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To cite this article: Emanuele Alesci and Leonardo Modesto 2012 *J. Phys.: Conf. Ser.* **360** 012036

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Hawking radiation from Loop Black Holes

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Abstract. We review a recent black hole particle production analisys in a regular spacetime metric obtained in a minisuperspace approach to loop quantum gravity. Repeating the Hawking analysis, which leads to a thermal flux of particles at the future infinity, one finds an infinite evaporation time and unitarity recovered due to the regularity of the singularity free spacetime and to the characteristic behavior of the surface gravity.

1. Introduction

Since classical black holes radiate particles [1, 2], their mass slowly decrease and finally disappear, leaving a paradox. Supposing that matter which enters the black hole was in a pure quantum state at early time, then the existence of an horizon transforms such matter into the mix thermal state of Hawking radiation at late time for an external observer, and the complete evaporation of the singularity destroys the correlations between the radiation and the information "swallowed" by the black hole that would allow to reconstruct the original pure quantum state. This is the Hawking scenario [3]. The role of the singularity is crucial in this paradox. Its absence in the presence of quantum gravitational effects has consequences for the entire global structure [4], and its removal is essential for resolving the black hole information loss problem [5, 6]. It is thus promising that a resolution of the big bang as well as the black hole singularites [7, 8, 9] has been achieved in a simplified version of LQG [10], known as loop quantum cosmology [11]. A regular static black hole metric (LBH) with quantum gravity corrections inspired by LQG was recently derived in [12, 13]. Here we review, the Hawking calculation of particle creation, based on an extension of the procedure in [14] [15], in a LBH background [16] to show that in this simplified model, the whole process, collapse and complete evaporation, is unitary.

2. The LBH spacetime: Collapse, Particle Creation, Evaporation and Unitarity

The quantum gravitationally corrected Schwarzschild metric of the LBH is

$$ds^2 = -G(r)dt^2 + \frac{dr^2}{F(r)} + H(r)d\Omega^2 \ , \qquad G(r) = \frac{(r - r_+)(r - r_-)(r + r_*)^2}{r^4 + a_o^2}$$

$$F(r) = \frac{(r - r_+)(r - r_-)r^4}{(r + r_*)^2(r^4 + a_o^2)} \ , \qquad H(r) = r^2 + \frac{a_o^2}{r^2} \tag{1}$$

with $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. Here, $r_+ = 2m$ and $r_- = 2mP^2$ are the two horizons, and $r_* = \sqrt{r_+ r_-} = 2mP$. P is the polymeric function with $P \ll 1$, such that r_- and r_* are very

close to $r = 0$. $a_o = A_{\min}/8\pi$, $A_{\min}$ being the area gap of LQG. In the limit $r \to 0$ the solution does not have a singularity: but instead has another asymptotically flat Schwarzschild region. The Penrose diagram [16] shows two horizons and two pairs of asymptotically flat regions.

If we proceed by combining the static metric with a radially ingoing null-dust, we obtain a dynamical space-time for a black hole formed from such dust. In the present model this process, usually described by the Vaidya metric [17], will have corrections, negligible in the asymptotic region, but crucial to avoid the formation of a singularity in the strong-curvature region.

Inserting the LBH metric in the wave-equation for a massless scalar field in a general spherically symmetric curved space-time one obtains obtains the radial equation for the field $\left[\frac{\partial^2}{\partial r^{*2}} + \omega^2 - V(r(r^*))\right]\psi(r) = 0$ where r^* is the tortoise coordinate implicitly defined by $\frac{dr^*}{dr} := \frac{1}{\sqrt{GF}}$. The potential $V(r)$ (see [16]) is zero at $r = r_+$ and r_- as for the classical Reissner-Nordström black hole and can be approximated near the horizons via $V(r^*) \propto e^{2\kappa_+ r^*}$, for $r \to r_+$ or $r^* \to -\infty$, $V(r^*) \propto e^{-2\kappa_- r^*}$, for $r \to r_-$ or $r^* \to +\infty$, $V(r^*) \to 0$, for $r \to 0$ or $r \to +\infty$. For this metric (1) we find the following values for the surface gravity on the inner and outer horizons:

$$\kappa_- = \frac{4m^3 P^4 (1 - P^2)}{16m^4 P^8 + a_o^2}, \quad \kappa_+ = \frac{4m^3 (1 - P^2)}{16m^4 + a_o^2}, \tag{2}$$

