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Testing Chern-Simons modified gravity with observations of extreme-mass-ratio binaries

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Abstract. Extreme-Mass-Ratio Inspirals (EMRIs) are one of the most promising sources of gravitational waves (GWs) for space-based detectors like the Laser Interferometer Space Antenna (LISA). EMRIs consist of a compact stellar object orbiting around a massive black hole (MBH). Since EMRI signals are expected to be long lasting (containing of the order of hundred thousand cycles), they will encode the structure of the MBH gravitational potential in a precise way such that features depending on the theory of gravity governing the system may be distinguished. That is, EMRI signals may be used to test gravity and the geometry of black holes. However, the development of a practical methodology for computing the generation and propagation of GWs from EMRIs in theories of gravity different than General Relativity (GR) has only recently begun. In this paper, we present a parameter estimation study of EMRIs in a particular modification of GR, which is described by a four-dimensional Chern-Simons (CS) gravitational term. We focus on determining to what extent a space-based GW observatory like LISA could distinguish between GR and CS gravity through the detection of GWs from EMRIs.

1. Introduction

It is known that surrounding the central MBH of many quiescent galaxies there are around $10^7 - 10^8$ stars forming a cusp/core (see e.g. [1]). As a consequence of mass segregation and large scattering encounters between stars in the cluster, the largest stellar-mass compact objects (SCOs), like neutron stars, $m \approx 1.4M_{\odot}$, white dwarfs, $m \approx 0.6M_{\odot}$, and stellar-mass black holes, $m \approx 1 - 50M_{\odot}$, may be perturbed onto an orbit that passes close enough to the central MBH and become gravitationally bound to it. It is expected that the capture of a SCO by a MBH may be a frequent phenomenon in the Universe. The MBHs of interest for a GW observatory like LISA [2] have masses in the range $M_{\bullet} \sim 10^5 - 10^7 M_{\odot}$. In addition, there are observations suggesting the existence of Intermediate-Mass Black Holes (IMBHs), whose masses are in the range $M_{\bullet} \sim 10^2 - 10^4 M_{\odot}$ (see e.g. [3]). These may form EMRI-like systems, either an IMBH-MBH inspiral or an SCO-IMBH system. Space-based detectors like LISA will be sensitive to the inspiral of SCOs into IMBHs.

Once the SCO is captured, the binary loses energy and angular momentum through the emission of GWs. This drives the inspiral of the SCO until it plunges into the MBH. Due to the fact that the mass of the SCO is much smaller than the mass of the MBH, with typical mass ratios lying in the range $\mu = m/M_{\bullet} \sim 10^{-7} - 10^{-3}$, the inspiral can be modelled as a slow

adiabatic process in which the orbital time scale is much shorter than the timescale on which the orbit shrinks.

During the last year of evolution before plunge, the orbit of the SCO spends of the order of 10^5 cycles in the strong field region of the MBH, mapping its geometric structure (which can be characterized in terms of mass and current multipolar moments). This map is encoded in the shape and timing of the GWs emitted [5]. If we can decode this map, EMRI detections will allow us to study different aspects of the MBHs located at the centers of galaxies, the stellar dynamics around galactic centers, etc. In particular, EMRI GW signals may be very useful to test the spacetime geometry of MBHs [6, 7, 8, 9, 10, 11, 12, 13] and even alternative theories of gravity (see, e.g. [14, 15, 13, 16, 17, 18]), which is the subject of this article.

Due to the myriad of alternative theories of gravity available, the question that arises here is which kind of theory do we choose to test? Alternatively, what kind of phenomena can we expect to test using EMRI observations? In this paper, we will focus on Dynamical Chern-Simons Modified Gravity (DCSMG) which is a 4D effective theory of gravity that arises in certain lowenergy limits of *string theory* and can also arise in *Loop Quantum Gravity* (see [19] for a review of DCSMG). DCSMG has been proposed as an explanation for the cosmic baryon asymmetry [20] and makes predictions for the polarisation of the cosmic microwave background [21]. In addition, it has potentially observable signatures in EMRI GW observations [22].

In this paper we summarize the necessary techniques that we have developed to estimate the capability of a LISA-like GW detector to distinguish between GR and DCSMG using GW signals from EMRIs, extending and improving the results of [22]. A more detailed account of these developments will appear elsewhere [23]. Due to the large difference between the masses of the components in the binary, an EMRI signal can be modelled accurately using black hole perturbation theory (see e.g. [24]), in which the SCO is represented as a particle orbiting in a given MBH spacetime background. In this framework, at the lowest order of approximation the orbit of the particle is a geodesic of the MBH spacetime, but radiation reaction (RR) effects, i.e. the effects that arise from the interaction of the SCO with its own gravitational field, accelerate the trajectory and then drive the inspiral of the SCO. Depending on the theory of gravity that describes the system, the RR effects drive the inspiral in different ways .

This paper is organised as follows. In Sec. 2 we summarize the formulation of our EMRI model in DCSMG. In Sec. 3 we review the basics of signal analysis employed in our study. Finally, in Sec. 4, we present results concerning whether a GW detector like LISA will be able to discriminate between an EMRI in GR and one in DCSMG.

