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Seniority isomers in nuclei

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Abstract. Seniority isomers are nuclear states with an electromagnetic decay that is hindered by selection rules related to the seniority quantum number. A simple analysis is presented of their possible formation with reference to the nickel isotopes $^{70-76}$Ni and the $N = 50$ isotones from molybdenum to cadmium. It is shown that the existence of seniority isomers in a $j = 9/2$ shell is predominantly governed by the quadrupole pairing matrix element of the nucleon-nucleon interaction. The analysis is generalized to shells with larger j.

Isomers are metastable quantum states. In nuclei isomers generally adopt a configuration which is different from those of states at lower energy and their decay is therefore hindered. The type of configuration change determines the nature of the isomer and hence one distinguishes, for example, shape isomers, spin isomers, and K isomers [1]. Seniority isomers exist by virtue of a conserved quantum number, seniority (denoted as v), and its associated selection rules. Seniority, introduced by Racah in the context of atomic physics [2], refers to the number of particles that are not in pairs coupled to angular momentum $J = 0$. Generally, and in particular in semi-magic nuclei, states with low seniority occur at low energy. For example, the ground state of an even-even semi-magic nucleus has approximately $v \approx 0$ (all nucleons in pairs coupled to $J = 0$) while its yrast levels with angular momenta $J = 2, 4, 6, \dots$ have $v \approx 2$ (containing one 'broken' pair with $J \neq 0$) [3]. Seniority isomerism is expected to occur in semi-magic nuclei because electric quadrupole (E2) transitions between $v = 2$ states are small when the valence shell is close to half-filled. This result is a consequence of the fact that the matrix elements of even tensor operators—and hence also of the quadrupole operator—between states with the same seniority vanish at mid shell [3].

Examples of seniority isomers have been found in the $N = 50$ isotones with protons dominantly confined to the $\pi g_{9/2}$ shell. In particular, the $J^\pi = 8^+$ levels in ^{94}Ru ($Z = 44$) and ^{96}Pd ($Z = 46$) have half-lives of 71 and 2.2 μs, respectively, resulting from a combination of slow E2 decay and a small energy difference with the $J^\pi = 6^+$ level below it. A review is given by Grawe *et al.* [4]. On the basis of similar arguments one would expect the same phenomenon to occur in the neutron-rich nickel ($Z = 28$) isotopes ^{72}Ni and ^{74}Ni with neutrons dominantly confined to the $\nu g_{9/2}$ shell but this does not seem to be the case [5]. In this contribution a simple explanation of this observation is given. The occurrence of seniority isomers is analyzed in nuclei where valence nucleons of one type (*i.e.*, either neutrons or protons) are confined to a single shell with $j = 9/2$ and this analysis is subsequently generalized to shells with larger j.

The starting point of the discussion is that the seniority classification is a very good approximation for n identical nucleons in a $j = 9/2$ shell (see also chapter 21 of ref. [3]). This is a trivial statement if only one state exists for a given particle number n and angular momentum J, in which case seniority must be exact. If two states with different seniority and the same

Table 1. Coefficients $a^{\lambda}_{\alpha vJ}$ in the expansion (1) of diagonal energies of the $(9/2)^4$ system.

J^π	v	$\lambda = 0$	$\lambda = 2$	$\lambda = 4$	$\lambda = 6$	$\lambda = 8$
0^+	0	$\frac{8}{5}$	$\frac{1}{2}$	$\frac{9}{10}$	$\frac{13}{10}$	$\frac{17}{10}$
	4	—	$\frac{13}{66}$	$\frac{735}{286}$	$\frac{961}{330}$	$\frac{459}{1430}$
2^+	2	$\frac{3}{5}$	$\frac{142}{99}$	$\frac{67}{55}$	$\frac{442}{495}$	$\frac{102}{55}$
	4	—	$\frac{134}{99}$	$\frac{241}{143}$	$\frac{842}{495}$	$\frac{901}{715}$
4^+	2	$\frac{3}{5}$	$\frac{67}{99}$	$\frac{746}{715}$	$\frac{1186}{495}$	$\frac{918}{715}$
	4_{s}	—	$\frac{68}{33}$	1	$\frac{13}{15}$	$\frac{114}{55}$
	4	—	$\frac{47}{99}$	$\frac{300}{143}$	$\frac{1214}{495}$	$\frac{697}{715}$
6^+	2	$\frac{3}{5}$	$\frac{34}{99}$	$\frac{1186}{715}$	$\frac{658}{495}$	$\frac{1479}{715}$
	4_{s}	—	$\frac{19}{11}$	$\frac{12}{13}$	1	$\frac{336}{143}$
	4	—	$\frac{61}{198}$	$\frac{545}{286}$	$\frac{479}{198}$	$\frac{391}{286}$
8^+	2	$\frac{3}{5}$	$\frac{6}{11}$	$\frac{486}{715}$	$\frac{87}{55}$	$\frac{1854}{715}$
	4	—	$\frac{9}{22}$	$\frac{405}{286}$	$\frac{45}{22}$	$\frac{609}{286}$

