

OPEN ACCESS

A conventional form of dark energy

To cite this article: K Kleidis and N K Spyrou 2011 *J. Phys.: Conf. Ser.* **283** 012018

View the [article online](#) for updates and enhancements.

You may also like

- [Gravitational waves with dark matter minispikes: Fourier-domain waveforms of eccentric intermediate-mass-ratio-inspirals](#)
Xiao-Jun Yue and Zhoujian Cao
- [CONFRONTING GALACTIC AND EXTRAGALACTIC -RAYS OBSERVED BY FERMI-LAT WITH ANNIHILATING DARK MATTER IN AN INERT HIGGS DOUBLET MODEL](#)
Kamakshya Prasad Modak and Debasish Majumdar
- [Structure, Kinematics, and Observability of the Large Magellanic Cloud's Dynamical Friction Wake in Cold versus Fuzzy Dark Matter](#)
Hayden R. Foote, Gurtina Besla, Philip Mocz et al.



ECS
The
Electrochemical
Society
Advancing solid state &
electrochemical science & technology

DISCOVER
how sustainability
intersects with
electrochemistry & solid
state science research

A conventional form of dark energy

K Kleidis^{1,2} and N K Spyrou¹

¹Department of Astronomy, Aristoteleion University of Thessaloniki, 541.24 Thessaloniki, Greece

²Department of Mechanical Engineering, Technological Education Institute of Serres, 621.24 Serres, Greece

E-mail: kleidis@astro.auth.gr & spyrou@astro.auth.gr

Abstract. Motivated by recent results, indicating that the dark matter (DM) constituents can be *collisional*, we assume that the DM itself possesses also some sort of *thermodynamical properties*. In this case, the Universe matter-content can be treated as a gravitating fluid of positive pressure, and, therefore, together with all the other physical characteristics, *the energy of this fluid's internal motions* should be taken into account as a source of the universal gravitational field. In principle, this form of energy can compensate, also, the *extra (dark) energy*, needed to compromise spatial flatness, while, the post-recombination Universe remains ever-decelerating. What is more interesting, is that, at the same time (i.e., in the context of the collisional-DM approach), the theoretical curve, representing the distance modulus as a function of the cosmological redshift, fits the *Hubble diagram* of an extended sample of SN Ia events quite accurately. However, as we demonstrate, this is not the case for someone who, although living in a Universe filled with collisional DM, insists in adopting the traditional, *collisionless-DM* approach. From the point of view of such an observer, the distant light-emitting sources seem to lie farther (i.e., they appear to be dimmer) than expected, while, the Universe appears to be either accelerating or decelerating, depending on the value of the cosmological redshift.

1. Introduction

The current cosmological picture includes two major unresolved issues: (i) According to the observational data on the temperature-variations of the cosmic microwave background, the Universe can be described, adequately, by a spatially-flat Robertson-Walker (RW) model (see, e.g., [1]) and, therefore, it must contain a considerably-larger amount of energy, than the equivalent to the total rest-mass density of its matter content does (see, e.g., [2]). (ii) The various cosmologically-distant indicators appear to be dimmer than expected [3], [4], something that has been accommodated in the context of a recent phase of accelerated expansion [5], [6].

In order to compromise the above-mentioned observational results within a unified theoretical framework, an extra, dark energy (DE) component, of negative pressure, has been introduced [7], [8]. Other physically-motivated models have also appeared in the literature, including alternative-gravity theories [9], [10], braneworld scenarios [11], [12], Chaplygin gas [13], Cardassian Cosmology [14], theories of compactified internal dimensions [15], mass-varying neutrinos [16], [17], etc. However, each and everyone of these attempts suffers from the so-called *coincidence problem*, i.e., why is the Universe transitioning from deceleration to acceleration so recently; the *inflection point* is being (observationally) set at the cosmological redshift $z_t = 0.46 \pm 0.13$ (see, e.g., [18] and references therein).

The cosmological constant, Λ , could provide a phenomenological description of the DE [19]. In this case, the coincidence problem does not exist, since the *transition redshift*, z_t , depends on the value of Λ , which, consequently, can be determined observationally. The particle-physics vacuum does contribute an effective cosmological constant, but with an energy density 10^{123} times larger than what is observed [20]. This gross mismatch between theory and observation is one of the deepest physical enigmas.