Throughout this paper, we follow the conventions of Misner, Thorne and Wheeler [25]: Greek letters will represent spacetime indexes and Latin letters in the middle of the alphabet i, j, ...,will represent spatial indexes only. Partial derivatives are denoted by $\partial_{\alpha}h = \partial h/\partial x^{\alpha} = h_{,\alpha}$, while, for any quantity h, covariant derivatives are denoted by $\nabla_{\alpha}h = h_{;\alpha}$. In addition, we use the Einstein summation convention and geometrised units, i.e. G = c = 1 and the gravitational constant is given by $\kappa = 1/(16\pi G)$. Finally, we use a bar, e.g. \bar{h} , to denote background quantities.

2. Formulation of our EMRI model

We approximate our system following the procedure described in [22], in which an EMRI is modelled as a binary composed of a point particle moving in a modified Kerr MBH background geometry. That is, the MBH geometry is slightly modified as a consequence of the CS gravitational modifications to GR. The motion of the particle, at the lowest order of approximation, is given by a geodesic of this CS-modified MBH geometry. The action describing the system is given by:

$$S = \kappa S_{EH} + \frac{\alpha}{4} S_{CS} + \frac{\beta}{2} S_{\vartheta} + S_{\text{mat}} , \qquad (1)$$

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where the different terms are:

$$S_{EH} = \int d^4x \sqrt{-g}R , \qquad (2)$$

$$S_{CS} = \int d^4x \sqrt{-g} \vartheta ^* RR , \qquad (3)$$

$$S_{\vartheta} = -\int d^4x \sqrt{-g} \left[g^{\mu\nu} (\nabla_{\mu}\vartheta) (\nabla_{\nu}\vartheta) \right] , \qquad (4)$$

$$S_{\rm mat} = \int d^4 \sqrt{-g} \, \mathcal{L}_{\rm mat} \,, \tag{5}$$

which represent the Einstein-Hilbert action [Eq. (2)], the CS gravitational correction [Eq. (3)], the CS scalar field action [Eq. (4)], and the action of the matter degrees of freedom [Eq. (5)]. The coupling constant, α , is associated with the Pontryagin density, $*RR := \frac{1}{2} \epsilon^{\gamma \delta \mu \nu} R^{\alpha}_{beta \mu \nu} R^{\beta}_{\alpha \gamma \delta}$, which is coupled to the CS scalar field, ϑ . The coupling constant, β , determines the strength of the coupling of the CS scalar field with the stress-energy distribution.

Following [26], we can construct the modified MBH geometry and the CS scalar field using the slow-rotation $(a/M_{\bullet} \ll 1)$, where M_{\bullet} and $a = |\mathbf{S}/\mathbf{M}_{\bullet}|$ are the MBH mass and spin respectively) and weak-coupling approximation. Then, the spacetime metric, $g_{\mu\nu}$, and the CS scalar field, ϑ , can be written as the sum of a background and a perturbative quantity characterized by the perturbative parameter v:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \upsilon h_{\mu\nu} + \mathcal{O}(\upsilon^2), \qquad (6)$$

$$\vartheta = \bar{\vartheta} + \upsilon\vartheta + \mathcal{O}(\upsilon^2). \tag{7}$$

The MBH metric has the following non-zero components in Boyer-Lindquist-like coordinates (t, r, θ, ϕ) [26]:

$$\bar{\mathbf{g}}_{tt} = -\left(1 - 2M_{\bullet}\frac{r}{\rho^2}\right),\tag{8}$$

$$\bar{\mathbf{g}}_{rr} = \frac{\rho^2}{\Delta} \,, \tag{9}$$

$$\bar{\mathbf{g}}_{\theta\theta} = \rho^2,\tag{10}$$

$$\bar{\mathbf{g}}_{\phi\phi} = \frac{\Sigma}{\rho^2} \sin^2 \theta \,, \tag{11}$$

$$\bar{\mathbf{g}}_{t\phi} = \left[\frac{5}{8} \frac{\xi}{M_{\bullet}^4} \frac{M_{\bullet}^5}{r^4} \left(1 + \frac{12M_{\bullet}}{7r} + \frac{27M_{\bullet}^2}{10r^2} \right) - 2M_{\bullet}^2 \frac{r}{\rho^2} \right] \frac{a}{M_{\bullet}} \sin^2 \theta , \qquad (12)$$

where $\Sigma = (r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta$, $\rho^2 = r^2 + a^2 \cos^2 \theta$ and $\Delta = r^2 f + a^2$, with $f = 1 - 2M_{\bullet}/r$. The CS parameter ξ is given by

$$\xi := \frac{\alpha^2}{\beta\kappa} \,. \tag{13}$$

Notice that the only metric component that contains the CS parameter ξ is $\bar{\mathbf{g}}_{t\phi}$ and that it behaves as r^{-4} for $M_{\bullet}/r \ll 1$, decaying faster than the rest of metric contributions and becoming negligible at large distances. Under the preceding approximations, the CS scalar field is given by [26]:

$$\bar{\vartheta} = \frac{5}{8} \frac{\alpha}{\beta} \frac{a}{M_{\bullet}} \frac{\cos\theta}{r^2} \left(1 + \frac{2M_{\bullet}}{r} + \frac{18M_{\bullet}^2}{5r^2} \right) \,. \tag{14}$$

