] \) is positive definite then the system is stable and the closed loop matrix \([A] - [B][K]\) is stable. The feedback control gain matrix can be obtained after substitution of \([P]\) in Eq. (9).

3. Objective function
A new objective function has been developed to find the optimal sensor/actuator configuration based on the \( H_\infty \) norm by measuring the open loop average dB gain for all sensors at all the required modes to be attenuated as a results of unity amplitude disturbance input voltage at a single predefined actuator selected at the position of maximum strain energy for all chromosomes and generations in the genetic algorithm. The open loop transfer function of a system in the frequency domain is

\[ G(\omega) = C(j\omega I - A)^{-1}B \] (10)

The peak of the transfer function magnitude is the \( H_\infty \) norm of a single mode \( \omega_i \).

\[ H_\infty = \max(G(\omega)) = G(\omega_i) \] (11)

\[ J(x,y) = \frac{1}{n_m n_{sa}} \sum_{i=1}^{n_m} \sum_{j=1}^{n_{sa}} 20 \log_{10}(G_j(\omega_i)) \] (12)

\[ J_{opt}(x,y) = \min(J(x,y)) \] (13)

Where \( x, y \in \) plate dimension \(50 \times 50 \) cm, \( n_m \) and \( n_{sa} \) refer to number of modes and sensor/actuator pairs respectively.

4. Genetic algorithm
In 1975, Holland invented the genetic algorithm, a heuristic method based on “survival of the fittest” or the principle of natural evolution. It has been continuously improved and is now a powerful method for searching optimal solutions. The fundamental unit in the genetic algorithm is a population of individuals, each defined by a chromosome containing a number of genes. The effectiveness or “fitness” of each individual is calculated according to some rule using the values of the genes. The members of the population with the highest fitness values are allowed to “breed” to form the next generation and the process continues until convergence is achieved. In this case, the ten “genes” are the locations of the ten sensor/actuator pairs, defined by an integer number (integer-coded chromosomes) (1-100), and the fitness function is the linear quadratic index. This process is directly analogous to the survival of the fittest concept in Darwinian natural selection, in which the more successful individuals in a population are inclined to breed and so form the next generation. By this means the genes that code for desirable characteristics, and so give the individuals possessing them a high degree of fitness, are transmitted down the generations at the expense of less useful genes, which
die out. A placement strategy using the genetic algorithm including a conditional filter has been investigated by Daraji and Hale and is used in this work [6].

5. Results and discussion

5.1. Research problem

A cantilever flat plate dimensioned $500 \times 500 \times 1.9$ mm was mounted rigidly from the left hand edge. The plate is described to one hundred elements $10 \times 10$ sequentially from left to right and down to up as shown in figure 2. Global optimal placement of ten piezoelectric sensor/actuator pairs is investigated to suppress the first six modes of vibration. This case is chosen to give direct comparison with the published works [4, 6] to test the reliability and robustness of this work. The plate and piezoelectric properties are listed in table 1.

Table 1. Plate and piezoelectric properties

<table>
<thead>
<tr>
<th>Properties</th>
<th>Plate</th>
<th>Piezoelectric PIC225</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus, GPa</td>
<td>210</td>
<td>--------</td>
</tr>
<tr>
<td>Density, kg/m$^3$</td>
<td>7810</td>
<td>7180</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
<td>--------</td>
</tr>
<tr>
<td>Thickness, mm</td>
<td>1.9</td>
<td>0.5</td>
</tr>
<tr>
<td>Length, width, mm</td>
<td>500</td>
<td>50</td>
</tr>
<tr>
<td>$e_{31}, e_{32}, e_{33}$, C/m$^2$</td>
<td>--------</td>
<td>-7.15, -7.15, 13.7</td>
</tr>
<tr>
<td>$C_{11}^e, C_{12}^e, C_{13}^e, C_{55}^e$, GPa</td>
<td>--------</td>
<td>123, 76, 7, 70.25, 22.226</td>
</tr>
<tr>
<td>$\mu_{44}^e$, F/m</td>
<td>--------</td>
<td>1.5 $\times 10^{-8}$</td>
</tr>
</tbody>
</table>

$C^e$, $e$ and $\mu^e$ refer to elasticity, piezoelectric and permittivity constants

5.2. Natural frequencies

The first six natural frequencies and mode shapes for the cantilever plate were determined using a Matlab m-code program based on the present model and validated by the ANSYS finite element package using two-dimensional SHELL63 elements, three-dimensional SOLID45 and also experimentally. The results are shown in table 2. They converged with the mesh refining to constant values, showing that the mesh of $10 \times 10$ shell63 elements gave good result accuracy for the first six natural frequencies compared with finer meshes and three-dimensional solid45 elements, as well as with experimental results as shown in table 2.

