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# Image processing of full-field strain data and its use in model updating

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Abstract. Finite element model updating is an inverse problem based on measured structural outputs, typically natural frequencies. Full-field responses such as static stress/strain patterns and vibration mode shapes contain valuable information for model updating but within large volumes of highly-redundant data. Pattern recognition and image processing provide feasible techniques to extract effective and efficient information, often known as shape features, from this data. For instance, the Zernike polynomials having the properties of orthogonality and rotational invariance are powerful decomposition kernels for a shape defined within a unit circle. In this paper, full field strain patterns for a specimen, in the form of a square plate with a circular hole, under a tensile load are considered. Effective shape features can be constructed by a set of modified Zernike polynomials. The modification includes the application of a weighting function to the Zernike polynomials so that high strain magnitudes around the hole are well represented. The Gram-Schmidt process is then used to ensure orthogonality for the obtained decomposition kernels over the domain of the specimen. The difference between fullfield strain patterns measured by digital image correlation (DIC) and reconstructed using 15 shape features (Zernike moment descriptors, ZMDs) at different steps in the elasto-plastic deformation of the specimen is found to be very small. It is significant that only a very small number of shape features are necessary and sufficient to represent the full-field data. Model updating of nonlinear elasto-plastic material properties is carried out by adjusting the parameters of a FE model until the FE strain pattern converges upon the measured strains as determined using ZMDs.

## 1 Introduction

Finite element model validation involves the use of inverse methods based on structural responses – i.e. comparing numerical predictions to experimental measurements to check the extent to which the model represents the real structure. Typical structural responses include natural frequencies, mode shapes and strain components. The conventional measurement methods are point-wise, e.g. a grid of accelerometers for measuring vibration mode shapes; electrical strain gauges for measuring strain at points of interest. The number of measured points and their selection [1-3] are important issues.

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The availability of a number of optical techniques [4] including digital image correlation, automated photoelasticity, electronic speckle pattern interferometry and thermoelastic stress analysis permit the full-field measurement of the structural responses. The captured full-field responses enable easy qualitative assessment of the similarities between the predictions and measurements [5]. Finite element model updating requires the calculation of structural-response sensitivities with respect to updating parameters. It is important that the problem of estimating the updated parameters is well conditioned, usually by ensuring that the system of equations is over determined. However, the full-field data are always of great number and highly redundant. It is necessary to compress the full-field data in a sensible way.

A number of image processing techniques have been investigated by Wang et al. [6-8] to compress the full-field data and applied in FE model updating by using vibration mode shapes. The image moment is one of the most popular techniques. Hu [9] introduces the concept of geometric image moment in 1960 by using the bivariate power functions as kernel functions for the integral transform. In 1980 Teague proposed orthogonal image moments by adopting orthogonal polynomials as kernel functions, i.e. the Zernike polynomials for the polar coordinates and the Legendre polynomials for Cartesian coordinates. Orthogonality leads to advantages such as transformation invariance, independence and easy reconstruction. However, evaluation the orthogonal moment of a digital image involves discretisation of the polynomials. This inevitably introduces computational errors and can be overcome by using discrete orthogonal polynomials. Mukundan et al [10] suggests the Tchebichef polynomials which are ideal for extracting global information from digital images. Yap et al [11,12] propose the Krawtchouk polynomials and Hahn polynomials. The Krawtchouk polynomials are suitable for detecting and describing the local features. The Hahn polynomials behave in-between the Tchebichef and the Krawtchouk polynomials by proper setting of the parameters of the functions. The discrete orthogonal polynomials mentioned previously are defined on uniform lattices. It is possible to adopt discrete orthogonal polynomials defined on non-uniform lattices [13] – this might be usefully for extracting shape features from the full-field measurement captured non-uniformly - e.g. the Racah polynomials.

The selection of an image moment descriptor is problem dependent. e.g. the Zernike moment descriptor is preferable for extracting cyclically symmetric patterns. Proper selection may result in effective and succinct descriptors. However, it is no always possible to perfectly select the image decomposition kernels. Appropriate modifications of existing kernel functions are feasible to solve the particular problem in hand. The orthogonality of the polynomials is satisfied only on their corresponding definition domain. In real applications, the full-field measured domains do not match perfectly with the domain of the chosen descriptor. The Gram-Schmidt orthogonalisation (GSO) process is a useful tool to regain the orthogonality [14]. Once efficient shape features are determined, the comparison of redundant full-field data with concise shape feature vectors can be made and discrepancies assessed in the geometric space.

