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Towards Cosmological Models by Compactification on a Non-geometric Twisted Torus

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Abstract. We study a type IIB superstring compactification in the presence of non-geometric fluxes. We study the non supersymmetric solutions to the equations of motion. The corresponding vacuum is found by employing a genetic algorithm constrained by the Bianchi identities. We explore the space of the non supersymmetric vacua and look for possible De Sitter solutions. We find that such vacua are possible if supersymmetry is broken by all the three moduli.

1. Introduction

After performing a string compactification from ten dimensions to four, the effective theory corresponds to a gauge supergravity theory coupled to light scalar fields [1]. These scalar fields parametrize the size and shape of the compact space and define the coupling constants and masses of the remaining fields of the effective theory. So in order to construct a phenomenological attractive theory they must acquire values which are compatible with the observed values. In the last years several ways have been proposed in order to stabilize all the moduli, mainly by turning on fluxes on the compact space [3, 4] and by considering the presence of exotic ingredients such as orientifolds, and generalized structures on the internal space. Some of them break supersymmetry in a localized way [5]. Some other examples of moduli stabilization are by mean of non perturbative effects on the Kähler potential or on the superpotential [5, 6, 7]. One interesting approach is to consider the possibility to obtain a classical de Sitter vacuum in order to explain the accelerated universe we observe [2].

The parameters of the effective gauged supergravity theory are associated either to the fluxes or to features of the background in which the compactification is performed. However in some cases fluxes which have a natural interpretation in a theory may not have a obvious interpretation in its T-dual part [8, 9, 10, 11, 12].

In the context of moduli compactification by fluxes it was argued that a compactification on space manifolds with negative curvature [13] leads to a lift in the vacua and suggests that De Sitter space can be constructed through these manifolds. However the stability of the vacua is in many cases only metastable [14]. In particular the twisted torus is an example of a negatively curvature manifold. The twist on the torus can be interpreted as a geometric flux in the context of Type IIB string theory which is T dual to a Type IIA in which the geometric fluxes

are mapped to a kind of flux which cannot be interpreted globally as a geometrical feature of the space time manifold. So this compactification which cannot be interpreted as a space time manifold leads to the notion of non geometric fluxes.

In order to get an intuition of the non-geometric fluxes consider for example a compactification on a six dimensional torus T^6 threaded with NS-NS 3-form flux H_{abc} where $a, b, \dots \in 1, 2, \dots, 6$ with nonzero values in the compact dimensions. The corresponding dimensional reduction is understood as a class of generalized Scherk-Schwarz dimensional reduction [15, 16, 17, 18, 19]. Under a T-duality, for example in direction a , the H -flux is mapped to a geometric flux associated with a twist in the torus topology. In the presence of this geometric flux, the metric on the twisted torus acquires a contribution which can be written as $(dx^a - f_{bc}^a x^c dx^b)^2$, where f_{bc}^a is integrally quantized and characterizes the geometric flux of the compactification.

Even in the presence of geometric flux f_{bc}^a , however, we can perform another T-duality on direction b , since the metric can be chosen to be independent of the coordinate x^b . Carrying out this T-duality explicitly leads to a dual torus, which is locally geometric, but it cannot be described globally in terms of a fixed geometry, due to the appearance of non-geometric duality transformation in the boundary conditions which patch together local descriptions of the compactified space. This non-geometric flux which arise after a second T-duality transformation, is labeled as Q_c^{ab} . It is known that the related superpotential depends on all the moduli of the compactification, implying that it is possible to look at classical level, for De sitter solutions. Therefore, we shall study type IIB string compactifications on a six dimensional torus threaded with non-geometric fluxes, besides the usual 3-form fluxes, in order to look for positive valued vacua.

Our work is organized as follows: in section 2, a type IIB compactification on a non-geometric torus is presented by preserving only one of the supersymmetries. In section 3 the constraints that the fluxes must satisfy are presented together with a set of solutions of these constraints. In section 4 we show a systematic analysis of the equations of motion by breaking supersymmetry through one or several moduli. In section 5, the effective scalar potential is constructed for each solution previously found. Finally in section 6 we give some final comments and perspectives of the work.

2. Flux compactification

We consider a Type IIB compactification with O3-planes transverse to the compact directions in order to avoid tadpole conditions in the presence of 3-form fluxes [20, 21]. The final $N = 1, D = 4$ contents include three complex moduli, namely: axio-dilaton S , a complex structure moduli τ and a Kähler moduli U .

