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$q - \bar{q}$, tetraquark and mixing in a dynamical model of the strong interaction

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Abstract. We perform a variational Monte Carlo simulation of a system consisting of two quarks and two antiquarks. Using a dynamical model of the strong interaction (string-flip model) we explore the formation of mesons and/or a tetraquark for a set of different density conditions. We characterize the single properties of these states and the modifications upon the mixing. The competition among the different configurations and its implications on several observables are exhibited.

1. Introduction

The Standard Model of Particle Physics tell us that hadronic matter is composed of quarks, and that these bound states must be color-singlets. The known states that can be color-singlets are the mesons ($q - \bar{q}$), baryons ($3q$) and their corresponding antiparticles. However, glueballs, hybrids and multiquark states are also color-singlets and are predicted as bound states in models inspired by Quantum Chromodynamics (QCD). In particular, mesons with exotic quantum numbers are non-($q - \bar{q}$) objects. The multiquark ($q - q - \bar{q} - \bar{q}$) or tetraquark state is the simplest on its kind and has been under study theoretically and experimentally, in order to find clear signatures of its existence [1, 2, 3]. In this work we present a preliminary study on the formation of this state from the presence of two initially uncorrelated q-qbar states, which are externally forced to approach to each other. We analyze the system using a QCD-inspired model (*String-Flip*)[4], incorporating a many-body quark potential able to confine quarks within color-singlet clusters, quantum correlations between quarks and a 3-Dimensional evolution of the system. At low densities the model describes a system of quarks as isolated hadrons, whereas at high densities, the system resembles a free Fermi gas of quarks. In the present case we consider all the quarks (antiquarks) as made of a single light flavor and all with the same color (anticolor) quantum number.

2. Variational wave function

The system wave function requires to account for the correlations between identical particles and the clustering formation in a color singlet way. These can be achieved by considering a variational wave function of the following form:

$$\Psi_\lambda(\vec{r}) = e^{-\lambda V(\vec{r})}\Phi_{FG}, \tag{1}$$

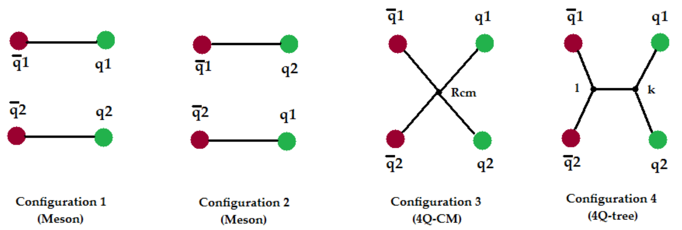

Figure 1. Possible configurations of the system. Two meson systems and two tetraquark systems

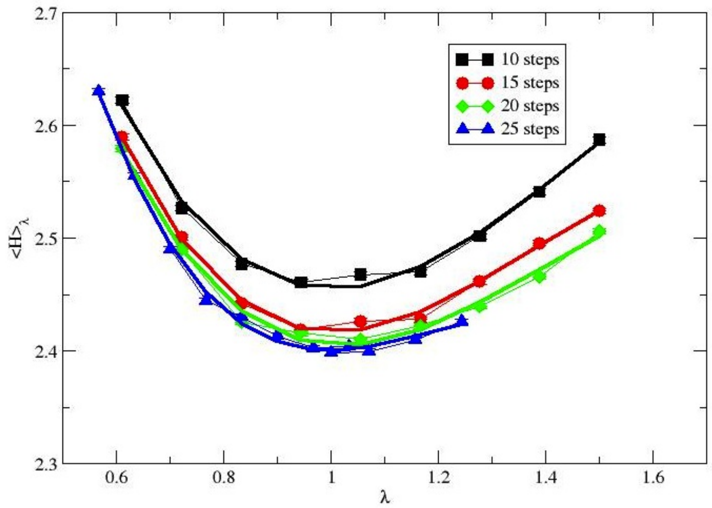

Figure 2. Example of the minimization of the energy as a function of the variational parameter and the efficiency of the $4Q - tree$ potencial algorithm for a given number or steps. We can observe the convergence on the system lowest energy as the number of steps increases.

