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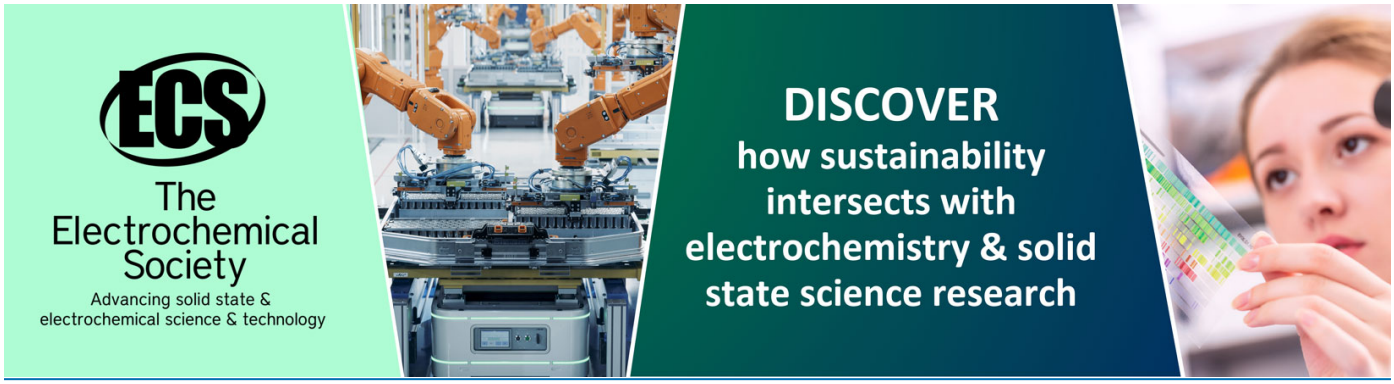

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Consistent probabilities in sLQC

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Abstract. We describe a complete, coherent framework within which probabilities may be consistently extracted from amplitudes for quantum histories in loop quantum cosmology. The decoherence functional is constructed for a loop quantization of a flat Friedmann-Lemaître-Robertson-Walker cosmological model, thereby permitting consistent quantum predictions to be made in a mathematically precise model of quantum cosmology. Consistent families of quantum histories are exhibited. Singularity avoidance by generic quantum states in loop-quantized models is contrasted with the failure of parallel Wheeler-DeWitt-quantized universes to avoid the big-bang singularity. It is shown that all loop quantum states approach arbitrarily large volume Wheeler-DeWitt states in both the asymptotic "past" and "future" of emergent time. The critical role played by decoherence of histories in these predictions is illustrated.

1. Introduction

The program of quantum cosmology is the application of a quantum theory of gravity[1] to arrive at quantum predictions for the behavior of cosmological models. Because the universe is homogeneous and isotropic on large scales, these models exhibit a high degree of symmetry. Quantum cosmology therefore serves in part also as a proving ground for candidate theories of quantum gravity because it allows ideas and techniques from the putative full theory to be applied to simplified models in which the classical symmetries are imposed prior to quantization. Loop quantum cosmology[2] (LQC) is a recent example of the successful application of this principle to loop quantum gravity (LQG). Perhaps the most striking result of LQC is the removal of the singularities of the corresponding classical model in a robust range of examples, in which the classical infinite-density "big bang" is replaced by a quantum "bounce" at the Planck density.

Easy to overlook in any application of a quantum theory of gravity to a cosmological model is that, in principle, the universe is a *closed system* – there is by definition nothing outside of it. In quantum mechanics, however, the orthodoxy is that probabilities may only be assigned to physical quantities which are *measured* by external observers, leading to the collapse of the wave function and the destruction of interference between alternatives. The canonical illustrative example is the two slit experiment. If the slit through which the particle passed is not measured, there is interference between the wave functions for the branches $\psi_{\text{upper}}(y)$ and $\psi_{\text{lower}}(y)$ passing through each slit and ending up at position y on the screen, and the probability $p(y)$ for ending up at y is not simply the sum of the probabilities for passing through each slit separately,

[1] See Ref. [1] for an overview of many of the currently viable candidates.
[2] See Ref. [2] for a thorough recent review.

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$p(y) = |\psi_{\text{upper}}(y) + \psi_{\text{lower}}(y)|^2 \neq |\psi_{\text{upper}}(y)|^2 + |\psi_{\text{lower}}(y)|^2$. In this sense it is not logically consistent to assert the particle "did" pass through one slit or the other if the slit was not measured: the putative probabilities simply do not add up.