Today, the need for an extra (dark) energy component has been confirmed, also, by other observational data, including galaxy-clusters' dynamics [21], the integrated Sachs-Wolfe effect [22] and baryon acoustic oscillations [23]. These results cry out for a unified theoretical explanation and, in the absence of a (so far) viable one, we should keep our options open, also, to alternative interpretations (see, e.g., [24] - [27]).

Nowadays, a lot of accumulated evidence suggests that, more than 80% (by mass) of the matter in the Universe consists of non-luminous and non-baryonic material [28]. Its name, dark matter (DM), reflects our ignorance on the exact nature of this constituent. Although we do not know for certain how the DM came to be formed, a sizeable relic abundance of weakly-interacting massive particles (WIMPs) is generally expected to have been produced as a by-product of the Universe's hot youth (see, e.g., [29] p. 369). Apart from their exact nature, the scientific community used to argue that, the WIMPs should be collisionless [30]. However, many recent results from high-energy-particle tracers, such as the ATIC [31] and PAMELA [32], combined with those of the Wilkinson microwave anisotropy probe (WMAP) survey [33], have revealed an unusually-high electron - positron production in the Universe, much more than what is anticipated by supernovae explosions or cosmic-ray collisions. These results have led many scientists to argue that, among the best candidate-sources of these high-energy events are the annihilations of WIMPs (see, e.g., [34] - [43], for an extensive, though incomplete list), i.e., that the DM constituents can be collisional [44] - [47]. If this is true, it could affect our perception on the nature of DE (in connection, see, e.g., [48] - [51]).

Indeed, if the (collisional-) DM constituents interact with each other frequently enough, so that their (kinetic) energy can be re-distributed, i.e., the DM itself possesses also some sort of thermodynamical properties, a conventional extra-energy component does exist in the Universe: *It is the energy of the internal motions of the collisional-DM fluid.*

On this basis, it is worth examining the evolution and the dynamical characteristics of a cosmological model, in which (in principle) there is no DE at all. The matter-energy content of this model consists only of two components: The DM (dominant) and the baryonic matter (subdominant), both having the abundances attributed to them by the WMAP survey [2].

Accordingly, we shall demonstrate that these two components are (by themselves) sufficient (i) to reproduce that (today) $\Omega = 1$, (ii) to account for the *observed* dimming of the distant light-emitting sources and (iii) to explain the *apparent* accelerated expansion of the Universe. All the above, simply provided that these two constituents (basically the dark one) form a gravitating perfect fluid of total rest-mass density ρ and a *positive* total pressure, p . In this case, together with all the other physical characteristics, the energy of this fluid's internal motions should (also) be taken into account as a source of the universal gravitational field.

This paper is organized as follows: In Section 2 we consider a cosmological model driven by an ideal fluid of thermodynamically-involved DM with positive pressure. As we find out, in this case, the extra energy (needed to compromise spatial flatness) can be compensated by the energy of the internal motions of this fluid, while, the post-recombination Universe remains ever-decelerating. However, as we demonstrate in Section 3, this is not the case for someone who (although living in a Universe filled with collisional DM) insists in adopting the traditional (collisionless-DM) approach. From the point of view of such an observer, besides the need of an extra-energy component (for confronting the CMB-based observational results), the distant light-emitting sources seem to lie farther (i.e., they appear to be dimmer) than expected, while

the Universe appears to be either accelerating or decelerating, depending on the value of the cosmological redshift.

2. Collisional-DM Cosmology

It is generally accepted that, the study of the CMB has been proved a powerful tool in exploring the *post-recombination Universe*. According to the various CMB-oriented observational data, the Universe has emerged out of the radiation epoch as a spatially-flat RW model (see, e.g., [1])

$$ds^2 = S^2(\eta) \left[c^2 d\eta^2 - (dx^2 + dy^2 + dz^2) \right], \quad (1)$$

where η is the conformal time and $S(\eta)$ is the scale factor. As a consequence, the value of the Hubble parameter at the present epoch is, by definition, given by

$$H_0^2 = \frac{8\pi G}{3} \rho_c \quad (2)$$

(see, e.g., [52] p. 77), where G is Newton's constant of gravitation and ρ_c is the *critical* rest-mass density for closing the Universe. The evolution of such a model depends on the nature of the source that drives the universal gravitational field, i.e., its matter-energy content.