With these tools one can evaluate the particle production looking for the Bogulibov transformations needed to connect the positive ingoing f_ω and outgoing p_ω frequency modes of the field, between the initial I^- and final I^+ stationary regions with natural time $v = t + r^*$ and $u = t - r^*$ respectively. Assuming the frequency to be discrete

$$p_\omega = \sum_{\omega'} A_{\omega\omega'} f_{\omega'} + B_{\omega\omega'} f_{\omega'}^* \quad, \qquad A_{\omega\omega'} = (p_\omega, f_{\omega'}) \quad \& \quad B_{\omega\omega'} = -(p_\omega, f_{\omega'}^*) \tag{3}$$

which satisfy the following matrix relations $AA^\dagger - BB^\dagger = 1$, $AB^T - BA^T = 0$ and the matrix elements are $A_{\omega\omega'} = -i \int_{I^-} dv H d\Omega (p_\omega \partial_v f_\omega^* - f_{\omega'}^* \partial_v p_\omega)$,. If any of the $B_{\omega,\omega'}$ are non zero the particle content of the vacuum state at I^- (which we indicate with the ket $|\text{in}\rangle$) respect to the Fock space at I^+ is non trivial: $\langle\text{in}|\hat{N}_\omega^{I^+}|\text{in}\rangle = \sum_{\omega'} |B_{\omega,\omega'}|^2$, where $\hat{N}_\omega^{I^+}$ is the particle number operator at frequency ω at I^+. In contrast, if all the coefficient $B_{\omega,\omega'}$ are equal to zero then the positive frequency mode basis f_ω and p_ω are related by a unitary transformation.

To evaluate A we need to know the behavior of the modes p_ω at I^-. This has been found with a *geometric optic approximation* in [16], and using it, is possible to calculate A. From the relations between A and B [17, 18] one gets, the number of particles in the ωth mode at I^+ :

$$\langle\text{in}|\hat{N}_\omega^{I^+}|\text{in}\rangle = (BB^\dagger)_{\omega,\omega} = \frac{1}{e^{2\pi\omega/\kappa_+} - 1}. \tag{4}$$

which is the Planck distribution of thermal radiation for bosons at temperature $T_{BH} = \kappa_+/2\pi$.

The evaporation proceeds through the Hawking emission at r_+, and the black hole's Bekenstein-Hawking temperature is [13] $T_{BH}(m) = \frac{(2m)^3(1-P^2)}{4\pi[(2m)^4+a_o^2]}$. T_{BH} coincides with the Hawking temperature in the limit of large masses but goes to zero for $m \to 0$. From the luminosity one can deduce the mass loss and this implies that the black hole needs an infinite time to completely evaporate. In the complete evaporation process, we neglected the backreaction for any value of the mass because at $v \approx +\infty$, when $m \lesssim m_P$, $dm/dv << m$ and then, contrary to the classical case, such approximation is valid also in the final stages of evaporation.

Combining the black hole formation and evaporation, considering the creation of (massless) particles on the horizon such that locally energy is conserved, one obtains the complete dynamics as depicted in the resulting causal diagram Fig.1, fundamental to study the black hole paradox.

The paradox presents essentialy two features: the loss of information, strictly linked to the existence of an horizon, and the not unitary evolution, that is essentially related to the presence of a singularity. The effective LBH spcetime, seems to be a good candidate to solve both problems. The spacetime is in fact singularity free and the Bogoulibov coefficients evolve with the mass allowing a unitary tranformation between the initial and final vacuum. For small but non zero black hole mass we have thermal radiation but when the mass goes to zero, in an infinite amount of time, the unitarity is restored. More concretely looking at the surface gravity κ_+, we see the mass goes to zero together with the temperature. This is the crucial difference with the classical explosive case where the temperature goes to infinity when the mass reduces to zero. From the Planck spectrum in (4) we deduce that in an infinite amount of time

$$\lim_{m \to 0} (BB^\dagger)_{\omega,\omega} = 0 \quad \forall \, \omega, \tag{5}$$

because $\kappa \to 0$ for $m \to 0$. Since $(BB^\dagger)_{\omega,\omega}$ is positive semi-define, this vanish iff $B = 0$. Since $B = 0$ the positive frequency *mode basis f_ω and p_ω are related by the unitary transformation A.* The quantum state at I^+ is described by a thermal *density matrix* $\rho = \prod_\omega (1 - e^{-2\pi\omega/\kappa_+}) \sum_{N=0}^{+\infty} e^{-2\pi\omega N/\kappa_+} |N_\omega\rangle \langle N_\omega|$, where $|N_\omega\rangle$ is the state at I^+ with N particles in the mode ω: when the balck hole mass goes to zero it reduces to a pure state

$$\lim_{m \to 0} \rho = \prod_\omega |0_\omega\rangle \langle 0_\omega| \equiv |\text{in}\rangle \langle \text{in}|, \tag{6}$$

Up to now we have restriced our attention to the surface I^+. When the black hole mass is non zero the pre state $|\text{in}\rangle$ could appear mixed but it is still a pure state since we have to add the modes crossing the event horizon H^+. Because we have ignored this sector we have lost all the possible correlations with the quanta entering into the black hole. If we properly take into account those modes, the purity of the $|\text{in}\rangle$ state is restored. Following [17] we find an explicit relation between $|\text{in}\rangle$ and the N-particles states with frequency ω at I^+ and H^+, respectively $|N_\omega^{I^+}\rangle$, $|N_\omega^{H^+}\rangle$: $|\text{in}\rangle = \prod_\omega \sqrt{1 - e^{-\frac{2\pi\omega}{\kappa_+}}} \sum_{N=0}^{+\infty} e^{-\frac{\pi N\omega}{\kappa_+}} |N_\omega^{I^+}\rangle \otimes |N_\omega^{H^+}\rangle$. This relation shows we have an independent emission of quantum entangled states of outgoing and ingoing radiation.