We consider the standard tree-level Kähler potential given by

$$K = -3\ln(-i(\tau - \bar{\tau})) - 3\ln(-i(U - \bar{U})) - \ln(-i(S - \bar{S})). \quad (1)$$

The contribution of the non-geometric fluxes to the superpotential is given in [8], and it is obtained by adding a term to the the well known Gukov-Vafa-Witten [22]

$$W = \int (F_3 - SH_3 - QU) \wedge \Omega. \quad (2)$$

By applying the Buscher rules to a type IIB torus compactification in the presence of a NS-NS 3-form flux, the metric of the non-geometric torus is derived and it is given by

$$ds^2 = \frac{1}{1+Q^2z^2}(dx^2 + dy^2) + dz^2.$$

This background is characterized by the non-geometric flux Q_a^{bc} .

The $N = 1$ four dimensional effective gauged supergravity scalar potential is given by the standard scalar potential [23]. The equations for a supersymmetric vacuum are $D_iW = 0$ which correspond to the F terms. For simplicity, we shall consider a six-dimensional torus made of three identical copies. This reduces the number of moduli which, as recently discussed, increases the chances to obtain De Sitter vacua [13]. Hence, by defining the complex coordinates on each torus as $z^a = x^\alpha + \tau x^i$ where $\alpha = 1, 3, 5$ and $i = 2, 4, 6$, the T-dual invariant superpotential is given by

$$\begin{aligned} W = & a_0 - 3a_1\tau + 3a_2\tau^2 - a_3\tau^3 + S(b_0 - 3b_1\tau + 3b_2\tau^2 - b_3\tau^3) \\ & + 3U(c_0 + (\check{c}_1 + \hat{c}_1 + \check{c}_1)\tau + (\check{c}_2 + \hat{c}_2 + \check{c}_2)\tau^2 - c_3\tau^3) \end{aligned} \quad (3)$$

where the coefficients are the components of each fluxes over integrated over a cycle. The coefficients are related with a flux as is indicated in the Table 1.

Table 1. Fluxes in the duality invariant superpotential.

Term	IIB Flux integer	Integer flux
1	\bar{F}_{ijk}	a_0
τ	$\bar{F}_{ij\gamma}$	a_1
τ^2	$\bar{F}_{i\beta\gamma}$	a_2
τ^3	$\bar{F}_{\alpha\beta\gamma}$	a_3
S	\bar{H}_{ijk}	b_0
U	$\bar{Q}_k^{\alpha\beta}$	c_0
$S\tau$	$\bar{H}_{\alpha jk}$	b_1
$U\tau$	$\bar{Q}_k^{\alpha j}, \bar{Q}_k^{i\beta}, \bar{Q}_\alpha^{\beta\gamma}$	$\check{c}_1, \hat{c}_1, \tilde{c}_1$
$S\tau^2$	$\bar{H}_{i\beta\gamma}$	b_2
$U\tau^2$	$\bar{Q}_\gamma^{i\beta}, \bar{Q}_\beta^{\gamma i}, \bar{Q}_k^{ij}$	$\check{c}_2, \hat{c}_2, \tilde{c}_2$
$S\tau^3$	$\bar{H}_{\alpha\beta\gamma}$	b_3
$U\tau^3$	\bar{Q}_γ^{ij}	c_3

Here we have adopted the notation given in [10] in which \bar{F}_{abc} is the integrated flux over the cycle abc . Taking the isotropic case where $\bar{F}_{ij\gamma} = \bar{F}_{k i \beta} = \bar{F}_{j k \alpha}$ there is only one representative flux for any given combination of indices. The same notation holds for the rest of fluxes.

As mentioned above, the presence of fluxes contributes with an amount of internal energy which must be cancelled by the addition of a negative tensioned object, in order to cancel the tadpole. These requirements lead to a set of constraints that the fluxes must satisfy. The R-R constraints for the fluxes are;

$$a_0 b_3 - 3a_1 b_2 + 3a_2 b_1 - a_3 b_0 = 16 \quad (4)$$

$$a_0 c_3 + a_1(\check{c}_2 + \hat{c}_2 - \tilde{c}_2) - a_2((\check{c}_1 + \hat{c}_1 - \tilde{c}_1) - a_3 c_0) = 0 \quad (5)$$

