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# Over spinning a black hole?

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**Abstract.** A possible process to destroy a black hole consists on throwing point particles with sufficiently large angular momentum into the black hole. In the case of Kerr black holes, it was shown by Wald that particles with dangerously large angular momentum are simply not captured by the hole, and thus the event horizon is not destroyed. Here we reconsider this gedanken experiment for black holes in higher dimensions. We show that this particular way of destroying a black hole does not succeed and that Cosmic Censorship is preserved.

## 1. Introduction

Black holes can be formed through the collapse of matter, through sufficiently high-energy collisions of particles or quantum fluctuations in the early universe. Basically any process capable of confining a large portion of matter in a small enough space. Once formed, black holes are hard to kill. Quantum processes aside, no known classical mechanism can destroy a black hole. One of such processes was considered by Wald [1] many years ago and revisited recently [2, 3, 4, 5, 6]. It consists in throwing a point particle at a (four-dimensional) Kerr black hole with just the right angular momentum to spin the black hole up in such a way that eventually the horizon is disrupted. Indeed, the angular momentum of Kerr black holes is bounded by  $J \leq M^2$ , thus *if it were possible* for the black hole to capture particles of high enough angular momenta, then one might exceed this bound, possibly creating a naked singularity. Wald showed this cannot happen, as the potentially dangerous particles (i.e., those with large enough angular momentum) are never captured by the black hole [1].

The purpose of this short letter is to extend Wald's analysis to other spacetimes, in particular the Myers-Perry family of rotating black holes in higher dimensions [7]. Such an analysis is interesting because it allows one to test Cosmic Censorship in a very simple, yet realistic scenario.

## 2. Effective potential for radial motion of a test particle

We are interested in analysing the motion of a test particle in a D-dimensional rotating black hole. More precisely, we will be considering an asymptotically flat stationary black hole described by the Myers-Perry solution [7]. Unlike Kerr black hole, where there is only one possible rotation axis, and therefore there is only one angular momentum parameter, in higher dimensions there are several choices of rotation axis and there is a multitude of angular momentum parameters,

each referring to a particular rotation plane [7]. Here, we will focus on two possible situations: (i) a black hole with all the angular momenta equal or (ii) a singly spinning black hole. In addition, we focus exclusively on the intuitively most dangerous process: particles falling in along the equator. Therefore, we use the effective “2+1” dimensional metric along the equatorial plane. It is as well easy to define the conserved energy,  $E$ , and the angular momentum  $L$  (per unit test-particle mass  $m_0$  in the case of time-like geodesics <sup>1</sup>) associated to the time-like and rotational Killing vectors of the background geometry.

The equatorial motion in the geometry just described can be reduced to the radial equation  $\dot{r}^2 = V_r$  [8, 9]. For a single rotational plane

$$r^2 V_r = \left[ r^2 E^2 + \frac{M}{r^{D-3}} (aE - L)^2 + (a^2 E^2 - L^2) - \delta_1 \Delta_D \right], \quad (1)$$

where  $\Delta_D = r^2 + a^2 - Mr^{5-D}$ . Similar equations can be written when all angular momenta are equal. For instance, for even  $D = 2(d+1)$  we find,

$$r^3 V_r = M (r^2 + a^2)^{(4-D)/2} \left[ r^2 \delta_1 + (L - aE)^2 \right] + r \left[ (r^2 + a^2)(E^2 - \delta_1) - L^2 \right], \quad (2)$$

while for odd  $D = 2d+1$  we obtain

$$r^2 V_r = M (r^2 + a^2)^{(3-D)/2} \left[ r^2 \delta_1 + (L - aE)^2 \right] + \left[ (r^2 + a^2)(E^2 - \delta_1) - L^2 \right]. \quad (3)$$

On the previous equations  $\delta_1 = 1, 0$  for timelike and null geodesics, respectively, and a dot stands for derivative respect to the proper time of the particle. The parameters  $a$  and  $M$  are related to the angular momentum and mass of the black hole (cf. Ref. [9])

The radial motion is completely governed by the potential  $V_r$ . If there are turning points outside the event horizon, then a particle coming from infinity can not reach the event horizon. Thus, the analysis we want to make is to study the maximum value of  $L$  for which there are either no turning points, or all of them lie inside the event horizon.

### 3. Spinning-up a black hole by throwing point particles

Now, we address the main issue of this paper: can we spin-up a black hole with mass  $\mathcal{M}_0$  and angular momentum  $\mathcal{J}_0$  in general  $D$  spacetime dimensions? For that, we throw in a particle of mass  $m_0$  with angular momentum  $\delta \mathcal{J} = m_0 L$  and energy  $\delta \mathcal{M} = m_0 E$ , such that  $\delta \mathcal{M} \ll \mathcal{M}_0$  and  $\delta \mathcal{J} \ll \mathcal{J}_0$ . Upon absorption of this particle, the dimensionless spin of the black hole