2. Consistent histories

There is, nonetheless, a framework for making consistent predictions for closed quantum systems for which no notion of external observers or measurements is available. This is known as the *consistent* or *decoherent histories* formulation of quantum theory [3, 4, 5]. The main idea is to take seriously the observation that the principle function served by external measurements, or, more generally, environmental decoherence [6] in producing well-defined probabilities is the destruction of interference between alternatives: probabilities may be assigned when, and only when, interference between alternatives vanishes. The central mathematical constructions of the theory are *class operators* C_h which specify physically distinct alternative histories h from an exclusive, exhaustive set of alternatives $\{h\}$; *branch wave functions* $|\psi_h\rangle$ constructed from the intitial states and the class operator for the history h, and which correspond to the particular quantum state which has "followed" the history h; and the *decoherence functional* $d(h, h')$ which measures the quantum interference between the histories h and h'. For theories defined on a Hilbert space with a homogeneous background time, class operators corresponding to histories $h = (\Delta a_{k_1}, \Delta a_{k_2}, \ldots)$ for which observable α_1 is in the range Δa_{k_1} at time t_1; observable α_2 is in the range Δa_{k_2} at time t_2; and so on, are defined by products of Heisenberg projections onto the ranges of eigenvalues of interest:

$$C_h = P^{\alpha_1}_{\Delta a_{k_1}}(t_1) P^{\alpha_2}_{\Delta a_{k_2}}(t_2) \cdots P^{\alpha_n}_{\Delta a_{k_n}}(t_n). \tag{1}$$

The corresponding branch wave function is given by

$$|\psi_h\rangle = C_h^\dagger |\psi\rangle. \tag{2}$$

The decoherence functional is then defined as

$$d(h, h') = \langle \psi_{h'} \,|\, \psi_h \rangle. \tag{3}$$

Probabilities $p(h)$ for the histories $\{h\}$ may then be sensibly defined – in the sense that $\sum_h p(h) = 1$ on an exclusive, exhaustive set of alternative histories – when the interference between h and h' vanishes for all pairs of histories in the set, as measured by the decoherence functional. This probability is given by the Lüders-von Neumann formula: $p(h) = \langle \psi_h \,|\, \psi_h \rangle$. When the decoherence functional is diagonal on a complete set of alternative histories, $\langle \psi_{h'} \,|\, \psi_h \rangle = 0$ for $h \neq h'$, that set is said to "decohere" or "be consistent", and the probabilities of the individual histories h are given by the diagonal elements of the decoherence functional: $d(h, h') = p(h)\, \delta_{h'h}$. The decoherence functional thus provides an objective, internally defined measure for when probabilities may be consistently defined in a closed quantum system. We now apply this construction to loop quantum cosmology.

3. Loop quantum cosmology

The model we consider is a loop quantization of a flat Friedmann-Lemaître-Robertson-Walker cosmology with a massless, minimally coupled scalar field ϕ as a matter source [7, 8, 2, 9]. We refer to these references for further definitions and technical details. The classical solutions of this model are $\phi = \pm \ln(V/V_c)/\sqrt{12\pi G} + \phi_c$, corresponding to disconnected expanding and collapsing branches. Within the framework of loop quantum cosmology, quantization in the harmonic gauge leads to an exactly solvable model dubbed "sLQC" [8]. In terms of the rescaled volume variable $\nu = \varepsilon V/2\pi\gamma l_p^2$, where V is the volume of a fiducial spatial cell and $\varepsilon = \pm 1$, solutions $\psi(\nu, \phi)$

to the quantum Hamiltonian constraint split up into identical positive and negative frequency sectors. Restricting to positive frequencies, these states satisfy $-i\,\partial_\phi\psi(\nu,\phi) = \sqrt{\Theta}\,\psi(\nu,\phi)$, where Θ is the gravitational part of the constraint. In sLQC, however, Θ is a *difference* operator, and solutions may be chosen to have support only on the lattice $\nu = 4\lambda n$, with n an integer, where $\lambda = \sqrt{4\sqrt{3}\pi\gamma}$ is related to the "area gap" of loop quantum gravity. In this way, volume in sLQC becomes discrete, and the positive definite inner product given by group averaging is $\langle\psi\,|\,\chi\rangle = \sum_\nu \psi(\nu,\phi)^*\chi(\nu,\phi)$. It is seen that ϕ may for convenience be viewed as an emergent background "clock", and we will define evolution with respect to it via the propagator $U(\phi) = \exp(i\sqrt{\Theta}\phi)$.