Table 2. Coefficients b_J in the expression (2) of the off-diagonal matrix elements of the $(9/2)^4$ system.

J^π	0^+	2^+	4^+	6^+	8^+
b_J	$-\frac{1}{10\sqrt{429}}$	$\frac{\sqrt{14}}{495\sqrt{13}}$	$-\frac{\sqrt{314}}{6435}$	$\frac{\sqrt{281}}{6435}$	$-\frac{\sqrt{19}}{2145}$

J occur, it can be shown that for any reasonable nuclear interaction $\hat{V}$ the off-diagonal matrix element is small compared to the states' energy difference. The argument can be illustrated with the example of the four-particle system $(9/2)^4$ for which the diagonal energies are

$$E[j^4\alpha vJ] = \sum_\lambda a^{\lambda}_{\alpha vJ}\,\nu_\lambda, \tag{1}$$

while the off-diagonal matrix elements are

$$b_J(65\nu_2 - 315\nu_4 + 403\nu_6 - 153\nu_8), \tag{2}$$

with the coefficients $a^{\lambda}_{\alpha vJ}$ and b_J listed in tables 1 and 2. In these expressions $\nu_\lambda \equiv \langle j^2\lambda|\hat{V}|j^2\lambda\rangle$ are the two-body interaction matrix elements and α is a quantum number additional to v and J. If we consider as an example the two $J^\pi = 2^+$ states, we find that the 'typical' splitting between the seniority $v = 2$ and $v = 4$ levels is about 1 MeV while the off-diagonal matrix element (2) is only of the order of a few tens of keV.

The proof that seniority mixing is negligible for the $J^\pi = 4^+$ and 6^+ states of a $(9/2)^4$ system is more subtle and is crucial for the existence of seniority isomers. There are *three* states for each of these angular momenta, two of which, with $v = 2$ and $v = 4$, are close in energy and could possibly strongly mix. The $v = 4$ members of these closely-spaced doublets are, however, the so-called 'solvable' $J^\pi = 4^+$ and 6^+ states discussed in refs. [6, 7, 8] which *have exact seniority*

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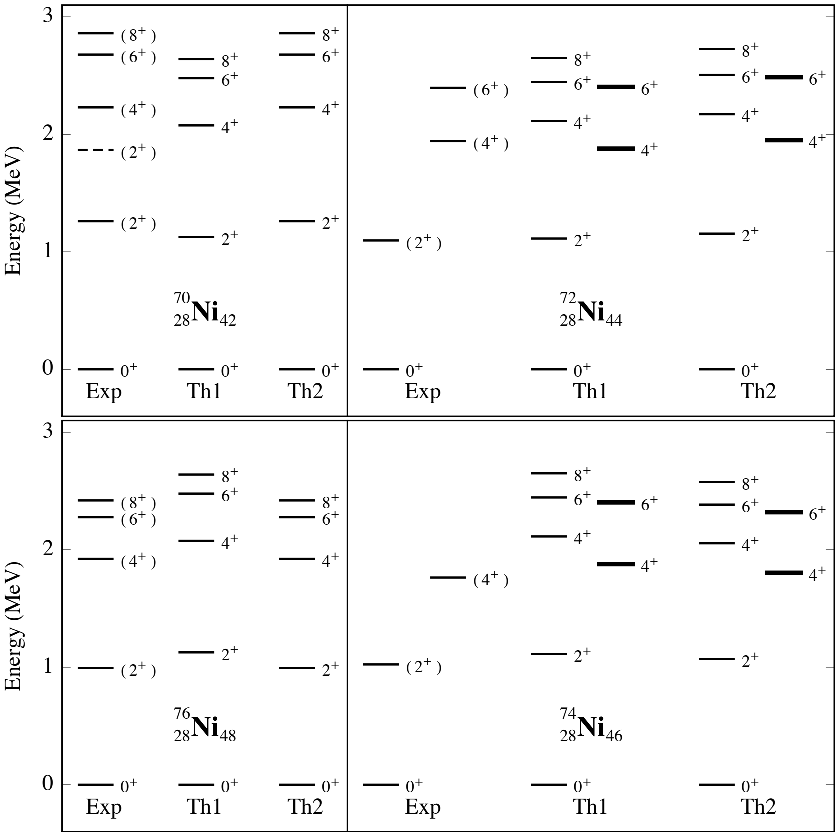