where λ is the single variational parameter, $V(\vec{r})$ is a many-body potential which will be described in the next section, and Φ_{FG} is the Fermi gas wave function given by a product of Slater determinants, one for each color-flavor combination of quarks, whose single wave functions correspond to that of a particle in a box. The structure of the wave function allows some simplifications in the calculations. After an integration by parts, the expectation value of the Hamiltonian operator for a given particle density can be evaluated as:

$$\langle H \rangle_\lambda = T_{FG} + 2\lambda^2 \langle W \rangle_\lambda + \langle V \rangle_\lambda \,, \tag{2}$$

where W is a kinetic term induced by the interaction and T_{FG} is the kinetic energy of a free Fermi gas.

3. Many body potential

The many-body potential is defined as the optimal clustering of quarks into color-singlets objects, where by optimal we mean the configuration that minimizes the potential energy. By this, we distinguish four possible configurations in the system:

- *Meson-like*: In this case we distinguish a pairing between color and anticolors, building two mesons. The system allows two possible pairings (configurations 1 & 2, Fig. 1) and the

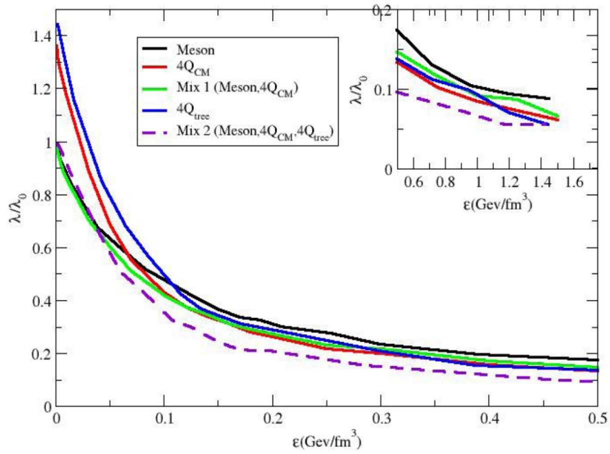

Figure 3. Evolution of the optimal variational parameter as a function of the energy density. The variational parameter is normalized to the theoretical solution of a harmonically bound quark-antiquark pair (isolated meson). We can observe that for the meson case the ratio converges to one as expected, while for the tetraquark cases this is up to 1.4 times larger and rapidly decreasing as the energy density increases. In the mixed cases the parameter is dominated by the meson configuration and thus the variational parameter resembles that of a mesonic system. All of them become very alike at high energy densities.

many-body potential made up of mesons is then given by:

$$V_{q\bar{q}} = \min_{P} \sum_{i=1}^{2} v[r_{q_i}, r_{\bar{q}_i}], \tag{3}$$

where $v[r_i, r_j] = \frac{1}{2}k(r_i - r_j)^2$, we use a harmonic potential to describe the interaction between particles. The minimum takes into account both permutations of the system.

- *Tetraquark-like*: In this case, we consider two different potentials in which we can confine four quarks into a color-singlet, denoted by $4Q - cm$ (configuration 3, Fig. 1) as linked through their center of mass. This is described by the following potential:

$$V_{4Q-cm} = \sum_{i=1}^{2} V\left[\vec{R}_{cm}, \vec{r_{q_i}}\right] + \sum_{i=1}^{2} V\left[\vec{R}_{cm}, \vec{r_{\bar{q}_i}}\right] \tag{4}$$

where $\vec{R}_{cm}$ is the vector position of the center of mass. Another possibility to link four quarks is by using the so called steiner-tree configuration, denoted by $4Q - Tree$ (configuration 4, Fig. 1). This is the most efficient way to confine four particles into a cluster [5, 6]. The potential has the following form:

$$V_{4Q-tree} = \sum_{i=1}^{2} V\left[\vec{k}, \vec{r_{q_i}}\right] + \sum_{i=1}^{2} V\left[\vec{l}, \vec{r_{\bar{q}_i}}\right] + V[\vec{k}, \vec{l}] \tag{5}$$

The implementation of such configuration requires an algorithm to search over the $\vec{k}$ and $\vec{l}$ vectors providing the minimal length of the links.