There are also constraints of the same nature on the NS fluxes which are obtained starting by the Bianchi identity $dH = 0$ and its T-dual constraints. There are a total of 8 constraints given by

$$c_0 b_2 - \check{c}_1 b_1 + \hat{c}_1 b_1 - \check{c}_2 b_0 = 0, \quad (6)$$

$$\check{c}_1 b_3 - \hat{c}_2 b_2 + \tilde{c}_2 b_2 - c_3 b_1 = 0, \quad (7)$$

$$c_0 b_3 - \check{c}_1 b_2 + \hat{c}_1 b_2 - \check{c}_2 b_1 = 0, \quad (8)$$

$$\check{c}_1 b_2 - \hat{c}_2 b_1 + \tilde{c}_2 b_1 - c_3 b_0 = 0, \quad (9)$$

$$c_0 \check{c}_2 - \check{c}_1^2 + \tilde{c}_1 \hat{c}_1 - \hat{c}_2 c_0 = 0, \quad (10)$$

$$c_3 \tilde{c}_1 - \check{c}_2^2 + \tilde{c}_2 \hat{c}_2 - \hat{c}_1 c_3 = 0, \quad (11)$$

$$c_3 c_0 - \check{c}_2 \hat{c}_1 + \tilde{c}_2 \check{c}_1 - \hat{c}_1 \tilde{c}_2 = 0, \quad (12)$$

$$\hat{c}_2 \tilde{c}_1 - \tilde{c}_1 \check{c}_2 + \check{c}_1 \hat{c}_2 - c_0 c_3 = 0. \quad (13)$$

For a formal derivation of these constraints, see [9].

3. Solutions of the equations of motion

Since we are not considering the presence of D-branes, there are no D-terms in the superpotential. and breaking SUSY is achieved only through the F terms. Our model consists only on three moduli, namely the complex structure τ , the Kähler modulus U and the axio-dilaton S . A complete solution of the supersymmetric cases is presented in [11] with AdS vacua as expected. However by breaking SUSY we shall explore the possibility to obtain classical De Sitter vacua.

Breaking SUSY through only one of the moduli can be achieved in three forms. First, breaking SUSY through the axio-dilaton implies that the F terms of the complex structure and Kähler moduli must be zero. However the condition of a zero Kähler derivative through S and U directions violates the tadpole condition. Second, breaking SUSY through the Kähler modulus leads to zero volume of the compact space or an infinite coupling constant e^ϕ where our perturbative approach is not valid. Finally SUSY breaking through the complex structure modulus implies

that the physical solutions must satisfy that $Im(P_2\bar{P}_3) = 0$. Solving this equation for the complex structure provides a way to a stabilize this modulus. The SUSY equations of motion for U and S can be solved, providing a stabilization of the axio-dilaton S and leaving unfixed the Kähler modulus. So the stabilization of the Kähler modulus depends on the minima that the scalar potential acquires. This possibility is studied in the following sections.

On the other hand, breaking SUSY through U and S leads to a violation of the Bianchi identity. This follows from the fact that the solution to the equation of motion implies that $P_1(\tau) = P_2(\tau) = P_3(\tau)$, so the fluxes associated with similar terms in τ must have the same value. However this condition leads to $a_1 = b_1 = 2c_1 - \tilde{c}_1$ and $a_2 = b_2 = \tilde{c}_2 - 2c_2$ which implies that the first Bianchi identity must be equal to zero which clearly violates the tadpole condition. SUSY breakdown through the Kähler moduli and the axio-dilaton leads to unphysical conditions which include a zero volume of the compact space and an infinite coupling constant as well.

Finally breaking SUSY through all the moduli implies that all the moduli must be stabilized dynamically through reaching the minima of the scalar potential. The values of the corresponding fluxes must satisfy the Bianchi identities and the moduli must acquire positive values.

In order to explore the space of solutions, we employ a genetic algorithm, and we look for the minimization of the scalar potential. The solutions of the equations of motion, without taking into account the case in which the scalar potential vanishes at the minimum, are summarized in the Table 2. It is important to stress out that a configuration of fluxes which satisfy the solutions does not satisfy automatically the tadpole conditions. Table 2 only remarks the fact that it is possible to find a consistent flux configuration.

Table 2. Solutions to the equations of motion.