In this paper, full-field strain measurements are made using a digital image correlation system on an aluminium plate with a circular hole under tensile loading as discussed in Section 2. The noise is considered to be the minimum measurement error from the calibration [17]. In Section 3, a special set of orthogonal decomposition kernels was constructed based on the classical circular Zernike polynomials. The modification to the Zernike polynomials includes coordinate transformation and weighting to the radial function. The parameters of the kernel functions are then optimised by using Newton's iterative method. Shape features of the full-field strain pattern are extracted by the modified Zernike polynomials. The results show that only a small number of feature terms are significant and necessary to be retained for reconstruction. The development of the strain response with respect to the tensile loading could also be characterised by succinct shape features. Noisy DIC measurements have virtually no effect on the estimated ZMDs since the characteristic dimension of the noise is short and the ZMDs of interest have long wavelengths. A FE model was parameterised and updating based on shape features in Section 4. Conclusions are drawn in Section 5.

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## 2 Full-field strain pattern measured by DIC

A quasi-static tensile test was carried out on the specimen as shown in Figure 1. Full-field measurements of strain were captured by using a Digital Image Correlation system (Istra 4D, Dantec Dynamics, Ulm, Germany) following the loading history shown in Figure 2. A two dimensional system was used with a single camera, which provides the in-plane components of displacement and strain. The resolution of the detector is 1040x1392 pixels, approximately 970x970 pixels of which covered the area of interest of the specimen. This produces a spatial resolution of 12.9 pixels per mm. The images were processed using a facet size of 25 pixels, with a 5 pixel offset between facets. The uncertainty for the DIC system for strains up to 2.11 mm/m was 1.4-1.7 percent [17]. Maximum principal strain patterns at 6.4, 12.9, 16.1 and 17.5kN, as circled along the loading history curve in Figure 2, are shown in the top row of Figure 3. Each map contains 23,622 data points at which a measurement was made. A finite element model for the specimen was simulated using ABAQUS as shown in Figure 4. To compare the detailed FE prediction with the full-field DIC measurements, shape feature decomposition from strain maps is carried out first.



**Figure 1.** Test specimen - 1.016mm thick Aluminum; (a) dimension of the aluminium specimen; (b) speckle pattern sprayed on the specimen



Figure 2. Loading history for the experimental test with the location at which data from the DIC were sampled shown as red circles.

## **3** Construction of shape decomposition kernels

A special set of orthogonal decomposition kernel functions were constructed based on the Zernike polynomials. In order to extract efficient and succinct shape features from the strain maps, modifications of the Zernike polynomials including coordinate transform and weighting functions



were introduced. Also, the Gram-Schmidt orthogonalisation process was considered to retain the orthogonality of the modified kernel functions.

**Figure 3.** Maps of maximum principal strain obtained using digital image correlation (*top*); their reconstructions from modified Zernike moment descriptors with 15 terms retained (*middle*) and the residual, i.e. the difference between the original and reconstructed maps (*bottom*). – color bar scale is strain in mm/m

## 3.1 Orthogonal decomposition

The general form of orthogonal decomposition of a full-field response  $S(x, \mathbf{\varsigma})$ , e.g. a strain map, can be expressed as

$$S(\mathbf{x}, \mathbf{\varsigma}) = \sum_{i}^{\infty} \mathfrak{D}_{i}(\mathbf{\varsigma}) \mathfrak{K}_{i}(\mathbf{x})$$
(1)

where  $\mathfrak{K}_i(\mathbb{X})_{i=1,2,\dots,\infty}$  is the orthonormal space,  $\mathbb{X}$  is the spatial coordinate (could be 1D, 2D or 3D),  $\boldsymbol{\varsigma}$  are the parameters (e.g. time, force amplitude etc.) and  $\mathfrak{D}_i(\boldsymbol{\varsigma})$  is projection of  $\mathcal{S}(\mathbb{X},\boldsymbol{\varsigma})$  onto the  $i^{th}$  kernel function  $\mathfrak{K}_i(\mathbb{X})$  obtained by

$$\mathfrak{D}_{i}(\boldsymbol{\varsigma}) = \int_{\Omega} \mathfrak{K}_{i}^{*}(\boldsymbol{x}) \mathcal{S}(\boldsymbol{x}, \boldsymbol{\varsigma}) d\boldsymbol{x}$$
(2)

where \* denotes the complex conjugate and  $\Omega$  denotes the domain of definition.