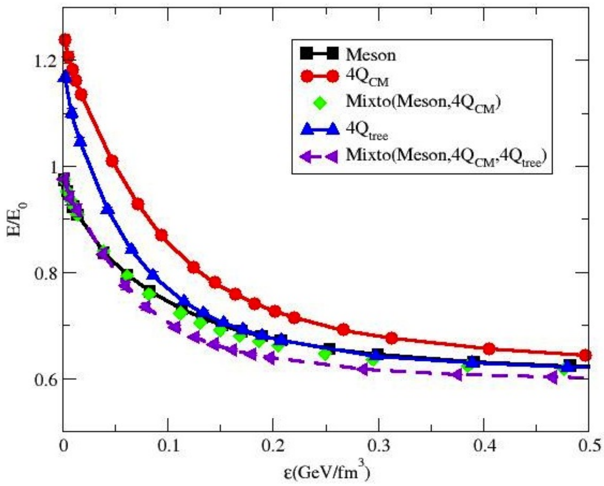

Figure 4. Evolution of the Energy per particle as a function of energy density. The energy is normalized to the theoretical solution of a harmonically bound quark-antiquark pair (isolated meson). We can observe that for the meson case the ratio converges to one as expected, while for the tetraquark cases this is around 1.2 times larger and rapidly decreasing as the energy density increases. In the mixed cases the energy is dominated by the meson configuration and thus the mixed system resembles that of a mesonic system. Note that the energy for the tetraquark is lower when considering the steiner tree configuration than when considering the center of mass configuration. This fact makes a substantial difference in the mixed case, showing that in the presence of the steiner tree configuration the mixed system is more stable than the pure mesonic state. All of them become very alike at high energy densities.

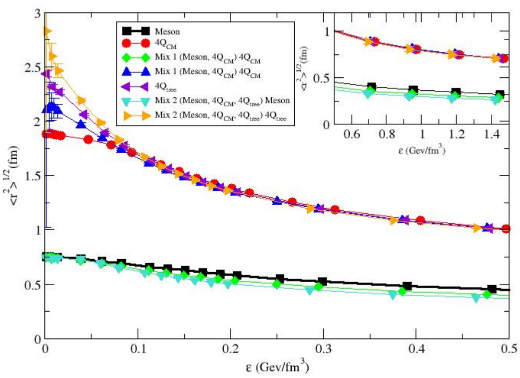

Figure 5. Evolution of the mean square radius as a function of the energy density. We can identify two typical structures, one for the meson and another for the tetraquark. They are well separated from each other regardless of the kind of potentials involved. The meson square radius is less sensitive to the energy density than that of the tetraquark, becoming more alike at high energy densities but still substantial differences remain

4. Results

To determine the variational parameter (λ) as a function of particle density (ρ) we proceed as follows: first we select the value of ρ in the box that confines the quarks, which, for a fixed

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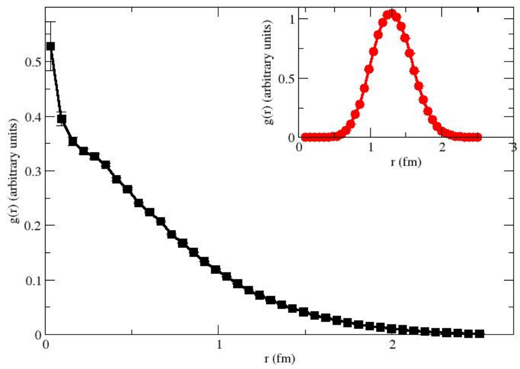