The numerical evolution of the system comprises two stages, namely the computation of the SCO trajectory around the MBH and then the computation of the corresponding waveform based on this trajectory. To generate the trajectory, it has been shown in [22] that in the test-mass limit the SCO must follow geodesics of the CS deformed MBH background. It turns out that the metric [Eqs. (8)-(12)] has almost identical symmetry properties as the Kerr metric (at least at the level of approximation used for its derivation). As a consequence, the geodesic motion of the particle is obtained by solving the following set of Ordinary Differential Equations (ODEs):

$$\dot{t} = \dot{t}_K + L_z \delta g_{\phi}^{CS}, \qquad (15)$$

$$\dot{\phi} = \dot{\phi}_K - E\delta g_{\phi}^{CS} , \qquad (16)$$

$$\dot{r}^2 = \dot{r}_K^2 + 2EL_z f \delta g_\phi^{CS}, \qquad (17)$$

$$\dot{\theta}^2 = \dot{\theta}_K^2, \tag{18}$$

where E is the energy and L_z is the z-component of the orbital angular momentum of the particle per unit mass. Here, a dot denotes differentiation with respect to the proper time and $(\dot{t}_K, \dot{r}_K, \dot{\theta}_K, \dot{\phi}_K)$ are the corresponding expressions for geodesics in a Kerr MBH background:

$$\rho^{2} \dot{t}_{K} = -\left(a^{2} E \sin^{2} \theta - a L_{z}\right) + \left(r^{2} + a^{2}\right) \left(\left(r^{2} + a^{2}\right) \frac{E}{\Delta} - \frac{a L_{z}}{\Delta}\right),$$
(19)

$$\rho^2 \dot{\phi}_K = -\left(aE - \frac{L_z}{\sin^2 \theta}\right) + (r^2 + a^2) \frac{aE}{\Delta} - \frac{a^2 L_z}{\Delta}, \qquad (20)$$

$$\rho^4 \dot{r}_K^2 = \left[(r^2 + a^2) E - a L_z \right]^2 - \Delta \left[\mathcal{Q} + (a E - L_z)^2 + r^2 \right], \qquad (21)$$

$$\rho^4 \dot{\theta}_K^2 = Q - \cot^2 \theta L_z^2 - a^2 \cos^2 \theta (1 - E^2), \qquad (22)$$

where Q denotes the Carter constant per unit mass squared. The correction δg_{ϕ}^{CS} appearing in Eqs. (15)-(18) is the CS correction, which is given by:

$$\delta g_{\phi}^{CS} = \frac{\xi a}{112 r^8 f} \left(70r^2 + 120rM_{\bullet} + 189M_{\bullet}^2 \right) \,. \tag{23}$$

In order to avoid the presence of turning points in the integration of these equations of motion, we introduce two new angle coordinates, ψ and χ , associated with the radial and polar motion respectively, which are defined by the equations:

$$r = \frac{pM_{\bullet}}{1 + e\cos\psi}, \qquad \cos^2\theta = \cos^2\theta_{\min}\cos^2\chi, \qquad (24)$$

where p and e are the semilatus rectum and the eccentricity of the orbit respectively, and θ_{\min} is the minimum value of the polar angle θ in the orbit. This value is related to the angle of inclination of the orbit, θ_{inc} (see Table 1), through

$$\theta_{\min} = \frac{\pi}{2} - \theta_{\rm inc} \,. \tag{25}$$

A given orbit can be characterized in terms of the constants of motion (E, L_z, \mathcal{Q}) or, alternatively, in terms of the orbital parameters (e, p, θ_{inc}) . To transform between these two sets of constants we use the equations at the turning points of the radial and polar coordinates. At $\theta = \theta_{min}$ we have that $\dot{\theta} = 0$, leading to an expression for \mathcal{Q} [22]:

$$\mathcal{Q} = \cos^2 \theta_{\min} \left[\frac{L_z^2}{\sin^2 \theta_{\min}} + a^2 (1 - E^2) \right] \,. \tag{26}$$

The additional two equations needed are those that define the radial turning points, that is, the pericenter, r_{peri} , and apocenter, r_{apo} , which we relate to (e, p) in the usual way:

$$r_{\rm peri} = \frac{pM_{\bullet}}{1+e}, \qquad r_{\rm apo} = \frac{pM_{\bullet}}{1-e}.$$
 (27)

Eq. (26) together with the equations $\dot{r}(r_{peri}) = \dot{r}(r_{apo}) = 0$ allow us to relate the values of the constants (E, L_z, Q) to those of the orbital parameters (e, p, θ_{inc}) .