F term	Tadpole condition	Physical solution
$D_\tau=0, D_S=0, D_U \neq 0$	No satisfied	$Im(S) = 0, Im(U) = 0$
$D_\tau=0, D_S \neq 0, D_U=0$	No satisfied	$Im(S) = 0, Im(U) = 0$
$D_\tau \neq 0, D_S=0, D_U=0$	Satisfied	$Im(P_2\bar{P}_3) = 0$
$D_\tau \neq 0, D_S \neq 0, D_U=0$	Satisfied	$Im(S), Im(U) = 0$
$D_\tau \neq 0, D_S=0, D_U \neq 0$	Satisfied	$Im(S), Im(U) = 0$
$D_\tau=0, D_S \neq 0, D_U \neq 0$	No satisfied	$a_i = b_i = c_i$

4. Discussion of the vacua

In order to get a set of solutions to the equations of motion which stabilize all the moduli, a genetic algorithm is employed.

Genetic algorithms are probabilistic global search and optimization methods that mimic the metaphor of natural biological evolution. The computational procedure operates on a set of initial values for the moduli (population) which gives a potential solution by applying the principle of survival of the fittest to produce successively better approximations to a minima of the scalar potential. At each generation of the numerical procedure, a new set of moduli is created by the process of selecting individuals according to their level of fitness in the moduli space and

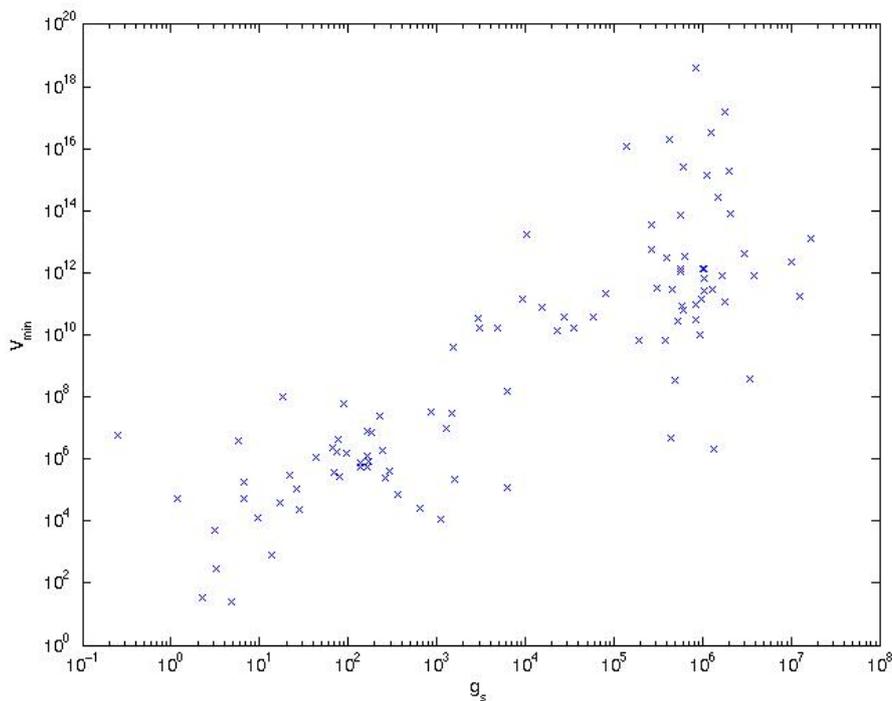


Figure 1. Logarithmic plot of g vs Λ for breaking SUSY through complex structure moduli.

reproducing them using feasible adaptation. This process leads to the evolution of moduli that are better suited to the minimum value of the scalar potential than the set of moduli from which they were created, so the algorithm successively improve the solution.

In addition we have introduced the SUSY constraints to the genetic algorithm . The numerical procedure begins with a solution to the RR and NS constraints. The solution is restricted to positive even fluxes. Once a solution to the constraints is obtained the genetic algorithm initializes with random values on the moduli. The iterative process proceeds to find a set of moduli which gives the minima value of the scalar potential. Then the constraints are check after each iteration until the maxima tolerance is reached.

In general the algebraic solution to the constraints is only valid if some of the fluxes is non zero. For example, solving the Equation (6) for a_3 is only valid if b_0 is different of zero. So the set of solutions generated turn off randomly fluxes only when the constraints equations does not yield to unphysical solutions.

In contrast to the supersymmetric solution studied in [9], where all the moduli are stabilized in a supersymmetric vacuum, we consider only two cases: the stabilization of four real moduli using the supersymmetric conditions for U and S while the remaining two real moduli, related to the complex structure τ are stabilized by minimizing the scalar potential, and the stabilization of all moduli by minimizing the scalar potential where supersymmetry is broken through all moduli.