Figure 6. We show the $q-\bar{q}$ (meson-meson) correlation function (upper panel) for an energy density of 0.1074 GeV/fm^3. The quark-antiquark correlation is large at short distances, as any quark can find an antiquark close to each other within a meson structure. The distance for the vanishing of the correlation can be understood then as the radial size of quark cloud of the meson. In the upper panel the meson-meson correlation takes its maximum around 1.3 fm. This can be seen as the typical distance among mesons at that energy density. Comparing with the quark-antiquark correlation function we can conclude that there is an small overlap between mesons.

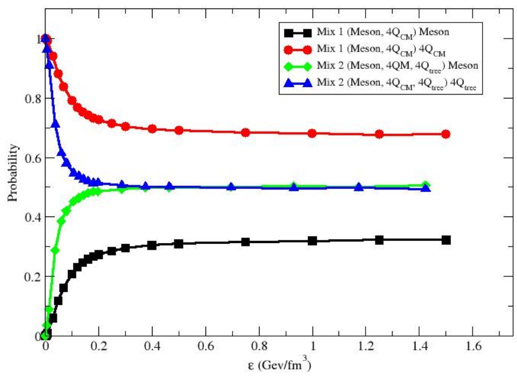

Figure 7. Probability to form mesons or a tetraquark as a function of the energy density. When the tetraquark is built up using a center of mass potential the probability of appearance increases from zero at low energy densities to around 30% at high energy densities. In the case that the tetraquark is formed by a steiner tree, its probability rapidly converges to 50%, making a perfect mixing with the mesonic state.

number of particles, means changing the box size. Then, we compute the energy of the system as a function of λ. The minimum of the energy determines the optimal λ.

$$\frac{\partial \langle \Psi_\lambda | H | \Psi_\lambda \rangle}{\partial \lambda} = \frac{\partial E_\lambda}{\partial \lambda} = 0 \tag{6}$$

For the $4Q-Tree$ configuration, we compute the $\vec{k}$ and $\vec{l}$ vectors using an algorithm that explores their magnitude and direction for a set of steps in spherical coordinates, taking $\vec{R}cm$ as a starting point. The algorithm then takes the ones producing the lowest potential energy. In Fig. 2 we show an example of the minimization of E_λ and exhibit the efficiency of the algorithm when including 10, 15, 20 and 25 steps. It is clear the energy convergence and the almost independence of the optimal λ parameter on them.

The optimal value for λ was determined for a set of densities. In Figure 3 we show the result as a function of the energy density. The energy density is used here after as a parameter to exhibit the evolution on a set of observables of the system: The total energy (Fig. 4), the square root of the mean square radius (Fig. 5), and the correlation function between particles (Fig. 6). In addition, we show the probability of having a meson or a tetraquark in the simulation (Fig. 7).

5. Conclusions

We have presented a preliminary study on the possible formation of a tetraquark state as produced when two mesons (namely two quarks and two antiquarks) are forced to approach to each other. Using a dynamical approximation which allows the competition among meson-like and tetraquark-like configurations. We have characterized each state and their modifications upon the mixing. We find that the total energy, mean square radius, and correlation function of a meson are susceptive to the effects of the mixing with a tetraquark state. The tetraquark state properties are also modified by the mixing, in particular the mean square radius. A substantial reduction on the ground state energy of the mixed system is achieved when including the steiner-tree configuration, making this smaller than the pure mesonic state as the energy density increases.

We have implemented a Monte Carlo simulation to study the tetraquark formation in a systematic way, we expect to explore the effects when considering different flavors, in particular it will be interesting to study the heavy-quark regime. An extended analysis, discussion and final results will be presented elsewhere.

Acknowledgments

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