Until now we have only considered geodesic motion. Constructing waveforms from geodesic trajectories can lead to a a confusion problem in the sense that it is possible that an EMRI waveform in GR matches an EMRI waveform in DCSMG but with different parameters, given that the motion in both cases is going to be characterized by just three fundamental frequencies of motion. This degeneracy is overcome when RR effects are taken into account, since these tend to break such confusion, as the RR effects drive the inspiral in different ways depending on the theory of gravity that describes the EMRI system. Nevertheless, it was shown in [22] that, at leading order, the GW emission in DCSMG takes the same form as in GR. So, as a first approximation we describe RR effects in DCSMG using expressions from GR. In particular we use the fluxes for the constants of motion (E, L_z, Q) provided in [27]. In principle one might think that this would lead to exactly the same inspiral evolution in DCSMG and GR, but it is not the case due to the different relations between the constants of motion (E, L_z, Q) and the orbital parameters (e, p, θ_{inc}) .

To compute the EMRI waveforms in DCSMG, we first solve the ODEs given in Eqs. (15)-(18) for the angle variables $(\psi(t), \chi(t), \varphi(t))$ (see e.g. [28]). The scheme for evolving a particular EMRI in DCSMG can be summarized as follows: For a given set of initial orbital parameters, $(e^{(0)}, p^{(0)}, \theta_{\rm inc}^{(0)})$, we find the associated initial constants of the motion, $(E^{(0)}, L_z^{(0)}, Q^{(0)})$, which are different from the ones that one would obtain in GR for the same orbital parameters. Then, the time evolution of the "constants" of motion, $(\dot{E}^{(0)}, \dot{L}_z^{(0)})$, the RR effect, is computed using the fluxes of [27]. From the values of the constants of motion, $(E^{(0)}, L_z^{(0)}, Q^{(0)})$, their evolution due to RR, $(\dot{E}^{(0)}, \dot{L}_z^{(0)}, \dot{Q}^{(0)})$, and the value of the geodesic radial period, T_r (the coordinate time to go from the apocenter to the pericenter and back again to apocenter), we obtain the values of the new constants of motion, $(E^{(1)}, L_z^{(1)}, Q^{(1)})$. Finally, from the new values of the constants of motion we find the new values of the orbital parameters $(p^{(1)}, e^{(1)}, \theta_{\rm inc}^{(1)})$. This algorithm is iterated along the whole EMRI evolution to obtain the SCO orbit around the MBH. The gravitational waveform is then generated using the multipolar formalism given by Thorne [29] up to quadrupolar order, by identifying the Boyer-Lindquist coordinates with spherical polar coordinates in a pseudo flat-space.

3. Signal analysis formalism

In this section we briefly review the formalism of signal analysis that we use in this paper. For our calculations we use the LISA noise sensitivity curve (see e.g. [30]) to represent a possible future space-based GW detector. At low-frequencies the response of the LISA detector can be decomposed as the response of a pair of independent, right-angle Michelson detectors, which we label by $\alpha = I, II$ in the standard way. Each of these data streams is a time series, $s_{\alpha}(t)$, which is a superposition of a possible gravitational wave signal $h_{\alpha}(t)$ and noise $n_{\alpha}(t)$, $s_{\alpha}(t) = h_{\alpha}(t) + n_{\alpha}(t)$. Under the assumptions that the noise in the detector is stationary, Gaussian and uncorrelated between the two data streams, and that the spectral density of the noise is the same in the two channels, the Fourier components of the noise, $\tilde{n}_{\alpha}(f)$, would have a Gaussian probability distribution and

$$\langle \tilde{n}_{\alpha}(f) \tilde{n}_{\beta}(f')^* \rangle = \frac{1}{2} \delta_{\alpha\beta} \,\delta(f - f') \,S_n(f) \,, \tag{28}$$

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where a tilde denotes the Fourier transform, an asterisk denotes complex conjugation, angle brackets denote the ensemble expectation value, and $S_n(f)$ is the one-sided noise spectral density of the data stream, which is the same for both channels.

The vector space of signals has a natural inner product associated with $S_n(f)$:

$$(\mathbf{a}|\mathbf{b}) = 2 \sum_{I,II} \int_0^\infty \frac{\tilde{a}^*_{\alpha}(f)\tilde{b}_{\alpha}(f) + \tilde{a}_{\alpha}(f)\tilde{b}^*_{\alpha}(f)}{S_n(f)} \mathrm{d}f.$$
(29)

The assumption of Gaussianity then means that the probability that the noise, \mathbf{n} , has a particular realisation, \mathbf{n}_0 , has the form

$$p(\mathbf{n} = \mathbf{n}_0) \propto e^{-(\mathbf{n}_0 | \mathbf{n}_0)/2},\tag{30}$$

and hence the likelihood that a signal **h** is present in the data stream **s** is proportional to $\exp(-(\mathbf{s} - \mathbf{h}|\mathbf{s} - \mathbf{h})/2)$. The 'best-fit' waveform will be the one that maximises ($\mathbf{s}|\mathbf{h}$) and, thus, provides the maximum likelihood parameter estimate. The expected signal-to-noise ratio (SNR) when filtering with the correct waveform is

$$SNR = \frac{(\mathbf{h}|\mathbf{h})}{rms(\mathbf{h}|\mathbf{n})} = \sqrt{(\mathbf{h}|\mathbf{h})}, \qquad (31)$$

where the second equality follows from the fact that the expectation value of $(\mathbf{a}|\mathbf{n})(\mathbf{b}|\mathbf{n})$ is $(\mathbf{a}|\mathbf{b})$ [31].