Breaking SUSY through the complex structure τ implies the existence of De Sitter solutions. The corresponding $g - \Lambda$ space distribution is shown in Figure 1. We find that small cosmological

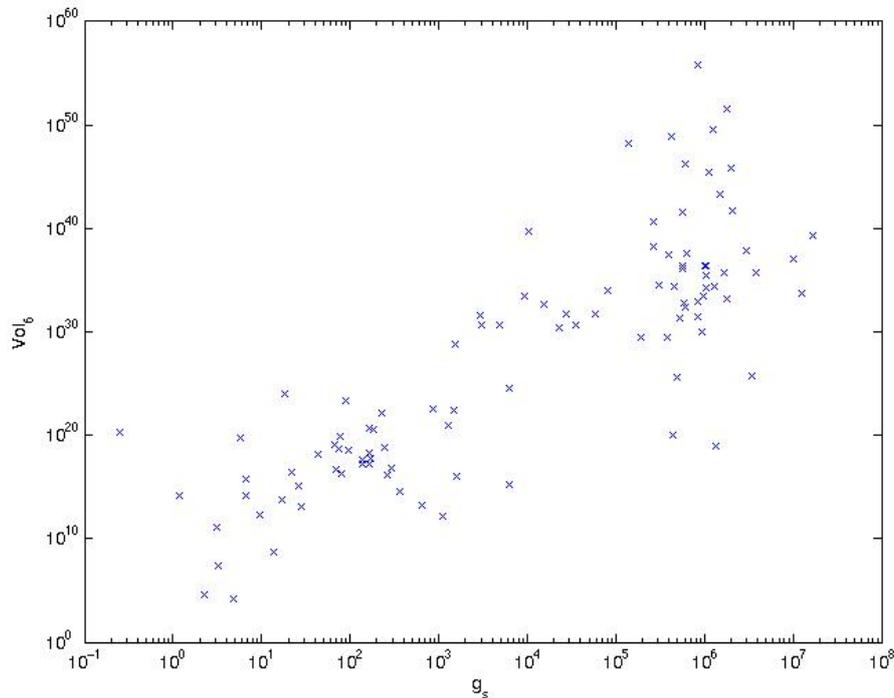


Figure 2. Logarithmic plot of g vs Vol_6 for breaking SUSY through complex structure moduli.

constants are very unfavored since we see that almost all these solutions are related with a large string coupling constant (fixed by the stabilization of the axio-dilaton), rendering our solutions unviable since they are far away from the perturbative regime where our approach is valid.

On the other hand, the vacua distribution in terms of the internal volume, lies in a region of small volume (small Kähler modulus) suggesting as well that the supergravity approximation is not valid.

In contrast to the above models, if SUSY is broken through all the moduli the $g - \Lambda$ distribution (Figure 3) seems to lie on a region of small coupling constant. The corresponding vacua have a positive value for all solutions. Finally as observed in Figure 4, the set of solutions in this case is constraint in a region in which the internal volume is big (an order of 10^4) implying that the supergravity approach we have taken is indeed valid. Therefore, we have found a rich region of De Sitter vacua in which all our approximations can be thrust. An extended analysis about their stability and the study of the existence of inflation conditions are under research [24].

5. Final comments

The present work shows a landscape of classical De Sitter vacua constructed by a Type IIB compactification on a six-dimensional torus threaded with NS-NS, RR and non-geometric fluxes and in the presence of an orientifold 3-plane. This constructions leads to a $\mathcal{N} = 1$ supersymmetric four-dimensional space-time. In effective theory there are three moduli related to the complex structure, Kähler modulus and the axio-dilaton field S of the corresponding three identical