In Fig.1 we can see that the evolution is unitary at a fundamental level, in fact the absence of singularity implies that any Cauchy surface Σ of the spacetime is a Cauchy surface for the whole spacetime and the Hawking scenario is avoided. If we use Σ as a "time label" for the spacetime describing the whole evaporation, we don't find any discontinuity in contrast to the Hawking scenario [18]. We can describe the state $|\text{in}\rangle$ in terms of a mass dependent splitting between internal and external spacetime of the final Cauchy surface $\Sigma_f^m = \Sigma_{\text{int}}^m \bigcup \Sigma_{\text{out}}^m$: which is based on the decomposition of the final one particle Hilbert space in terms of the ones internal and external to the horizon, $H_{\text{fin}}^m = H_{\text{int}}^m \oplus H_{\text{out}}^m$. For $m = 0$, $H_{\text{fin}}^0 = H_{\text{r}=0} \oplus H_{\text{I}^+}$ and the initial vacuum reduces to $|0\rangle_{in} = |0\rangle_{\text{out}} \otimes |0\rangle_{\text{int}} = |0\rangle_{\text{fin}}$ and the density matrix to the pure state (6).

We can better understand the properties of the radiation introducing an abserver O in the classical region of the dynamical spacetime describing the collapse. (Figure 1). O_2 at $u << u_c$ will not see any remarkable radiation being too far from the event horizon. Any O_3 for $-\infty > u \geq u_c$ will see thermal radiation as an incoming flux of particles from the region where the matter collapsed. The essential point is the presence of an horizon: the pure state is perceived as a flux of particles because the observer is obliged to trace over the internal degrees of freedom, losing the information about the particles inside the black hole. O_3 will experience a thermal particle bath of increasing or decreasing temperature depending on the stage of the evaporation. Finally O_1 (r= constant) sees a gas of particles of increasing or decreasing temperature moving in time towards i^+ and at i^+ the initial vacuum state evolves in itself when $\kappa \to 0$. We wish to mention the fact that the observer in i^+ is in causal contact with both the radiation outcoming towards I^+ and the one ingoing to $r = 0$, this fact in principle allows him to detect the previously hidden correlations needed to restore purity at all the times.
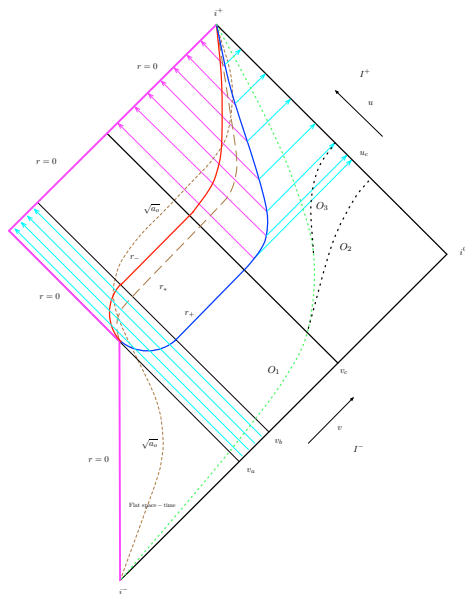
Figure 1. Penrose diagram for the formation and evaporation of the LBH. The red and dark blue solid lines depict the two trapping horizons r_- and r_+. The brown, dotted line is the curve of $r = \sqrt{a_o}$ and the brown, long dashed one is r_*. In the region $v < v_a$ we have a flat and empty region. For all times $v > v_a$, the inner and outer trapping horizons are present. These horizons join smoothly at $r = 0$ in an infinite time. A black hole begins to form at $v = v_a$ from null dust which has collapsed completely at $v = v_b$ to a static state with mass m_0. It begins to evaporate at $v = v_c$, and the complete evaporation takes an infinite time. The observer at I^+ sees particle emission set in at some retarded time u_c. The region with $v > v_c$ is then divided into a static region for $u < u_c$, and the Vaidya region for $u > u_c$, which is further subdivided into an ingoing and an outgoing part. The ingoing negative flux during evaporation (magenta arrows) has a flipped sign region.

3. Conclusions

We have calculated the particle creation by an entirely singularity-free LBH. The Schwarzschild surface gravity (proportional to the temperature) diverges when the black hole mass goes to zero. For the LBH instead it goes to zero with the mass but in an infinite time. The approximation used in [16] is that the massless scalar field obey the usual wave equations on an effective LBH metric, solution of Einstein equations with an effective energy-tensor with contributions that violate the positive energy condition and prevent the formation of a singularity. Even if the model is very simple and the analysis similar to the classical one, the result is that in an infinite time *the whole process, collapse and complete evaporation, is unitary.*

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