A given template family $\mathbf{h}(t; \vec{\lambda})$ will depend on a set of parameters $\vec{\lambda}$. If we expand $\exp(-(\mathbf{s} - \mathbf{h}|\mathbf{s} - \mathbf{h})/2)$ around the best-fit parameters, $\vec{\lambda}_0$, by writing $\vec{\lambda} = \vec{\lambda}_0 + \delta \vec{\lambda}$, we can derive a probability distribution function for the parameter uncertainties

$$p(\delta\vec{\lambda}) = \mathcal{N}\exp\left(-\frac{1}{2}\Gamma_{jk}\delta\lambda^{j}\delta\lambda^{k}\right), \qquad (32)$$

where $\mathcal{N} = \sqrt{\det(\Gamma/2\pi)}$ is the normalization factor and Γ_{ij} is the Fisher information matrix [32]

$$\Gamma_{jk} = \left(\frac{\partial \mathbf{h}}{\partial \lambda^j} \middle| \frac{\partial \mathbf{h}}{\partial \lambda^k}\right) \,. \tag{33}$$

The variance-covariance matrix for the waveform parameters is therefore given by the inverse of the Fisher matrix

$$\langle \delta \lambda^j \delta \lambda^k \rangle = \left(\Gamma^{-1} \right)_{jk} + O(1/\text{SNR})$$
(34)

and so we can estimate the precision with which we will be able to measure a particular parameter, λ_i , using LISA observations by computing the component Γ_{ii}^{-1} of this inverse matrix.

In our simulations, the space of EMRI waveforms in DCSMG, $\{h(t; \lambda^i) | i = 1, ..., N\}$ is characterised by the following 15 parameters (see Table 1 for a description of these parameters):

$$\vec{\lambda} = \{M_{\bullet}, a, \mu, e_0, p_0, \theta_0^{\text{inc}}, \hat{\xi}, \theta_S, \phi_S, \theta_K, \phi_K, D_L, \psi_0, \chi_0, \phi_0\}.$$
(35)

The Fisher matrices for EMRI waveforms have very large condition numbers (the ratio of the largest to the smallest eigenvalues) and so we use an LU decomposition to invert them, where the Fisher matrix [Eq. (34)] is written as the product of a lower triangular matrix and an upper triangular matrix [33]. In addition, to verify that the error estimates that we obtain are reliable, we employ the maximum-mismatch criterion proposed in [34].

4. Results

The main goal of our study is to determine how well a GW detector like LISA could estimate the parameters of an EMRI system (see Table 1) in DCSMG and in particular how well it could constrain the CS parameter ξ and hence distinguish between an inspiral in DCSMG and one in GR. To that end, we have followed the approach of [22] to model the motion of the SCO in the modified DCSMG MBH geometry, but this time we have included RR effects. To evolve the inspiral we have followed the algorithmic scheme described above. Once the trajectory of the SCO has been computed, we obtain the waveforms by using the multipolar formalism described by Thorne in [29], but only keeping the quadrupolar term. To compute the Fisher matrix, the waveform derivatives, $\partial_i \mathbf{h} = \partial \mathbf{h}/\partial \lambda^i$, are obtained numerically through a five-point finite difference rule. Finally, the response of the detector and the Doppler modulation for a detector like LISA are computed using the expressions in [30] and in [35].

Table 1. Summary of the EMRI parameter space employed in our analysis. The angles (θ_S, ϕ_S) and (θ_K, ϕ_K) are spherical coordinates and t_0 stands for the initial time in the computation. For the parameters with physical dimensions, we indicate the units used in round brackets.

M_{ullet}	MBH mass.
$a = \mathbf{S} / M_{\bullet} (M_{\bullet}^2)$	MBH spin.
$\mu = m/M_{\bullet}$	Mass-ratio.
e_0	Eccentricity of the particle orbit at t_0 .
p_0	Semilatus rectum at t_0 .
$ heta_0^{ m inc}$	Inclination of the orbit at t_0 .
$\hat{\xi} = \xi \times S \ (M^6_{\bullet})$	Product of the Chern-Simons parameter and the spin.
θ_S	Polar angle with respect to the ecliptic.
$\tilde{\phi_S}$	Azimuthal angle with respect to the ecliptic.
$\hat{\theta_K}$	Spin polar angle with respect to the ecliptic.
ϕ_K	Spin azimuthal angle with respect to the ecliptic.
D_L (Gpc)	Distance to the EMRI from the ecliptic baricentre.
ψ_0^-	Phase angle associated with the radial motion at t_0 .
χ_0	Phase angle associated with the polar motion at t_0 .
ϕ_0	Boyer-Lindquist azimuthal coordinate, ϕ , at t_0 .

Table 2. Parameters of the different systems used in our work, where we have fixed p = 10, $\theta_{\text{inc}} = 0.85$, $\theta_S = 1.1$, $\phi_S = 0.3$, $\theta_K = 1.4$, $\phi_K = 0.25$, $D_L = 1 \text{ Gpc}$, $\psi_0 = 0.25$, $\chi_0 = 1$, and $\phi_0 = 0.1$.