In determining the Universe matter-energy content, within the context of a collisional-DM cosmology, we assume that, in principle, there is no DE at all. Instead, the DM, together with the small *baryonic contamination* (the latter being tightly bounded to the former), constitute a gravitating perfect fluid (i.e., a fluid that is, practically, homogeneous and isotropic at large scale), which possesses also some sort of thermodynamical properties. In this case, together with all the other physical characteristics, the energy of this fluid's internal motions should be taken into account as a source of the universal gravitational field and, therefore, the post-recombination Universe is no longer driven by *dust*, but, by a fluid of *positive pressure*

$$p = w\rho c^2, \quad (3)$$

where ρ is the rest-mass density, c is the velocity of light and $0 \leq w = \left(\frac{c_s}{c}\right)^2 \leq 1$ is a dimensionless constant, which measures the square of the *speed of sound*, c_s , in terms of c^2 .

The motions of the (luminous and dark) elements in the interior of such a fluid are governed by the equations

$$T^{\mu\nu}_{;\nu} = 0, \quad (4)$$

where *Greek* indices refer to the four-dimensional space-time (in connection *Latin* indices refer to the three-dimensional spatial section), the *semicolon* denotes covariant derivative, and $T^{\mu\nu}$ is the energy-momentum tensor of the perfect fluid, which takes on the standard form

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - pg^{\mu\nu}, \quad (5)$$

where $u^\mu = dx^\mu/ds$ is the four-velocity ($u_\mu u^\mu = 1$) at the position of a fluid's volume element, $g^{\mu\nu}$ are the contravariant components of the Universe metric tensor and ε is the fluid's *total-energy* density, which, in this case, is decomposed as

$$\varepsilon = \rho c^2 + \rho\Pi \quad (6)$$

(see, e.g., [53] pp. 81 - 84 and 90 - 94), where $\rho\Pi$ is the *energy density of the internal motions* of the fluid.

In a maximally-symmetric (i.e., homogeneous and isotropic) cosmological setup, the geodesic motions and the hydrodynamic flows are, practically, *indistinguishable*. Hence, a comoving

observer of the cosmic expansion also traces the hydrodynamic flow of the homogeneous cosmic fluid and the *Weyl's postulate* is valid (see, e.g., [54] p. 91). Accordingly, the dynamical evolution of the model (1) is governed by the Friedmann equation (with $\Lambda = 0$) of the classical Friedmann-Robertson-Walker (FRW) cosmology

$$H^2 = \frac{8\pi G}{3c^2} \varepsilon, \quad (7)$$

where

$$H = \frac{S'}{S^2} \quad (8)$$

is the *Hubble parameter* in terms of $S(\eta)$ and the *prime* denotes differentiation with respect to η . Nevertheless, inherently, there is an essential difference between our model and the rest of the classical FRW cosmologies: In our case, the basic matter-constituents (although they may look like particles receding from each other) are, in fact, the volume elements of a collisional-DM fluid, i.e., *they possess some sort of internal structure*; and, hence, *thermodynamical content*, as well, and so, the functional form of ε in Eq. (7), is no longer given by $\varepsilon = \rho c^2$, but, by Eq. (6).

On the other hand, in terms of the metric tensor (1), the conservation law $T^{0\nu}_{;\nu} = 0$, yields

$$\varepsilon' + 3 \frac{S'}{S} (\varepsilon + p) = 0, \quad (9)$$

which, upon consideration of Eqs. (3) and (6), results in

$$\rho = \rho_0 \left(\frac{S_0}{S} \right)^3, \quad (10)$$

where ρ_0 and S_0 can be considered as denoting the corresponding present-time values. Eq. (10) represents the conservation of the *total mass*, in a cosmological model where matter dominates over radiation, i.e., for every η within the *post-recombination* epoch.

Finally, in a cosmological model filled with collisional (i.e., thermodynamically-involved) DM, the *first law of thermodynamics* for *adiabatic flows*

$$d\Pi + p d\left(\frac{1}{\rho}\right) = 0 \quad (11)$$

(see, e.g., [55]), results in

$$\Pi = \Pi_0 + w c^2 \ln \left(\frac{\rho}{\rho_0} \right), \quad (12)$$

where Π_0 is the energy per unit mass, Π , of this fluid's internal motions, at the present epoch. Accordingly, the total-energy density of the Universe matter-energy content is written in the form

$$\varepsilon = \rho c^2 \left[1 + \frac{\Pi_0}{c^2} + w \ln \left(\frac{\rho}{\rho_0} \right) \right]. \quad (13)$$