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#### 3.2 The circular Zernike moment descriptor

The Zernike shape features, called the Zernike moment descriptor (ZMD), are obtained when using the orthogonal Zernike polynomials  $\{V_{i, i=1,2,...}\}$  as the decomposition kernel functions  $\Re_i^*(\mathbf{x})$  in equation (2). It may be expressed as

$$z_i = \iint_{x^2 + y^2 \le 1} V_i^*(x, y) \mathcal{S}(x, y) \mathrm{d}x \mathrm{d}y \tag{3}$$

where \* denotes the complex conjugate, and

$$V_i \equiv V_{n,m}(x, y) \equiv V_{n,m}(\rho, \vartheta) = R_{n,m}(\rho) e^{im\vartheta}$$
(4)

where  $R_{n,m}(\rho)$  represents the radial polynomials which are defined as

$$R_{n,m}(\rho) = \sum_{s=0}^{\frac{n-|m|}{2}} (-1)^s \frac{(n-s)!}{s! \left(\frac{n+|m|}{2}-2\right)! \left(\frac{n-|m|}{2}-2\right)!} \rho^{n-2s}$$
(5)

and

 $i = \sqrt{-1}$ 

- n Non-negative integer, representing the order of the radial polynomial;
- *m* Positive and negative integers subject to constraints n |m| even,  $|m| \le n$  representing the repetition of the azimuthal angle;
- $\rho$  Length of vector from the origin to (x, y);
- $\vartheta$  The azimuthal angle between vector  $\rho$  and the x -axis in counter-clockwise direction;

 $R_{n,m}$  The radial polynomial.

Zernike polynomials satisfy the orthogonality condition within the unit circle [15],

$$\iint_{x^2 + y^2 \le 1} V_{p,q}(x, y) V_{n,m}^*(x, y) dx dy = \frac{\pi}{n+1} \delta_{n,p} \delta_{m,q}$$
(6)

where and  $\delta_{n,p}$  and  $\delta_{m,q}$  are Kronecker deltas.

#### 3.3 Modifications to the decomposition kernel functions

The difference between the real and imaginary parts of the conventional Zernike polynomials (equation (4)) is only the phase angle. Retaining the real part is sufficient to capture the axissymmetrically distributed patterns – e.g. the strain patterns shown in Figure 3. Also, the real basis ensures that the reconstructed strain map from arbitrary number of Zernike moments will be real. Thus, use of the term *Zernike polynomial* is understood to mean the real part of the conventional Zernike polynomial for the rest of this paper.

It is not the best option to apply the circular Zernike polynomials directly to extract shape features from the specimen. Coordinate transform and weighting function enable the modified kernel functions to better capture the strain distribution around the hole in the specimen. The Gram-Schmidt orthogonalisation process ensures the orthogonality of the modified Zernike polynomials is satisfied on the domain of the specimen – e.g. the boundary shown in Figure 5. Figure 6(a) shows the 14<sup>th</sup> circular Zernike polynomial (i.e.  $V_{i=14}$ ). Figure 6(b) shows the

Figure 6(a) shows the 14<sup>th</sup> circular Zernike polynomial (i.e.  $V_{i=14}$ ). Figure 6(b) shows the orthogonalised 14<sup>th</sup> Zernike polynomial on the specimen's domain. It is clear that the high value regions on Figure 6(b) are not close enough to the circular hole, as are the measured strain patterns.

Coordinate transformation of the radial function can 'push' the high value regions towards the central hole, expressed as

$$R'_{n,m}(\rho) \equiv R_{n,m}(\rho') = R_{n,m}(\rho^{\nu}) \tag{7}$$





**Figure 4.** Finite element mesh for a square plate with circular a hole – simulating the region marked by dashed square in **Figure 1**(b).

**Figure 5.** Rectangular test specimen with a circular hole normalised inside an unit circle:  $a^2 + b^2 = 4$ ;  $\varepsilon < \min\left(\frac{a}{2}, \frac{b}{2}\right)$ 

where  $R_{n,m}(\cdot)$  is the radial function of equation (5),  $\rho' \equiv \rho'(\rho, v) = \rho^v$  with  $v \in \mathbb{R}$ . e.g. the coordinate transformed and orthogonalised 14<sup>th</sup> Zernike polynomial is shown in Figure 6(c). However, it is seen that in the strain remains high around the four edges, which is undesirable. To reduce this effect, the radial functions may be modified by multiplying by a decaying weighting function  $w(\rho, t)$  as

$$R_{n,m}^{w}(\rho,t) \equiv w(\rho,t)R_{n,m}'(\rho) = \rho^{t}R_{n,m}'(\rho), \qquad t \in \mathbb{R}^{-}$$
(8)

Figure 6(d) shows the weighted 14<sup>th</sup> Zernike polynomial.