Figure 1. The low-energy spectra of the nickel isotopes $^{70-76}$Ni. The left columns show the observed levels while the columns 'Th1' and 'Th2' contain the results of a $(\nu g_{9/2})^n$ shell-model calculation with constant or linearly varying two-body matrix elements, respectively. The two solvable $J^\pi = 4^+$ and $J^\pi = 6^+$ levels with seniority $v = 4$ are shown with thick lines; the dashed line corresponds to an intruder level.

$v = 4$ *for any interaction* (denoted as $v = 4_{\rm s}$ in table 1). As a consequence, breaking of seniority only arises through the mixing matrix elements (2) between the $v = 2$ and the higher-lying $v = 4$ states and, by the same argument as above, this mixing is found to be small.

The spectra of the nickel isotopes from ^{70}Ni to ^{76}Ni are compared in fig. 1 to the observed levels [9, 10]. The nucleus ^{70}Ni should, in the approximation of an isolated $\nu g_{9/2}$ shell, display a two-neutron-particle spectrum $(\nu g_{9/2})^2$ with excited states with $J^\pi = 2^+, 4^+, 6^+$ and 8^+. This is indeed found to be the case except for the additional (intruder) $J^\pi = (2^+)$ level at 1867 keV. The nucleus ^{76}Ni displays a two-neutron-hole spectrum $(\nu g_{9/2})^{-2}$ with the same yrast sequence as in ^{70}Ni but no known intruder state. The two-particle and the two-hole spectra fix the differences $\nu_{\lambda-0} \equiv \nu_\lambda - \nu_0$. This is done at two levels of sophistication by taking either constant matrix elements that are the average of those in ^{70}Ni and ^{76}Ni (Th1) or by letting them vary linearly from ^{70}Ni to ^{76}Ni (Th2). In the latter approximation the spectra of the two-particle and the two-hole nuclei are exactly reproduced [except for the intruder $J^\pi = (2^+)$ state in ^{70}Ni]; the description of the two intermediate isotopes, ^{72}Ni and ^{74}Ni, should be rather accurate, albeit very phenomenological.

The two solvable $J^\pi = 4^+$ and $J^\pi = 6^+$ levels with seniority $v = 4$ are shown with thick lines. A noteworthy feature of the calculated spectra of the isotopes ^{72,74}Ni is the occurrence of *two* levels for $J^\pi = 4^+$ and for $J^\pi = 6^+$ which are very close in energy, especially in the latter case.