Now, with the aid of Eqs. (2), (10) and (13), Eq. (7) reads

$$\left(\frac{H}{H_0} \right)^2 = \Omega_M \left(\frac{S_0}{S} \right)^3 \left[1 + \frac{\Pi_0}{c^2} + 3w \ln \left(\frac{S_0}{S} \right) \right], \quad (14)$$

where $\Omega_M = \frac{\rho}{\rho_c}$ is the rest-mass density parameter. According to Eq. (14), at the present epoch (where $S = S_0$ and $H = H_0$), the value of Π_0 is given by

$$\Pi_0 = \left(\frac{1}{\Omega_M} - 1 \right) c^2. \quad (15)$$

In view of Eqs. (6) and (15), at the present epoch, the total-energy density parameter results in

$$\Omega = \frac{\varepsilon_0}{\varepsilon_c} = \frac{\rho_0 c^2}{\rho_c c^2} + \frac{\rho_0 \Pi_0}{\rho_c c^2} = \Omega_M + \Omega_M \frac{\Pi_0}{c^2} = 1, \quad (16)$$

i.e., *the extra (dark) energy, needed to flatten the Universe, can be compensated by the energy of the internal motions of a collisional-DM fluid.*

Upon consideration of Eq. (15), Eq. (14) is written in the form

$$\left(\frac{H}{H_0}\right)^2 = \left(\frac{S_0}{S}\right)^3 \left[1 + 3w\Omega_M \ln\left(\frac{S_0}{S}\right)\right]. \quad (17)$$

Eq. (17) can become particularly transparent (and useful), if we take into account that, since $0 \leq w \leq 1$ and $\Omega_M \simeq 0.3$ (see, e.g., [2]), the combination $w\Omega_M$ can be quite small, i.e., $w\Omega_M \ll 1$. In this case, to terms linear in $w\Omega_M$, Eq. (17) results in

$$H \simeq H_0 \left(\frac{S_0}{S}\right)^{\frac{3}{2}(1+w\Omega_M)} \quad (18)$$

and can be solved, to determine the scale factor of the collisional-DM model (1), as follows

$$S = S_0 \left(\frac{\eta}{\eta_0}\right)^{\frac{2}{1+3w\Omega_M}}, \quad (19)$$

where we have defined the present-time value, η_0 , of the conformal time, η , as

$$\eta_0 = \frac{2}{(1+3w\Omega_M)H_0 S_0}. \quad (20)$$

For $w \neq 0$, Eq. (19) is the *natural generalization* of the corresponding collisionless-DM model, the well-known Einstein-de Sitter (EdS) Universe ($S \sim \eta^2$) (see, e.g., [52] pp. 77, 83 and 142 - 144).

In the collisional-DM model (1), the *cosmological redshift* parameter is defined as

$$z + 1 = \frac{S_0}{S} \quad (21)$$

and, therefore, Eq. (18) is written in the form

$$H = H_0(1+z)^{\frac{3}{2}(1+w\Omega_M)}. \quad (22)$$

We cannot help but noticing the surprising functional similarity of Eq. (22) to the corresponding result regarding *a dark-energy fluid* with equation of state in the form of Eq. (3) (cf. Eqs. (13) and (14) of [18]). In our case, however, $w \geq 0$ and, therefore, on the approach to $z = 0$, $H(z)$ decreases monotonically. In other words, a cosmological model filled with collisional DM necessarily decelerates its expansion.

This can be readily verified, by expressing the corresponding deceleration parameter, q , in terms of H and z , as

$$q(z) = \frac{dH/dz}{H(z)}(1+z) - 1 \quad (23)$$

(cf. Eq. (16) of [18]), which, in view of Eq. (22), yields

$$q(z) = \frac{1}{2}(1+3w\Omega_M) > 0, \quad (24)$$

independently of z , even if $w = 0$. In other words, the model of the gravitating perfect fluid source seems to be inadequate for confronting the *apparent* accelerated expansion of the Universe. The actual reason is that, it does not have to. As we shall demonstrate in the next Section, in a Universe filled with collisional DM, both the observed dimming of the distant light-emitting sources and the apparent accelerating expansion of the Universe, can be due to the misinterpretation of several cosmologically-relevant parameters by those observers who, although living in a collisional-DM Universe, insist in adopting the collisionless-DM approach.