$$j \equiv \frac{\mathcal{J}}{\mathcal{M}^{\frac{D-2}{D-3}}}, \quad (4)$$

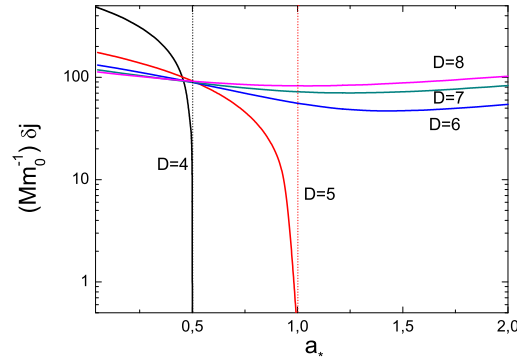
changes to

$$j = j_0 + \delta j, \quad (5)$$

where the subscript stands for initial parameters of the black hole and

$$\delta j = \frac{m_0}{\mathcal{M}_0} \left( \frac{L}{\mathcal{M}_0^{\frac{1}{D-3}}} - E j_0 \frac{D-2}{D-3} \right). \quad (6)$$

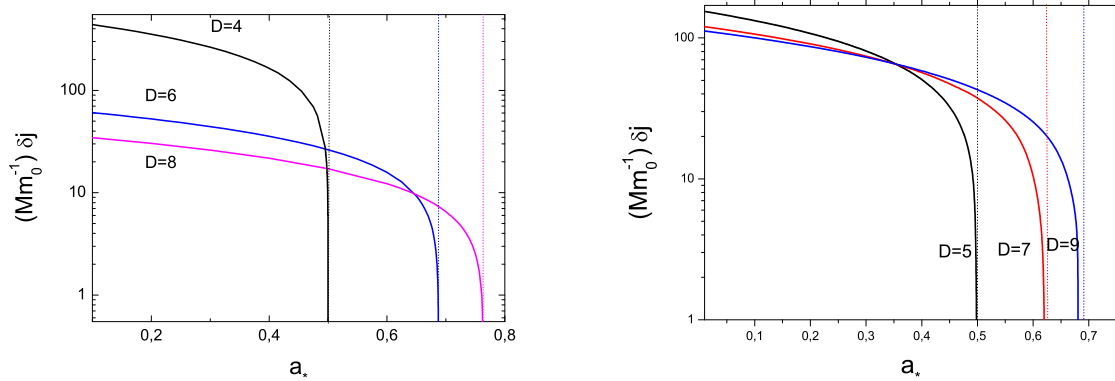
<sup>1</sup> For massless particles, the quantities  $E$  and  $L$  may be regarded as the energy and angular momentum.



**Figure 1.** This figure shows the maximum increase in spin,  $\frac{M}{m_0}(\delta j)_{\max}$  caused by a particle with  $E/m_0 = 100$  falling into a Myers-Perry black hole with a single rotation parameter. The dimensionless rotation parameter  $a_*$  is defined as  $a_* \equiv \frac{a}{M^{1/(D-3)}}$ . Notice that it is not possible to spin-up an extremal black hole (for  $D = 4, 5$ , the extremal value is marked with a dotted line).

### 3.1. Single rotation parameter

As in four dimensions, in  $D = 5$  we can also spin the black hole to the extremal limit and not further than that [10, 9]. What about general  $D$ ? We have numerically searched for the critical angular momentum, and computed  $\delta j$  in Eq. (6). The results, which are summarised in Fig. 1, are clear: neutral black holes in four and five spacetime dimensions with a single rotation cannot be spun-up past extremality. For larger  $D$ , there is no extremal limit, and the black holes can be spun-up to an arbitrarily high rotation.



**Figure 2.** This figure shows the maximum increase in spin,  $\frac{M}{m_0}(\delta j)_{\max}$  caused by  $d$  particles with  $E/m_0 = 100$  falling into a Myers-Perry black hole with all rotation parameters equal. The left panel considers even  $D$ , the right panel refers to odd spacetime dimensions  $D$ . It is not possible to spin-up an extremal black hole, marked with a dotted line. The dimensionless rotation parameter  $a_* \equiv \frac{a}{M^{1/(D-3)}}$ .

### 3.2. All angular momenta equal

As we are dealing with a Myers-Perry black hole with equal angular momenta, by throwing only one point particle, we would end up with a Myers-Perry black hole with different angular momenta where the condition of all angular momenta being equal does not apply and we could conclude erroneously a violation of the cosmic censorship. How can we avoid this situation? Instead of throwing in one test particle, we consider  $d$  particles following similar geodesics along the  $d$  orthogonal rotation planes. The final black hole will also have all angular momenta equal. In this situation the expression (6) should read

$$\delta j = \frac{m_0}{\mathcal{M}_0} \left( \frac{L}{\mathcal{M}_0^{\frac{1}{D-3}}} - dEj_0 \frac{D-2}{D-3} \right), \quad (7)$$

and the results are presented in Fig. 2. In full analogy with the singly spinning case in  $D = 5$  in which the spin is bounded, we cannot exceed the extremal limit by throwing in test particles.

## 4. Conclusions

We have shown that a D-dimensional Myers-Perry black hole is immune to the throwing of point particles: in the geodesic approximation employed here, particles which are captured by the black hole have an angular momentum which is sufficiently low so as to be harmless; in fact sufficiently low that they are never able to spin-up the geometry past the extremal value.

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