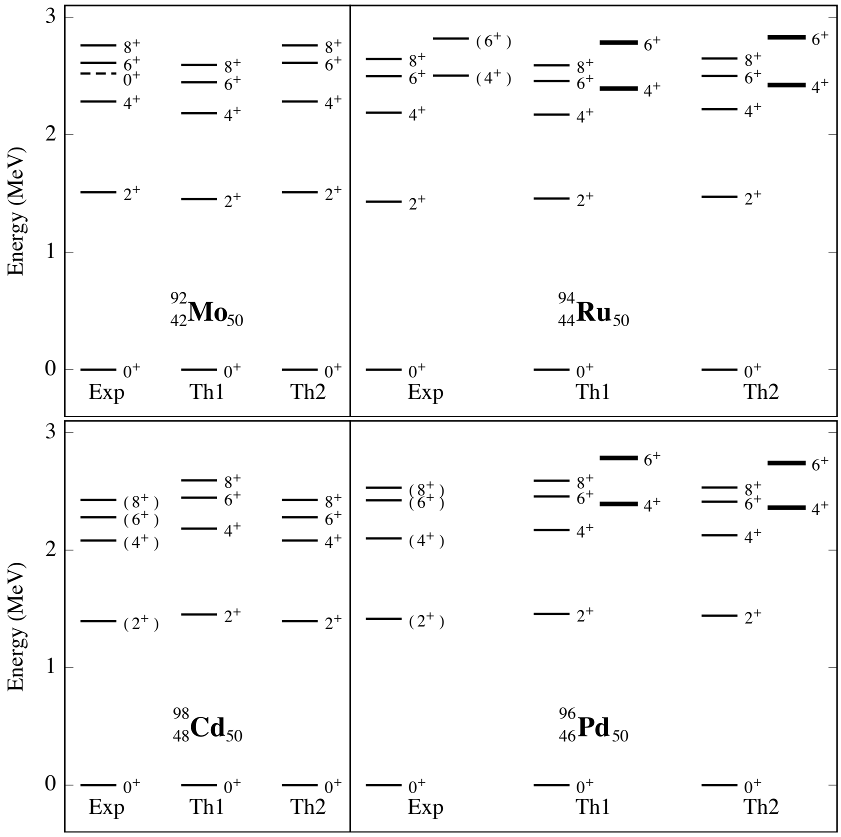

Figure 2. The low-energy spectra of the $N = 50$ isotones ^{92}Mo, ^{94}Ru, ^{96}Pd and ^{98}Cd. The left columns show the observed levels while the columns 'Th1' and 'Th2' contain the results of a $(\pi g_{9/2})^n$ shell-model calculation with constant or linearly varying two-body matrix elements, respectively. The two solvable $J^\pi = 4^+$ and $J^\pi = 6^+$ levels with seniority $v = 4$ are shown with thick lines; the dashed line corresponds to an intruder level.

This is a direct consequence of the solvability of one member of each doublet which cannot mix with the close-lying, predominantly $v = 2$ state with the same spin. At least for the $J^\pi = 6^+$ levels, this feature is still clearly present in the large-scale shell-model calculations of Lisetskiy *et al.* [11].

A similar analysis in the same approximation can be performed for the $N = 50$ isotones with protons in the $\pi g_{9/2}$ shell. In fig. 2 the results of the calculation are compared with the observed spectra [12, 13, 14, 15]. In ^{94}Ru (4_2^+) and (6_2^+) levels are observed at energies of 2503 and 2818 keV, respectively [13]; these are possible candidates for the solvable states which are calculated at 2422 and 2828 keV in the approximation 'Th2'. It would be of interest to confirm the spin assignment and to determine the E2 transition probability between the two levels.

There is a striking difference between the calculated four-particle (or four-hole) spectra of the nickel isotopes and those of the $N = 50$ isotones: the solvable $J^\pi = 4^+$ and $J^\pi = 6^+$ states are yrast in ^{72}Ni and ^{74}Ni while they are yrare in ^{94}Ru and ^{96}Pd. For this reason one may conjecture that the observed yrast (4^+) and (6^+) levels in ^{72}Ni and the observed yrast (4^+) level in ^{74}Ni are the solvable states in question, as is done in fig. 1. This drastically changes the E2-decay pattern as is illustrated in fig. 3. On the left-hand side is shown the 'typical' decay with small $B(\mathrm{E2})$ values between $v \approx 2$ states which is characteristic of the seniority classification in nuclei near mid-shell ($n \approx j + 1/2$) and which is at the basis of the explanation of seniority isomers [4].

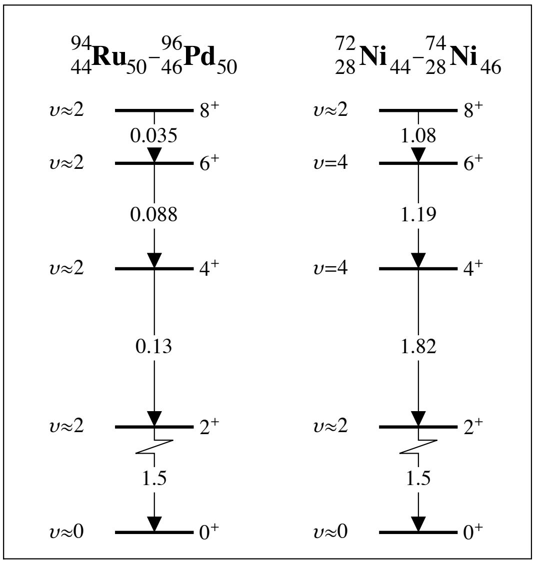

Figure 3. E2 decay between yrast states of the $(9/2)^4$ system as expected in the $N = 50$ isotones (left) and the $Z = 28$ nickel isotopes (right). The numbers between the levels denote calculated B(E2) values expressed in units of $B(\mathrm{E2}; 2^+_1 \to 0^+_1)$ of the two-particle system, assuming a seniority-conserving interaction. All levels have $v \approx 2$, except the $J^\pi = 4^+$ and 6^+ states in ^{72,74}Ni which have exact $v = 4$.