General solutions to the constraint may be written

$$\Psi(\nu,\phi) = \int_{-\infty}^{\infty} dk\,\tilde{\Psi}(k)e_k^{(s)}(\nu)e^{i\omega_k\phi}, \tag{4}$$

where $\omega_k = \sqrt{12\pi G}|k|$ and the $e_k^{(s)}(\nu)$ are symmetric eigenfunctions of the evolution operator Θ with eigenvalues ω_k^2. Exact expressions for the $e_k^{(s)}(\nu)$ are available [7, 10, 11, 12], and much about them is known. Two pertinent facts are (i) the $e_k^{(s)}(\nu)$ approach at large volume a symmetric superposition of the corresponding expanding and contracting eigenfunctions of the Wheeler-DeWitt quantization of the same physical model; and (ii) the $e_k^{(s)}(\nu)$ exhibit an ultraviolet cutoff at $|k| = |\nu/2\lambda|$, decaying exponentially to zero for $|k|$ larger than that limit [13, 12].

Observables include $p_\phi = -i\hbar\partial/\partial\phi$, the momentum conjugate to the scalar field and a constant of the motion (since it commutes with $\sqrt{\Theta}$ via the constraint), and the volume at scalar field value ϕ^*, given by $\hat{\nu}_{|\phi^*}\,\psi(\nu,\phi) = U(\phi^*-\phi)^\dagger\,\hat{\nu}\,U(\phi^*-\phi)\psi(\nu,\phi)$. Defining Heisenberg-picture projection operators via the propagator $U(\phi)$, class operators, branch wave functions, and the decoherence functional in this model may be defined just as in Eqs. (1), (2) and (3). It is then straightforward to show, for example, that the probability the universe is found in a range of volume $\Delta\nu$ at ϕ is given by $p_{\Delta\nu}(\phi) = \sum_{\nu\in\Delta\nu}|\Psi(\nu,\phi)|^2$ [14].

It is known that these quantum cosmological models are non-singular, in the sense that the density is bounded above by approximately the Planck density [7, 8, 12], unlike the Wheeler-DeWitt quantization of the same model, in which states are invariably "sucked in" to the classical singularity in one of the limits $\phi \to \pm\infty$ [8, 9]. Indeed, numerical solutions [7] show that quasiclassical states peaked on a contracting classical solution "bounce" at small volume and thereafter remain peaked on an expanding classical solution.

In Ref. [9] the Wheeler-DeWitt quantization of the same model was studied from the consistent histories point of view. Taking the cue from the fact that the sLQC eigenstates are symmetric superpositions of expanding and contracting Wheeler-DeWitt eigenstates at large volume, the question of whether the quantum "bounce" in sLQC could be due to this superposition was investigated. A calculation of the probability that the volume of such a universe becomes arbitrarily small naively suggested that a bounce might indeed be possible for such a superposed state. A more careful consistent histories analysis showed, however, that this was not the case, and that Wheeler-DeWitt universes are invariably singular. This arose from the recognition that the physical statement that the universe "bounces" is an assertion about a *sequence* of values of ϕ – precisely the situation in which decoherence matters, just as in the two-slit experiment. It was then shown that the branch wave function for a bouncing cosmology is in fact zero in the limit $\phi \to \pm\infty$ in which the histories {bounce,singular} decohere.

Here, we show in a similar way that, unlike in the Wheeler-DeWitt model, *all* states in sLQC – quasiclassical or not – achieve arbitrarily large volume volume in both the "past" ($\phi \to -\infty$) and "future" ($\phi \to +\infty$). This is fairly easy to see. Indeed, if one chooses any arbitrary volume ν^* and defines $\Delta\nu^* = [-\nu^*, \nu^*]$ and $\overline{\Delta\nu^*}$ to be its complement, the probability the universe has volume less than ν^* at ϕ is $p_{\Delta\nu^*}(\phi) = \sum_{\nu\in\Delta\nu^*}|\Psi(\nu,\phi)|^2$. Because of the discreteness of

volume in sLQC and the UV cutoff in the $e_k^{(s)}(\nu)$, rapid oscillation of the exponential factor in Eq. (4) sends this probability to zero in both limits $\phi \to \pm\infty$. Since ν^* is arbitrary, this says the probability the universe assumes arbitrarily large volume as both $\phi \to \pm\infty$ is unity.