**Figure 6.** Representation of (a) the  $14^{\text{th}}$  ( $V_{i=14}$ , n = 4, m = 2) Zernike polynomial; (b) Gram-Schmidt orthonormalisation of the same polynomial, i.e.  $P'_{14}$ ; (c) as for (b) but with a coordinate transformation applied before the orthonormalisation; and (d) as for (c) but with a decaying weighting function applied after the coordination transformation and prior to the orthonormalisation.

To optimise the two parameters [v, t] in equations (7) and (8), the Newton's iterative method was adopted to minimise the mean-square-root of the errors between the original strain pattern and the reconstructed. Figure 7 shows the convergence of the optimisation process. The converged parameters for the kernel functions are  $\mu^* = [v, t]^T = [3.300, -5.8785]^T$  from an initial guess  $\mu_0 = [v_0, t_0]^T = [2.500, -3.300]^T$ .

The coordinate transformed, weighted, orthogonalised and optimised Zernike polynomials (viz. the modified Zernike polynomials) are adopted to extract shape features from the stain maps. Figure 8 shows several modified Zernike polynomials.

## 3.4 The modified Zernike moments

The modified Zernike moments, i.e. substituting the modified Zernike polynomials to the kernel function in equation (2), of the maximum principal strain component from the FE model at 6.4kN are shown in Figure 9. It is seen that only a small number of ZMD terms are significant. Retaining the first few of the largest terms may be sufficient to describe the original shape. i.e.

$$S(x,y) \approx \hat{S}(x,y) = \sum_{r=1}^{K} z_r P_r$$
(9)

where  $\{z_{r, r=1,2,\dots,K}\}$  are the *K* most significant descriptors sorted by decreasing amplitude and  $P_{r, r=1,2,\dots,K}$  are the corresponding modified Zernike polynomials.



**Figure 7.** Convergence of the kernel parameter  $\mu^* = [v, t]^T$  by Newton's method from an initial guess of [2.5, -3.3].



**Figure 8.** Individual decomposition kernels for the strain map in pattern (a) sorted in four rows from the top by decreasing magnitude of the Zernike moment descriptor.

The second row in Figure 3 shows the reconstructed strain patterns from the 15 most significant Zernike moments. It can be observed that not much information of the DIC measured strain patterns are lost by representation by 15 terms of Zernike features. It is possible to characterise the global development of the strain pattern using the Zernike feature vector containing the ZMD - i.e.  $\{z_r(\varsigma), r=1,2,...,K\}$  where  $\varsigma$  denotes the tensile loading.

Figure 11 shows the development of the 11 most significant modified ZMDs with respect to the tensile loading. It is clear that the developments of these ZMDs tend to be similar. For the lower tensile forces, the ZMDs increase approximately linearly. They increase sharply at around 17kN and then once again tend to be linear. This reflects exactly the elasto-plastic material property of the specimen.

According to the Parseval's theorem, the energy of the shape pattern may be expressed the square term,

$$\int_{\Omega} \mathcal{S}^2(x, y; \varsigma) \mathrm{d}x \mathrm{d}y = \sum_{r=1}^{\infty} |z_r(\varsigma)|^2 \tag{10}$$

The relative energies (i.e. dividing the individual ZMD's energy by the total energy) of the five most significant ZMDs (no. 1, 6, 14, 26 and 44) are shown in Figure 10. It is seen that the energy percentage of ZMD no. 1 remains around 85% below 13kN. This implies that the modified Zernike polynomials no.1 (i.e. the mean value) dominates and does not change because the whole plate is under linear elastic deformation. ZMD no.1 decreases after the onset of plastic deformation during which the relative energies of higher order ZMDs increase. The relative energies of ZMDs no. 6 and

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14, which show highly similar patterns to the stress concentration of the specimen, both increase from around 10kN. This may be the indicator of the onset of plastic yielding. Furthermore, the relative energies of higher order ZMDs, e.g. no 26 and 44 as shown in Figure 10, increase at higher loadings – around 17kN. These modified Zernike polynomials dominate the region slightly further away from the central hole than the lower order ones. Therefore, the relative energy of ZMDs no 26 and 44 increase significantly at higher loading because the plastic deformation developed to the region where they dominate. Thus, the shape descriptors extracted by the modified Zernike polynomials are shown to be effective and efficient in characterising the full-field stain pattern. Also, it is feasible to use these succinct shape features as the output of finite element model updating process, which will be discussed next.