This situation applies to ^{94}Ru and ^{96}Pd where the $v \approx 2$ states are yrast. On the right-hand side of fig. 3 is shown the E2 decay pattern as it is calculated in ^{72}Ni and ^{74}Ni where the yrast $J^\pi = 4^+$ and 6^+ levels have $v = 4$, leading to fast E2 decay in the yrast band. As a consequence, the $J^\pi = 8^+$ and 6^+ levels are unlikely to be isomeric in this case.

Given that approximate analytic expressions are available for the energies of all relevant states, conditions for the existence of isomers can easily be established. Coefficients $a^\lambda_{\alpha vJ}$ crucial to the existence of isomers are related to the quadrupole pairing matrix element which has $\lambda = 2$ (see table 1). Although conditions can be derived in general, results become more transparent by averaging over matrix elements with $\lambda \neq 0, 2$ and introducing $\bar{\nu} \equiv (\nu_{4-0} + \nu_{6-0} + \nu_{8-0})/3$. (This procedure is justified since the three matrix elements are close in value for any reasonable nuclear interaction.) With the help of table 1 one shows that the condition to have yrast $J^\pi = 4^+$ and 6^+ levels with $v = 2$ is

$$\nu_{2-0} \gtrsim \frac{388}{685}\bar{\nu} \approx 0.57\bar{\nu}. \tag{3}$$

A second set of conditions for the existence of seniority isomers is obtained by requiring that the $v = 4$ state with angular momentum $J - 2$ is higher in energy than the $v = 2$ state with angular momentum J. This leads to the conditions

$$\nu_{2-0} \gtrsim \frac{553}{850}\bar{\nu} \approx 0.65\bar{\nu}, \qquad \nu_{2-0} \gtrsim \frac{32}{65}\bar{\nu} \approx 0.49\bar{\nu}, \tag{4}$$

necessary for the existence of a $J^\pi = 6^+$ and a $J^\pi = 8^+$ isomer, respectively. Since both conditions (3) and (4) are necessary, we conclude that $\nu_{2-0} \gtrsim 0.57\bar{\nu}$ suffices for the existence of a $J^\pi = 8^+$ isomer while $\nu_{2-0} \gtrsim 0.65\bar{\nu}$ is required for the isomeric character of the $J^\pi = 6^+$. Obviously, these are only approximate conditions, derived under several simplifying assumptions, but they are borne out by the observations in the $N = 50$ isotones and the nickel isotopes, that is, a $J^\pi = 8^+$ isomer exists in the former [4] and so far none is observed in the latter [5].

Table 3. Coefficients $a^{\lambda}_{\alpha vJ}$ (with $\alpha=[II']$) in the expansion (1) of the diagonal energies of the $(11/2)^4$ system.

J^π	v	$[II']$	$\lambda=0$	$\lambda=2$	$\lambda=4$	$\lambda=6$	$\lambda=8$	$\lambda=10$
0^+	0	[00]	1.667	0.333	0.600	0.867	1.133	1.400
	4	[22]	—	0.408	2.695	0.061	2.234	0.602
2^+	2	[02]	0.667	1.231	0.952	0.803	0.693	1.654
	4	[22]	—	1.513	0.984	1.098	1.246	1.158
4^+	2	[04]	0.667	0.529	0.926	0.989	1.976	0.913
	4	[22]	—	1.908	0.873	0.792	0.609	1.818
6^+	2	[06]	0.667	0.309	0.684	1.851	1.028	1.461
	4	[24]	—	1.494	1.036	0.717	0.911	1.842
8^+	2	[08]	0.667	0.204	1.046	0.786	1.575	1.722
	4	[26]	—	0.579	1.078	1.082	1.873	1.388
10^+	2	[0, 10]	0.667	0.394	0.391	0.904	1.394	2.249
	4	[28]	—	0.623	0.622	1.340	1.372	2.043

These findings can be generalized to shells with larger j. For $j=9/2$ the lowest $v=4$ states with $J^\pi=4^+$ and 6^+ have a constant structure, independent of the interaction. They can be written as

$$\begin{aligned}|(9/2)^4, v=4_{\rm s}, J=4\rangle &= \sqrt{\frac{25500}{25591}}|(9/2)^4[22], v=4, J=4\rangle_{\rm GS}\\ &\quad -\sqrt{\frac{91}{25591}}|(9/2)^4[24], v=4, J=4\rangle_{\rm GS},\\ |(9/2)^4, v=4_{\rm s}, J=6\rangle &= \sqrt{\frac{27132}{27257}}|(9/2)^4[24], v=4, J=6\rangle_{\rm GS}\\ &\quad +\sqrt{\frac{125}{27257}}|(9/2)^4[26], v=4, J=6\rangle_{\rm GS}.\end{aligned} \tag{5}$$