3. Mistreating dark matter as collisionless

At the time the (unexpected) dimming of the SNe Ia standard candles was first discovered, the common perception about the *cosmos* was that the DM constituents are collisionless. The *physical content* of a collisionless-DM Universe (in which both the pressure and the energy of the internal motions are assumed to be negligible and, therefore, disregarded) is entirely different than that of the thermodynamically-involved-DM model (where $p, \Pi \neq 0$). In other words, the dynamical properties of a pressureless Universe are no longer described by $g_{\mu\nu}$, i.e., Eq. (1), but, rather, in terms of *another* metric tensor, $\tilde{g}_{\mu\nu}$, for which, the corresponding (spatially-flat) line-element is written in the form

$$d\tilde{s}^2 = R^2(\eta) \left[c^2 d\eta^2 - (dx^2 + dy^2 + dz^2) \right]. \quad (25)$$

Clearly, the evolution of such a model is given in terms of the scale factor $R(\eta)$ and, therefore, from the point of view of an observer who (mis)treats the DM as collisionless, $\tilde{g}_{\mu\nu}$ is the metric tensor upon which he/she should rely on, in interpreting observations.

However, we recall that, for such an observer, the accumulated evidence, in favor of spatial flatness, necessarily leads to the assumption of an extra (dark) energy component, in contrast to the collisional-DM case, where such an assumption is no longer necessary. Indeed, in the latter case, the appropriate candidate for the extra energy needed to flatten the Universe is already included in the model (the energy of the internal motions).

Furthermore, in the context of the collisionless-DM scenario, every effort to account for the (apparent) dimming of the SNe Ia standard candles, naturally, should also be based on $\tilde{g}_{\mu\nu}$ and the cosmologically-relevant parameters arising from it. Accordingly, a possible explanation could be that, recently, the Universe accelerated its expansion [3], [4]. Such an assumption, however, attributes unnecessarily-exotic properties to the extra amount of energy needed to compromise spatial flatness (e.g., it should be repulsive in nature, i.e., of negative pressure, etc.). Therefore, we cannot help but wondering, whether there is another (more conventional) explanation, that could be found (also) within the context of the collisional-DM model.

In fact, in what follows, we shall demonstrate that both the observed dimming of the distant light-emitting sources and the accelerated expansion of the Universe could be only apparent, based on the misinterpretation of several cosmologically-relevant parameters by someone who (although living in a Universe filled with collisional DM) insists in adopting the traditional (collisionless-DM) approach.

In order to explore such a possibility, we note that the collisional-DM treatment of the Universe (in terms of which $p \neq 0$ and the motions of its matter constituents are, in principle, hydrodynamic flows) can be related to the collisionless-DM approach (in terms of which $\tilde{p} = 0$ and the corresponding motions are, necessarily, geodesics) by means of a conformal transformation [56], [57]

$$\tilde{g}_{\mu\nu} = f^2(x^\kappa) g_{\mu\nu} \quad (26)$$

(in connection, see also [58], [59] pp. 24 - 29 and 54 - 61, [60]), where, upon consideration of *isentropic flows*, the conformal factor $f(x^\kappa)$ takes on the functional form [56]

$$f(x^\kappa) = \mathcal{C} \left(\frac{\varepsilon + p}{\rho c^2} \right) = \mathcal{C} \left[1 + \frac{1}{c^2} \left(\Pi + \frac{p}{\rho} \right) \right], \quad (27)$$

where $\mathcal{C}$ is an arbitrary (integration) constant. According to Eq. (27), $f(x^\kappa)$ is essentially the *specific enthalpy* of the ideal fluid under consideration. Recently, Verozub [61] extrapolated these results to include every Riemannian space-time and not just the metric attributed to a bounded, perfect-fluid source. In particular, he showed that the adiabatic hydrodynamic motion of an ideal-fluid element in a space-time with metric tensor $g_{\mu\nu}$, takes place along the geodesic lines of a Riemannian manifold with metric tensor given by the combination of Eqs. (26) and (27).

With the aid of the technique developed by Kleidis and Spyrou [56], we can determine the scale factor of the spatially-flat cosmological model (25), i.e., the scale factor of the Universe as it is realized by someone who, although living in a collisional-DM Universe (where $p, \Pi > 0$ and $\frac{dp}{d\eta} \neq 0$), *mistreats* the DM as collisionless ($\tilde{p} = 0$).