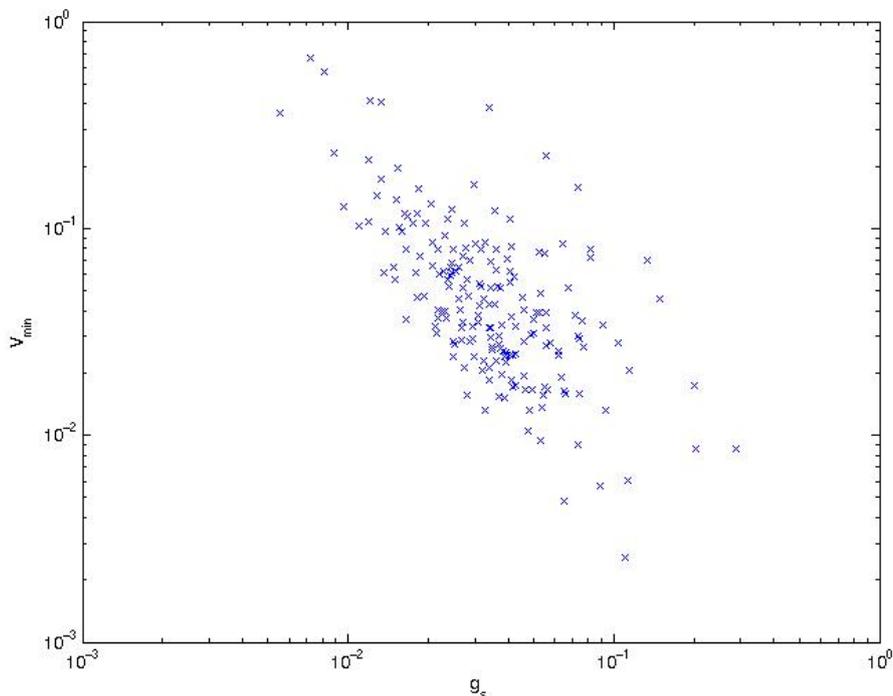


Figure 3. Logarithmic plot of g vs Λ for breaking SUSY through all the moduli.

two-dimensional torus.

We find that the viability of the solutions depends on which supersymmetric conditions are satisfied. There are only two cases in which physical solutions can be achieved and they correspond to the cases in which supersymmetry is broken through the complex structure or through all moduli.

The first case leads to vacua in which the string coupling constant is large and the internal volume is very small. In such scenario our perturbative approach and use of supergravity is not valid.

However, for the second case we find suitable physical conditions in which our approximations are valid. Although the cosmological constant is positive and small, still it seems that fine-tuning is required. For these cases, we consider it is worth to study whether they are compatible with inflation scenarios. For that it is necessary to study some extra features as their stability. We shall explore cosmological models within these vacua in a forthcoming work.

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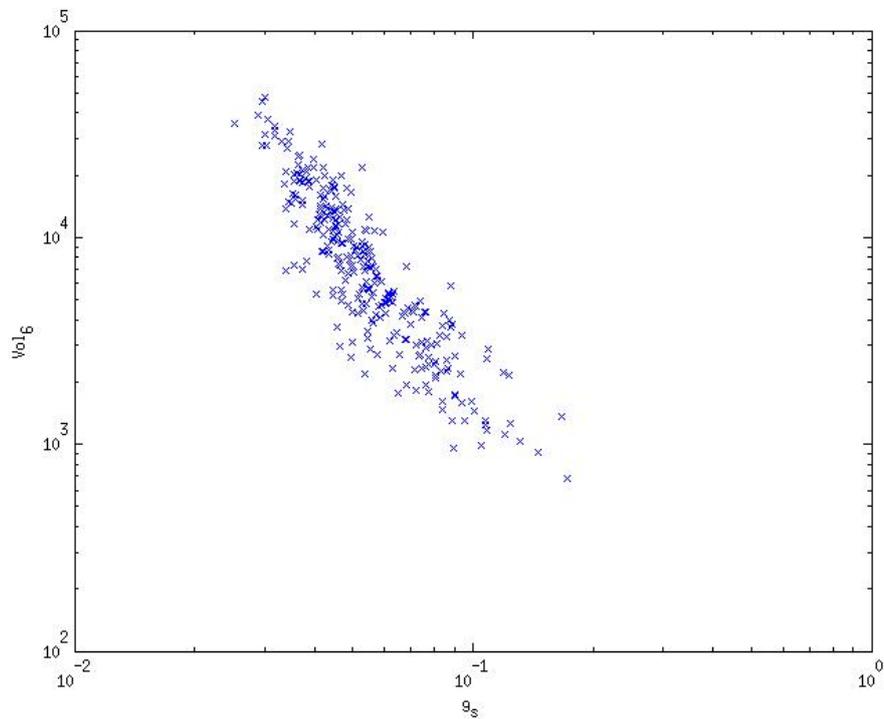


Figure 4. Logarithmic plot of g vs Vol_6 for breaking SUSY through all the moduli.

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