Table 2. ANSYS natural frequencies compared with present model and experimental results

<table>
<thead>
<tr>
<th>Element type</th>
<th>1$^{st}$</th>
<th>2$^{nd}$</th>
<th>3$^{rd}$</th>
<th>4$^{th}$</th>
<th>5$^{th}$</th>
<th>6$^{th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANSYS Shell63 (10 $\times$ 10)</td>
<td>6.59</td>
<td>16.17</td>
<td>41.32</td>
<td>52.37</td>
<td>59.79</td>
<td>104.70</td>
</tr>
<tr>
<td>ANSYS Solid45(50 $\times$ 50)</td>
<td>6.59</td>
<td>16.15</td>
<td>40.44</td>
<td>51.68</td>
<td>58.86</td>
<td>103.18</td>
</tr>
<tr>
<td>Present model</td>
<td>6.59</td>
<td>16.14</td>
<td>40.62</td>
<td>51.78</td>
<td>58.99</td>
<td>103.29</td>
</tr>
<tr>
<td>Experimental Frequency</td>
<td>5.90</td>
<td>16.90</td>
<td>37.30</td>
<td>51.60</td>
<td>58.20</td>
<td>101.00</td>
</tr>
</tbody>
</table>

5.3. Actuator location optimization

The genetic algorithm described in section 4 based on reference [6] was used to find optimal locations for ten piezoelectric actuators on 0.5m square cantilever plate mounted rigidly from the left edge. The progressive convergence of the population onto an optimal solution is shown in figure 1, in which the population is distributed around the circle with radius (r) representing the fitness value to be minimised. A unit amplitude voltage disturbance is applied at actuator location (01), forced for all chromosomes in all generations.
At the first generation (Figure 1a) the population is very diverse with representatives of high and low fitness and the range in between. After thirty generations (Figure 1b) the population is much less diverse, made up of individuals of high, though not yet optimal, fitness. After 500 generations (Figure 1c) the population has completely converged to a level of fitness higher than any individual in the first or thirtieth generations.

5.5. Vibration reduction

The present optimal placement configuration is tested to measure the average closed loop dB gain reduction over all sensor/actuator pairs over the first six modes using an optimal linear quadratic control scheme and the results compared with two references as shown in Figure 2. A unit amplitude voltage disturbance is applied at actuator number 11 for all three cases and the present configuration given an improvement of 28.5% reduction at $Q = 10^8$ and 24.35% at $Q = 10^9$ as shown in Table 3.

Table 3: Closed loop average dB gain reduction and number of fitness computational cost for three cases

<table>
<thead>
<tr>
<th>Case</th>
<th>$Q = 10^8$</th>
<th>$Q = 10^9$</th>
<th>Number of fitness calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref [4]</td>
<td>6.21dB</td>
<td>12.44dB</td>
<td>$5 \times 10^4$</td>
</tr>
<tr>
<td>Ref [6]</td>
<td>7.61dB</td>
<td>14.42dB</td>
<td>$3.19 \times 10^4$</td>
</tr>
<tr>
<td>Present work</td>
<td>7.98dB</td>
<td>15.06dB</td>
<td>$0.925 \times 10^4$</td>
</tr>
<tr>
<td>Percentage reduction improvement</td>
<td>28.5%</td>
<td>24.35%</td>
<td>81.5%</td>
</tr>
</tbody>
</table>

Figure 1. Population fitness progression over 500 generations. Each individual is represented as one of the points distributed around the circle with radii $r$ which represent its fitness value to be minimized.

Figure 2. Optimal sensor/actuator pairs configurations for a cantilever plate mounted rigidly from the left edge. (a) Ref [4], (b) Ref [6], (c) present work.
6. Conclusion
A new objective function is developed to optimize ten sensor/actuator pairs for a cantilever plate using a genetic algorithm and to reduce the computational cost. The objective function is taken as a minimization of an absolute average open loop dB gain for all sensor/actuator pairs and modes as a result of applying disturbance unit volt amplitude at a predefined actuator.

An isotropic plate bonded with a number of piezoelectric patches is modeled using isoparametric four nodes element and Hamilton’s principle based on first order shear deformation theory taking the effects of mass, stiffness and electromechanical coupling of piezoelectric pairs. The first six natural frequencies are determined based on the present model and validated with ansys package and experimentally.

The optimal configuration of ten sensor/actuator pairs is found symmetrically distributed about the plate axes of symmetry. The vibration reduction is tested for the present work compared with two references using optimal linear quadratic control. It has been found that the present optimal configuration gives a higher average dB gain reduction of 28.5% and 24.35% than published at $R = 1$ and $Q = 10^8$ and $10^9$ respectively. The new objective function achieves a higher vibration reduction and reduced computation cost.

References