**Figure 9.** Spectrum of the modified Zernike moment-descriptor obtained for the strain map – FE model.

**Figure 10.** Energy percentages of the Zernike moment descriptors number 1, 6, 14, 26 and 44

#### 4 FE model updating by using shape features

The FE model prediction can be improved by validating the model based on the DIC measured responses - i.e. the Zernike features of the full-field strain maps. The model is first parameterized and the updated by iterative process based on the sensitivity method [16] which may be expressed as

$$\boldsymbol{\theta}_{(i+1)} = \boldsymbol{\theta}_{(i)} + \left[ \mathbf{G}_{(i)}^T \mathbf{W}_{\varepsilon\varepsilon} \mathbf{G}_{(i)} + \mathbf{W}_{\theta\theta} \right]^{-1} \left\{ \mathbf{G}_{(i)}^T \mathbf{W}_{\varepsilon\varepsilon} \left\{ \mathbf{x}_{(m)} - \mathbf{x}_{(i)} \right\} - \mathbf{W}_{\theta\theta} \left\{ \boldsymbol{\theta}_{(i)} - \boldsymbol{\theta}_{(0)} \right\} \right\}$$
(11)

where  $\theta_{(i)}$  and  $\theta_{(i+1)}$  are  $p \times 1$  vectors of structural modification parameters at the current and next iterations;  $\mathbf{x}_{(m)}$  and  $\mathbf{x}_{(i)}$  denote the  $q \times 1$  responses of the measured and predicted output vectors (Zernike shape features of the maximum strain components at the four loading stages as labelled in Figure 2) at the current iteration;  $\mathbf{W}_{\varepsilon\varepsilon}$  is the weighting matrix for the errors of output vectors;  $\mathbf{W}_{\theta\theta}$  is the weighting matrix for the parameters;  $\mathbf{G}_{(i)}$  is the sensitivity matrix at the current iteration, written as

$$\mathbf{G}_{(j)} = \left[\frac{\partial x_k}{\partial \theta_\ell}\right]_{\mathbf{\theta} = \mathbf{\theta}_{(j)}}; k = 1, 2, \dots, q; \ell = 1, 2, \dots, p$$
(12)

It is usually necessary to scale equation (12). One of the most common ways of scaling is by dividing the prediction error by the prediction and the parameter error by the initial value of the parameter. The parameters for the present updating process are

$$\boldsymbol{\theta} = \{ E \quad \sigma_{y0} \quad \boldsymbol{\epsilon}_1 \quad \boldsymbol{\epsilon}_m \quad T_p \}^T \tag{13}$$

where represents *E* the Young's modulus,  $\sigma_{y0}$  initial yield stress,  $\epsilon_1$  plastic strain corresponding to 285MPa,  $\epsilon_m$  the plastic strain corresponding to the ultimate stress (310MPa) and  $T_p$  the thickness of the

plate. The nominal stress/strain curve for the FE model is shown in Figure 12 as the blue dotted line labelled with asterisks. All the five structural parameters are converged after only a few iterations. The convergence history is shown Figure 13 and the updated strain-stress curve is plotted in Figure 12 with red dashed line marked by circles.



Figure 11. Eleven largest Zernike moment descriptors as functions of applied load.



Figure 13. Parameter updating history

## **5** Conclusions

To validate a numerical models it is necessary to compare predictions with measurements. Full field measurement techniques enable the global assessment of structural responses. The captured data are usually of great number and redundant. It is good practice to compress the data in a sensible way before carrying out the comparisons. Image moment decomposition using orthogonal kernel functions provides a feasible way to compress the captured data. It is found that selecting the decomposition kernel functions is problem dependent. Also, it is possible to construct one's own kernel functions specifically for the task in hand. Appropriate kernel functions generally result in effective and succinct shape features. Therefore, the comparison between the full-field structural responses can then be transformed to the discrepancy assessment between the extracted shape feature vectors from measured and FE-predicted data, which is concise.

In this paper, a special set of Zernike polynomials were constructed to decompose the full-field strain patterns for a square plate with a circular hole under tensile test. The modifications of the kernel functions included coordinate transform, weighting, Gram-Schmidt orthogonalisation and optimisation. The shape features extracted by the modified Zernike polynomial from the strain maps are shown to be efficient and effective. Only a small number of terms of Zernike features are significant. These few

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terms encapsulate most information of the original shapes. The development of responses with respect to load are then characterised by the Zernike feature vector. Furthermore, the FE model was parameterised and then successfully updated by the Zernike features using an iterative updating process.

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