In view of Eq. (26), the scale factor of the Universe as it is realized by a supporter of the collisionless-DM scenario, $R(\eta)$, is related to the corresponding quantity of the collisional-DM model, $S(\eta)$, as follows

$$R(\eta) = f(x^\kappa)S(\eta), \quad (28)$$

where $f(x^\kappa)$ is given, in terms of z , by

$$f(z) = \frac{\mathcal{C}}{\Omega_M} (1 + w\Omega_M [1 + 3\ln(1+z)]) . \quad (29)$$

In Eq. (29), the arbitrary integration constant, $\mathcal{C}$, can be determined, by demanding that, *in the pressureless case, these two models should coincide*, i.e., $\tilde{g}_{\mu\nu} = g_{\mu\nu}$. In other words, for $w = 0 = p$, $R(\eta) = S(\eta, w = 0)$ - the EdS model; and, hence, $f(w = 0) = 1$.

For $p = 0$, the first law of thermodynamics results in $\Pi = \text{constant} = \Pi_0$, with Π_0 (in our case) being given by Eq. (15). Now, according to Eq. (27), the condition $f(w = 0) = 1$ leads to

$$\mathcal{C} = \Omega_M \quad (30)$$

and, therefore, Eq. (29) results in

$$f(z) = 1 + w\Omega_M [1 + 3\ln(1+z)] . \quad (31)$$

Next, using Eqs. (28) and (29), we shall express several cosmologically-relevant parameters of the collisional-DM model, in terms of their collisionless-DM counterparts.

3.1. The cosmological redshift

A supporter of the collisionless-DM scenario would define the corresponding cosmological redshift parameter, $\tilde{z}$, as

$$\tilde{z} + 1 = \frac{R(\eta_0)}{R(\eta)}, \quad (32)$$

which, upon consideration of Eq. (28), is written in the form

$$\tilde{z} + 1 = \frac{f(\eta_0)}{f(\eta)}(z + 1), \quad (33)$$

where

$$f(\eta_0) = \frac{R(\eta_0)}{S(\eta_0)} = 1 + w\Omega_M . \quad (34)$$

Taking into account Eq. (31), Eq. (33) yields

$$\tilde{z} + 1 = \frac{1 + w\Omega_M}{1 + w\Omega_M [1 + 3\ln(1+z)]} (z + 1), \quad (35)$$

which, to linear terms in $w\Omega_M$, results in

$$1 + \tilde{z} \simeq (1 + z)^{1-3w\Omega_M} . \quad (36)$$

Notice that, for every (fixed) value of the cosmological redshift z , i.e., as it is defined in the collisional-DM model, the corresponding collisionless-DM quantity $\tilde{z}$ is always a little bit *smaller* ($\tilde{z} < z$). In other words, on observing a light-emitting source of the collisional-DM model, an observer who adopts the collisionless-DM scenario will "*realize*" that this source lies farther (z) than he/she would expect ($\tilde{z}$).

3.2. The luminosity distance and the distance modulus

Nowadays, the most direct and reliable method to determine, observationally, the (relatively) recent history of the Universe expansion, is to measure the redshift and the apparent luminosity (equivalently, the apparent magnitude, m) of cosmologically-distant indicators (standard candles), whose absolute luminosity (equivalently, the absolute magnitude, M) is assumed to be known.

SN Ia events appear to be one of the most suitable cosmological standard candles. With the aid of these events, a number of scientific groups have attempted to find evidence in support of a recently-accelerating stage of the Universe [62] - [66]. In each and everyone of these surveys, the SN Ia events, at *peak luminosity*, appear to be *dimmer* (i.e., they seem to lie farther) than expected. This result was, eventually, accommodated within the context of the *concordance model*, by a DE fluid of negative pressure, with $\Omega_X \sim 0.7$ [67]. However, in view of Eq. (36), there may be another, more conventional, interpretation.

Photons travel along *null geodesics*, $d\tilde{s}^2 = 0 = ds^2$, which remain unaffected by conformal transformations. Accordingly, both in the collisional-DM treatment and in the collisionless-DM approach, the *radial distance* of a light-emitting source (in comoving coordinates) is the same, i.e.,

$$\tilde{r} = c(\eta_r - \eta_e) = r , \quad (37)$$

where η_r and η_e are the conformal times of *reception* and *emission* of light, respectively (usually, $\eta_r = \eta_0$).