(The states are identical to those of eq. (8) of ref. [7] but written in an orthonormalized basis.) The labels $[II']$ are intermediate angular momenta and indicate that the state has been obtained by anti-symmetrization of $|j^2(I)j^2(I');J\rangle$. The states resulting from this anti-symmetrization have been further ortho-normalized by a Gram-Schmidt procedure, as indicated by the subscript 'GS'. The expansion (5) shows that the dominant component of the solvable $J^\pi=4^+$ state with $v=4_{\rm s}$ is $(D^\dagger\times D^\dagger)^{(4)}|{\rm o}\rangle$, that is, two $J=2$ pairs coupled to total angular momentum $J=4$, where it is understood implicitly that this two-pair state is orthogonalized to the $v=2$ state. For $J^\pi=6^+$, the dominant component is $(D^\dagger\times G^\dagger)^{(6)}|{\rm o}\rangle$. This shows that specific two-pair states $[II']$ are a good approximation to the exact $(9/2)^4$ eigenstates and this property can be used to generalize to higher j. The coefficients $a^{\lambda}_{\alpha vJ}$ for the two-pair states of relevance in the $(11/2)^4$ system are given in table 3. Such coefficients are rational numbers [16]; since they involve large integers, approximate numerical values are given in the table.

Pure $(11/2)^n$ configurations are found in the $N=82$ isotones with protons above $Z=64$ confined to the $\pi h_{11/2}$ shell and taking ^{146}Gd as a closed core [3]. The energy spectrum of the two-particle nucleus ^{148}Dy fixes the two-body interaction. The exact shell-model diagonalization of this interaction for the four-particle nucleus ^{150}Er yields eigenstates that are close to the ones obtained in a two-pair approximation. All exact eigenstates have overlaps of more than 99% with the two-pair states shown in table 3, except for the $J^\pi=8^+_2$ state which turns out to be a complicated mixture of several two-pair states. Conditions for the existence of seniority isomers can be established for the $j=11/2$ shell in the same spirit as for $j=9/2$, and one finds

Table 4. B(E2) values (in units e^2fm^4) observed in $N = 82$ isotones compared with $(11/2)^n$ shell-model results that are exact (SM-1) or in a pair approximation (SM-2).

	$B(\mathrm{E2}; 10^+_1 \to 8^+_1)$			$B(\mathrm{E2}; 8^+_1 \to 6^+_1)$		
	Exp	SM-1	SM-2	Exp	SM-1	SM-2
$A = 148$	43 (4)	43^a	43^a	79 (23)	115	115
$A = 150$	11.4 (1.4)	10.6	10.7	~ 38	27	23
$A = 152$	0.96 (0.14)	0	0	—	0	0

[a] Normalized value.

that the $J^\pi = 10^+$ level is always isomeric while 6^+ and 8^+ are isomeric if $\nu_{2-0} \gtrsim 0.58\bar{\nu}$ and $\nu_{2-0} \gtrsim 0.48\bar{\nu}$, respectively, where $\bar{\nu} \equiv (\nu_{4-0} + \nu_{6-0} + \nu_{8-0} + \nu_{10-0})/4$. The isomeric character of the $J^\pi = 10^+$ and 8^+ levels is established experimentally [17] in ^{148}Dy, ^{150}Er and ^{152}Yb (only 10^+ in the latter), and the resulting B(E2) values are summarized in table 4. The pair approximation (SM-2) is seen to be close to the exact shell-model calculation (SM-1) which in turn is close to the observed E2 decays. It would be of interest to search for isomers in the heavier $N = 82$ isotones ^{154}Hf and ^{156}W. Given that this analysis *presupposes* an isolated $\pi h_{11/2}$ shell, the detection of seniority isomers would be an indication of the doubly magic nature of the nucleus ^{158}Os, akin to ^{146}Gd.

In summary, a simple analysis has been presented of the possible formation of seniority isomers in nuclei, which shows that their existence is predominantly governed by the quadrupole pairing matrix element of the nucleon-nucleon interaction.

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