System	M_{ullet}	a/M_{\bullet}	μ	e_0	$\hat{\xi}/M_{ullet}^6$
А	5×10^5	0.5	2×10^{-5}	0.5	10^{-2}
В	5×10^5	0.5	2×10^{-5}	0.5	0
С	5×10^5	0.25	2×10^{-5}	0.5	10^{-2}
D	5×10^5	0.25	2×10^{-5}	0.5	0
Е	10^{6}	0.25	10^{-5}	0.32	10^{-2}
F	10^{6}	0.25	10^{-5}	0.32	0

We have computed parameter measurement error estimates for different systems in DCSMG and the analogous systems in GR (that is, setting $\xi = 0 = \delta \xi$). In Table 2, we show the

parameters of the different (systems) used in this analysis, where the distance to the source is given in Gpc, the angles are given in radians and the rest of the parameters are given in MBH mass units, while the MBH mass itself is given in solar masses. The direction of the MBH spin is taken to be parallel to the z-direction of the EMRI reference frame.

As a test of our numerical set-up, in Table 3 we compare our numerical estimates in GR with the ones obtained in [30]. Since the parameter space employed in our analysis differs with the one chosen in [30], we just compare the results obtained for the parameters common to both works and, as we can see from Table 3, the two methods produce very similar results. However, as it is shown in the table, our estimate for the error in the MBH spin is two orders of magnitude smaller than the one obtained in [30]. In our analysis, we include the effect of the MBH spin directly in the geodesic equations, whereas in [30] the effect of the MBH spin is introduced as a post-Newtonian correction in their *post-Keplerian* equations of motion. Our analysis therefore includes higher order spin corrections which may explain some of the improvement that our results show. A similar improvement in spin measurement precision was observed in [33] who used a similar method based on exact Kerr geodesic trajectories.

Table 3. Parameter accuracy estimates for the inspiral of a $10M_{\odot}$ SCO into a 10^6M_{\odot} MBH. It is assumed that we have observed the last year of inspiral and that the system is an EMRI described by GR, i.e., $\xi = 0$. We compare the results we have obtained with the ones obtained for the same parameters by Barack and Cutler [30].

Parameter	GR (this work)	$\mathrm{GR}~([30])$
$\Delta \ln M_{\bullet}$	$8.6 imes 10^{-4}$	9.2×10^{-4}
$\Delta a/M_{\bullet} \ (M_{\bullet})$	$9.5 imes 10^{-6}$	$6.3 imes 10^{-4}$
$\Delta \ln \mu$	$1.4 imes 10^{-4}$	$9.2 imes 10^{-5}$
Δe	2.7×10^{-5}	2.8×10^{-4}
$\Delta \ln D_L^{(a)}, \Delta \ln (\mu/D_L)^{(b)}$	$6.5 \times 10^{-2} ^{(a)}$	$3.7 \times 10^{-2} {}^{(b)}$

We have considered two different kind of EMRIs, namely one composed of a MBH with $M_{\bullet} = 5 \times 10^5 M_{\odot}$ and a SCO with mass $m = 10 M_{\odot}$, and the other one composed of a MBH with mass $M_{\bullet} = 10^6 M_{\odot}$ and a SCO with mass $m = 10 M_{\odot}$. For the former, we consider an observation lasting for the six months before plunge (SNR=35) and for the latter we consider an observation lasting one year before plunge (SNR = 40). In Table 4, we show the error estimates obtained in our simulations. We typically find that an EMRI observation can determine the extrinsic parameters, namely $\vec{\lambda}_{\text{extrinsic}} = \{\theta_S, \phi_S, \theta_K, \phi_K, D_L, \psi_0, \chi_0, \phi_0\}$, to a fractional error of ~ 10^{-2} , while the intrinsic parameters: $\lambda_{\text{intrinsic}} = \{M_{\bullet}, a, \mu, e_0, \theta_0^{\text{inc}}\}$, are typically measured with a fractional error of ~ $10^{-4}, 10^{-5}, 10^{-5}, 10^{-5}, 10^{-5}$ respectively. For the CS parameter $\hat{\xi}$, we find that in the best case it can be measured with a fractional error $\sim 10^{-1}$. These results suggest that LISA might be able to distinguish between GR and DCSMG using certain EMRI observations. Notice that in this paper we are using the concept of the LISA mission as a reference and, as it is shown in [23], the results obtained are also valid for the updated eLISA or NGO (New Gravitational-Waves Observatory) [36] by normalising to a fixed SNR. In addition, we find that for a given value of ξ , it will be better estimated for systems with smaller MBH mass, M_{\bullet} , and spin, a. This is not surprising, since the leading-order term that contains the CS parameter in the DCSMG metric [Eq. (12)] is $\sim \hat{\xi}/M_{\bullet}^6(M_{\bullet}/r)^6$. By decreasing the MBH mass with fixed $\hat{\xi}$, this term becomes larger at fixed r/M_{\bullet} and by decreasing the MBH spin this term is larger relative to the Kerr $g_{t\phi}$ metric coefficient.