In this case, with the aid of Eq. (36), the formula determining the luminosity distance in a spatially-flat collisional-DM model,

$$d_L(z) = rS(\eta_0)(1 + z) , \quad (38)$$

can be expressed in terms of the corresponding collisionless-DM quantity,

$$\tilde{d}_L(\tilde{z}) = \tilde{r}R(\eta_0)(1 + \tilde{z}) , \quad (39)$$

as follows

$$\frac{d_L}{\tilde{d}_L} = \frac{1}{1 + w\Omega_M}(1 + z)^{3w\Omega_M} . \quad (40)$$

This relation is very interesting. It suggests that, in a Universe filled with collisional DM (i.e., as long as $w \neq 0$), there exists a *characteristic (transition) value* of the cosmological redshift

$$z_c = (1 + w\Omega_M)^{\frac{1}{3w\Omega_M}} - 1 , \quad (41)$$

such that, the luminosity distance of the various light-emitting sources located at $z > z_c$ is *always larger* than what is realized by a supporter of the collisionless-DM scenario. Therefore, an observer who treats the DM as collisionless, necessarily admits that, any standard candle located at $z > z_c$, lies *farther* than expected ($d_L > \tilde{d}_L$).

The same thing happens, also, in the case of the distance moduli corresponding to d_L and $\tilde{d}_L$. Let us denote by

$$\mu(z) = 5 \log \left(\frac{d_L}{Mpc} \right) + 25 \quad (42)$$

the *K-corrected* distance modulus ($m - M$) of a light-emitting source (see, e.g., [54], Eqs. (13.10) and (13.12), p. 359) in the collisional-DM model, where d_L is measured in *megaparsecs* (Mpc). In a similar fashion,

$$\tilde{\mu}(\tilde{z}) = 5 \log \left(\frac{\tilde{d}_L}{Mpc} \right) + 25 \quad (43)$$

is the distance modulus of the same source, as it is defined by someone who, although living in the collisional-DM model, insists in adopting the (traditional) collisionless-DM approach. Subtracting Eqs. (42) and (43) by parts, and using Eq. (40), we obtain

$$\Delta\mu = \mu - \tilde{\mu} = 15w\Omega_M \log(1+z) - 5 \log(1+w\Omega_M). \quad (44)$$

According to Eq. (44), any light-emitting source of the collisional-DM Universe located at $z > z_c$, from the point of view of an observer who treats the DM as collisionless, appears to be dimmer than expected, i.e., $\tilde{\mu} < \mu$.

We can not help but noticing, the prominent similarity between the characteristic value z_c and the *transition redshift*, z_t , that signals the *onset of dimming* of the SNe Ia standard candles, which, according to the supporters of the collisionless-DM approach, is interpreted as an entry into a phase of accelerated expansion. For $w\Omega_M = 0.1$, i.e., $w = \frac{1}{3}$ (the DM consists of relativistic particles), the characteristic transition value under consideration is set at $z_c = 0.37$, while, for lower values of w , z_c reaches up to 0.39. Each of these results lies within the observationally-determined range of values of the cosmological redshift, concerning the transition point, which distinguishes between accelerated and decelerated expansion of the Universe [6]. In other words, in a collisional-DM model, an *inflection point* (in the d_L versus z diagram) arises naturally (namely, z_c), without the need to assume a transition from deceleration to acceleration. Certainly, such a model no longer suffers from the *coincidence problem*.

Therefore, if the DM possesses some sort of thermodynamical properties, then, it is possible that: (i) The discrepancy between the *expected* value of the distance modulus ($\tilde{\mu}$) of a SN Ia standard candle and the corresponding *observed* one (μ), as well as (ii) the accompanying inflection point, z_t , that signals the transition from deceleration to acceleration, arise only because many cosmologists (although living in a collisional-DM model) rather insist in adopting the (traditional) collisionless-DM approach. We can readily demonstrate this, by overplotting Eq. (44) in the *Hubble* (μ versus z) *diagram* of a SN Ia dataset.

There is an extended sample of 192 SN Ia events, which has been used by Davis et al. [68] to scrutinize the viability of various DE scenarios. It is available, either at <http://www.ctio.noao.edu/essence> or at <http://braeburn.pha.jhu.edu/~ariess/R06>.

In order to overplot Eq. (44) on the μ versus z diagram of this dataset, first of all, we need to determine the luminosity distance $\tilde{d}_L(\tilde{z})$ (and, through that, the corresponding modulus, $\tilde{\mu}(\tilde{z})$), that is realized by someone who, although living in a collisional-DM model, insists in adopting the collisionless-DM approach. Such an observer, performs calculations in the (traditional) framework of a pressureless Universe, adopting the corresponding formula of the luminosity distance. In a spatially-flat model, such a formula is given by (see, e.g., [69])

$$\tilde{d}_L(\tilde{z}) = \frac{2c}{\tilde{H}_0} (1 + \tilde{z})^{1/2} \left[(1 + \tilde{z})^{1/2} - 1 \right], \quad (45)$$

representing the *luminosity distance in the EdS Universe*, the (conformally) pressureless counterpart of the collisional-DM model (19).