However, the statement that a quantum universe "bounces" is the statement that it has large volume *both* in the past and the future. As in the Wheeler-DeWitt case, it must be shown that the coarse-grained histories {bounce, singular} decohere before probabilities may be assigned. The class operator for the history in which the universe bounces is

$$C_{\text{bounce}} = \lim_{\substack{\phi_1 \to -\infty \\ \phi_2 \to +\infty}} P^{\nu}_{\overline{\Delta\nu_1^*}}(\phi_1) P^{\nu}_{\overline{\Delta\nu_2^*}}(\phi_2), \tag{5}$$

and for the complementary history C_{sing} in which the universe takes on zero volume at either or both of $\phi \to \pm\infty$, $C_{\text{sing}} = 1 - C_{\text{bounce}}$. In the same way as above it may be shown that $\lim_{\phi\to\pm\infty} P^{\nu}_{\Delta\nu^*}(\phi)|\Psi\rangle = 0$ and $\lim_{\phi\to\pm\infty} P^{\nu}_{\overline{\Delta\nu^*}}(\phi)|\Psi\rangle = |\Psi\rangle$. In this way, in the limit $\phi \to \pm\infty$, $|\Psi_{\text{bounce}}\rangle = C^{\dagger}_{\text{bounce}}|\Psi\rangle = |\Psi\rangle$ and $|\Psi_{\text{sing}}\rangle = C^{\dagger}_{\text{sing}}|\Psi\rangle = 0$, so that the histories {bounce, singular} decohere, and $p_{\text{bounce}} = 1$ and $p_{\text{sing}} = 0$ for all states in sLQC, completely the opposite of the case for the Wheeler-DeWitt quantization. We have thus shown in an objective, observer independent way, that a quantum bounce is inevitable in these models.[3]

4. Summary

We have described the consistent histories formulation of loop quantum cosmology, and showed how it can be used to arrive at consistent quantum predictions for histories of alternative outcomes without any need for observers or external measurements. By way of example, we have shown that arbitrary states in the theory – not merely quasiclassical ones – "bounce" from an arbitrarily large volume contracting Wheeler-DeWitt state in the "past" ($\phi \to -\infty$) to the corresponding symmetrically related large volume expanding Wheeler-DeWitt state in the "future" ($\phi \to +\infty$).

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References

[1] Kiefer C 2007 *Quantum gravity* 2nd ed (Oxford University Press, Oxford)
[2] Ashtekar A and Singh P 2011 *Class. Quant. Grav.* **28** 213001 (*Preprint* `1108.0893`)
[3] Omnès R 1994 *The interpretation of quantum mechanics* (Princeton University Press, Princeton)
[4] Griffiths R B 2008 *Consistent quantum theory* (Cambridge University Press, Cambridge)
[5] Hartle J B 1995 Spacetime quantum mechanics and the quantum mechanics of spacetime *Gravitation and Quantizations, Proceedings of the 1992 Les Houches Summer School* ed Julia B and Zinn-Justin J (North Holland, Amsterdam) (*Preprint* `gr-qc/9304006`)
[6] Schlosshauer M 2007 *Decoherence and the quantum-to-classical transition* (Springer-Verlag, Berlin)
[7] Ashtekar A, Pawlowski T and Singh P 2006 *Phys. Rev.* **D74** 084003 (*Preprint* `gr-qc/0607039`)
[8] Ashtekar A, Corichi A and Singh P 2008 *Phys. Rev.* **D77** 024046 (*Preprint* `0710.3565`)
[9] Craig D A and Singh P 2010 *Phys. Rev.* **D82** 123526–123546 (*Preprint* `1006.3837v1`)
[10] Ashtekar A, Campiglia M and Henderson A 2010 *Class. Quant. Grav.* **27** 135020 (*Preprint* `1001.5147v2`)
[11] Ashtekar A, Campiglia M and Henderson A 2010 *Phys. Rev.* **D82** 124043 (*Preprint* `1011.1024`)
[12] Craig D A 2012 Dynamical eigenfunctions and critical density in loop quantum cosmology (in preparation)
[13] Ashtekar A, Pawlowski T and Singh P 2006 *Phys. Rev.* **D73** 124038 (*Preprint* `gr-qc/0604013v3`)
[14] Craig D A and Singh P 2012 Consistent probabilities in loop quantum cosmology (in preparation)

[3] As an aside, we note that this is precisely the role of the $\phi \to \pm\infty$ limit in these calculations: to guarantee the results hold for all choices of state, and are therefore generic, certain predictions of the theory, rather than being particular to a sub-class of states such as quasi-classical ones.