Due to the relatively high computational cost of these calculations, a full (i.e., including all the

Parameter	А	В	С	D	Ε	F
$ \begin{array}{c} \Delta \ln M_{\bullet} \\ \Delta a (M_{\bullet}) \\ \Delta \ln \mu \end{array} $	$\begin{array}{c} 1.0 \times 10^{-4} \\ 1.5 \times 10^{-5} \\ 1.8 \times 10^{-4} \end{array}$	9.8×10^{-3} 9.5×10^{-6} 3.9×10^{-5}	$\begin{array}{c} 7.8 \times 10^{-4} \\ 7.5 \times 10^{-6} \\ 5.7 \times 10^{-5} \end{array}$	$7.6 \times 10^{-4} \\ 6.3 \times 10^{-6} \\ 2.4 \times 10^{-5} \\ 5.4 \times 10^{-5$	$\begin{array}{c} 8.6 \times 10^{-4} \\ 1.8 \times 10^{-5} \\ 3.2 \times 10^{-4} \end{array}$	$\begin{array}{c} 8.4 \times 10^{-4} \\ 9.1 \times 10^{-6} \\ 1.2 \times 10^{-4} \end{array}$
$\Delta e_0 \ \Delta heta_0^{ m inc} \ \Delta \ln \hat{\xi}$	9.2×10^{-5} 2.8×10^{-5} 2.1	9.2×10^{-5} 1.7×10^{-5}	$8.4 \times 10^{-5} 2.6 \times 10^{-5} 4.4 \times 10^{-1}$	8.4×10^{-5} 2.2×10^{-5}	1.9×10^{-5} 8.1×10^{-5} 5.1	1.7×10^{-5} 3.2×10^{-5}
$egin{array}{c} \Delta heta_S \ \Delta \phi_S \ \Delta heta_K \end{array}$	1.8×10^{-2} 1.7×10^{-2} 2.8×10^{-2}	1.7×10^{-2} 1.6×10^{-2} 2.8×10^{-2}	1.5×10^{-2} 1.5×10^{-2} 2.5×10^{-2}	1.4×10^{-2} 1.5×10^{-2} 2.5×10^{-2}	$\begin{array}{c} 1.1 \times 10^{-2} \\ 2.1 \times 10^{-2} \\ 2.3 \times 10^{-2} \end{array}$	$\begin{array}{c} 1.1 \times 10^{-2} \\ 2.0 \times 10^{-2} \\ 2.3 \times 10^{-2} \end{array}$
$\begin{array}{c} \Delta\phi_K\\ \Delta D_L \ (Gyr)\\ \Delta\psi_0 \end{array}$	$7.2 \times 10^{-2} \\ 2.7 \times 10^{-2} \\ 9.8 \times 10^{-2}$	$7.2 \times 10^{-2} 2.6 \times 10^{-2} 7.1 \times 10^{-2}$	$\begin{array}{c} 6.5 \times 10^{-2} \\ 2.2 \times 10^{-2} \\ 7.2 \times 10^{-2} \end{array}$	$\begin{array}{c} 6.5\times 10^{-2}\\ 2.3\times 10^{-2}\\ 6.0\times 10^{-2}\end{array}$	$\begin{array}{c} 8.6 \times 10^{-2} \\ 1.8 \times 10^{-2} \\ 1.0 \times 10^{-2} \end{array}$	$\begin{array}{c} 8.6 \times 10^{-2} \\ 1.9 \times 10^{-2} \\ 8.6 \times 10^{-2} \end{array}$
$\Delta \chi_0 \ \Delta arphi_0$	$\begin{array}{l} 8.6\times 10^{-2} \\ 8.6\times 10^{-2} \end{array}$	8.5×10^{-2} 8.3×10^{-2}	7.7×10^{-2} 7.6×10^{-2}	7.7×10^{-2} 7.5×10^{-2}	9.2×10^{-2} 1.1×10^{-1}	9.0×10^{-2} 1.1×10^{-2}

Table 4. Error estimates for half a year of evolution (SNR= 35) for Systems A, B, C, and D; and for one year of evolution (SNR= 40) for Systems E and F.

parameters) error estimation takes approximately three days using a single processor machine. For this reason, we have only studied a small number of points in the parameter space of all possible EMRI systems that we might detect. In this sense, these results could be improved by performing a more exhaustive study of the parameter space and also using a full Monte Carlo sampling. In addition, we could have considered longer gravitational waveforms, that cover the entire length of the LISA mission ($\sim 2-5$ years), which would be expected to improve our ability to measure the system parameters.

5. Conclusions

In this work we have presented a first study aimed at assessing the ability of a GW detector like LISA to discriminate between GR and DCSMG by performing parameter estimation studies of EMRIs in DCSMG. We have computed the waveforms generated by a SCO orbiting in a MBH geometry modified by CS corrections. The parameter error estimates have been computed using a Fisher matrix analysis. To validate our computational framework, we have studied a typical EMRI system in GR and we have found agreement between our results and previous results found in the literature. We have applied these parameter estimation studies to estimate the ability of LISA to distinguish between GR and DCSMG by estimating the CS parameter ξ . To that end, we have performed simulations of the GWs emitted during the last six months before plunge of an EMRI system for which the gravitational wave emission is at frequencies in the most sensitive part of the LISA band. Our results indicate that, for certain EMRI systems, a detector like LISA may be able to discriminate between GR and DCSMG. We have also seen that the error in estimating ξ decreases with the MBH mass and spin. We are now performing a more exhaustive study of the parameter space of possible EMRIs to better understand if these results hold in general [23].

Another subject that deserves further study is the question of the systematic errors that would arise by using GR waveform templates to detect EMRIs that are actually described by DCSMG. This can be done by using the formalism developed recently by Cutler and Vallisneri [37], which allows for the estimation of the magnitude of the model errors. Finally, the present work can be extended to other GW detectors, for instance, to study Intermediate-Mass-Ratio Inspiral detection with the future Einstein Telescope.

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References

- [1] Amaro-Seoane P and Preto M 2011 Class. Quant. Grav. 28 094017 (Preprint 1010.5781)
- [2] LISA (Laser Interferometer Space Antenna), http://www.esa.int, http://lisa.jpl.nasa.gov
- [3] Amaro-Seoane P, Gair J, Freitag M, Miller C M, Mandel I, Cutler C and Babak S 2007 Class. Quant. Grav. 24 R113 (Preprint astro-ph/0703495)
- [4] ET (Einstein Telescope), http://www.et-gw.eu/
- [5] Ryan F D 1995 Phys. Rev. D 52 5707
- [6] Collins N A and Hughes S A 2004 Phys. Rev. D69 124022 (Preprint gr-qc/0402063)
- [7] Glampedakis K and Babak S 2006 Class. Quant. Grav. 23 4167–4188 (Preprint gr-qc/0510057)
- [8] Barack L and Cutler C 2007 Phys. Rev. D75 042003 (Preprint gr-qc/0612029)
- [9] Yunes N, Sopuerta C F, Rubbo L J and Holley-Bockelmann K 2008 Astrophys. J. 675 604–613 (Preprint 0704.2612)
- [10] Brink J 2008 Phys. Rev. D78 102001 (Preprint 0807.1178)
- [11] Vigeland S J and Hughes S A 2010 Phys. Rev. D81 024030 (Preprint 0911.1756)
- [12] Hughes S A 2010 (*Preprint* 1002.2591)
- [13] Sopuerta C F 2010 GW Notes 4(4) 3–47 (Preprint 1009.1402)
- [14] Hughes S A 2006 AIP Conf. Proc. 873 233-240 (Preprint gr-qc/0608140)
- [15] Schutz B F 2009 Class. Quant. Grav. 26 094020
- [16] Babak S, Gair J R, Petiteau A and Sesana A 2011 Class. Quant. Grav. 28 114001 (Preprint 1011.2062)
- [17] Vigeland S, Yunes N and Stein L 2011 Phys. Rev. D 83 104027 (Preprint 1102.3706)
- $[18]\,$ Gair J R and Yunes N 2011 (Preprint 1106.6313)
- [19] Alexander S and Yunes N 2009 Phys. Rep. 480 1
- [20] Alexander S H, Peskin M E and Sheikh-Jabbari M M 2006 Phys. Rev. Lett. 96 081301 (Preprint hep-th/0403069)
- [21] Lue A, Wang L M and Kamionkowski M 1999 Phys. Rev. Lett. 83 1506 (Preprint astro-ph/9812088)
- [22] Sopuerta C F and Yunes N 2009 Phys. Rev. D 80 064006 (Preprint 0904.4501)
- [23] Canizares P, Gair J R and Sopuerta C F 2012 to be submitted
- [24] Drasco S and Hughes S A 2006 Phys. Rev. D 73 024027 (Preprint gr-qc/0509101)
- [25] Misner C W, Thorne K and Wheeler J A 1973 Gravitation (San Francisco: W. H. Freeman & Co.)
- [26] Yunes N and Pretorius F 2009 Phys. Rev. D **79** 084043 (Preprint 0902.4669)
- [27] Gair J R and Glampedakis K 2006 Phys. Rev. D 73 064037
- [28] Drasco S and Hughes S A 2004 Phys. Rev. D 69 044015 (Preprint astro-ph/0308479)
- [29] Thorne K S 1980 Rev. Mod. Phys. 52 299
- [30] Barack L and Cutler C 2004 Phys. Rev. D 69 082005 (Preprint gr-qc/0310125)
- [31] Cutler C and Flanagan E E 1994 Phys. Rev. D 49 2658 (Preprint gr-qc/9402014)
- [32] Fisher R A 1935 Roy. Stat. Soc. J. 98 35
- [33] Huerta E A and Gair J R 2009 Phys. Rev. D 79 084021 (Preprint 0812.4208)
- [34] Vallisneri M 2008 Phys. Rev. D 77 042001
- [35] Cornish N J and Larson S L 2003 Class. Quant. Grav. 20 163 (Preprint arXiv:gr-qc/0206017)
- [36] Amaro-Seoane P, Aoudia S, Babak S, Binetruy P, Berti E et al. 2012 (Preprint 1201.3621)
- [37] Cutler C and Vallisneri M 2007 Phys. Rev. D 76 104018