However, we need to stress that, in a collisional-DM Universe, the measured quantity (corresponding to the cosmological redshift) is z and not $\tilde{z}$. Therefore, in order to include, also, the function $\tilde{\mu}(\tilde{z})$ in the Hubble diagram of the SN Ia dataset used by Davis et al. [68], we have to express $\tilde{d}_L(\tilde{z})$ in terms of the *truly measured* quantity, z . It can be done (appropriately) by inserting Eq. (36) into Eq. (45), to obtain

$$\tilde{d}_L(z) = \frac{2c}{(1 - 4w\Omega_M) H_0} (1 + z)^{\frac{1}{2}(1-3w\Omega_M)} \left[(1 + z)^{\frac{1}{2}(1-3w\Omega_M)} - 1 \right]. \quad (46)$$

However, this is not the case for a supporter of the collisionless-DM scenario. Indeed, in depicting Eq. (43) - with $\tilde{d}_L(\tilde{z})$ being given by Eq. (45) - on the μ versus z diagram of a sample of SN events, such an observer (admitting that $w = 0$), unavoidably, misinterprets the measured quantity z as $\tilde{z}$ (also, the quantity H_0 is misinterpreted as $\tilde{H}_0$). In other words, the theoretical formula of the luminosity distance that is used by someone who, although living in a (spatially-flat) collisional-DM model, insists in adopting the collisionless-DM approach, rather is (falsely) written in the form

$$\tilde{d}_L(z) = \frac{2c}{H_0} (1 + z)^{1/2} \left[(1 + z)^{1/2} - 1 \right], \quad (47)$$

instead of that given by Eq. (46). In what follows, we admit that $H_0 = 70.5 \text{ Km/sec/Mpc}$ (Komatsu et al. 2009) and hence $2c/H_0 = 8509.8 \text{ Mpc}$.

Next, the theoretical curves, corresponding to the distance moduli $\mu(z)$ - for $w\Omega_M = 0.16$ - (green solid line), $\tilde{\mu}(z)$ - (also for $w\Omega_M = 0.16$) with $\tilde{d}_L(z)$ being given by Eq. (46) - (orange solid line) and $\tilde{\mu}(z)$ - with $\tilde{d}_L(z)$ being given by Eq. (47) - (dashed line), are overplotted in the Hubble diagram of the SN Ia dataset used by Davis et al. [68] (Fig. 1). We observe that, the (appropriately translated in terms of z) collisionless-DM quantity $\tilde{\mu}(z)$ - with $\tilde{d}_L(z)$ being given by Eq. (46) - (orange solid line) is quite far from fitting this sample of data, although, for $z \leq 1.75$, it is much closer to the μ versus z distribution of the SN Ia data available, than the falsely used quantity $\tilde{\mu}(z)$ - with $\tilde{d}_L(z)$ being given by Eq. (47) - (dashed line).

The situation changes, completely, when someone takes into account the thermodynamical content of a collisional-DM fluid with $w\Omega_M = 0.16$, thus using Eq. (44), instead of Eq. (43) alone. In this case, the function $\mu(z)$ (green solid line) fits the entire dataset under consideration quite accurately.

Taken together, these results suggest that, if the DM constituents interact with each other frequently enough (so that their energy is re-distributed and, hence, the DM fluid acquires some sort of thermodynamical properties), then, what is realized as "dimming" of the SNe Ia standard candles could be only "apparent", provided that the cosmologists no longer insist in adopting the collisionless-DM approach. In other words, in a Universe filled with collisional DM, the unexpected dimming of the distant light-emitting sources can be explained in a more conventional way, than that implemented within the context of the accelerated expansion.

3.3. The Hubble and the deceleration parameters

By virtue of Eq. (28), the Hubble parameter that is realized by a supporter of the collisionless-DM scenario, $\tilde{H}$, is written in terms of H as

$$\tilde{H} = H \frac{d}{dz} \left(\frac{1+z}{f} \right), \quad (48)$$

which, in view of Eqs. (22) and (31), results in

$$\tilde{H} = H_0 (1+z)^{\frac{3}{2}(1+w\Omega_M)} \frac{1 - 2w\Omega_M + 3w\Omega_M \ln(1+z)}{(1 + w\Omega_M [1 + 3 \ln(1+z)])^2}. \quad (49)$$

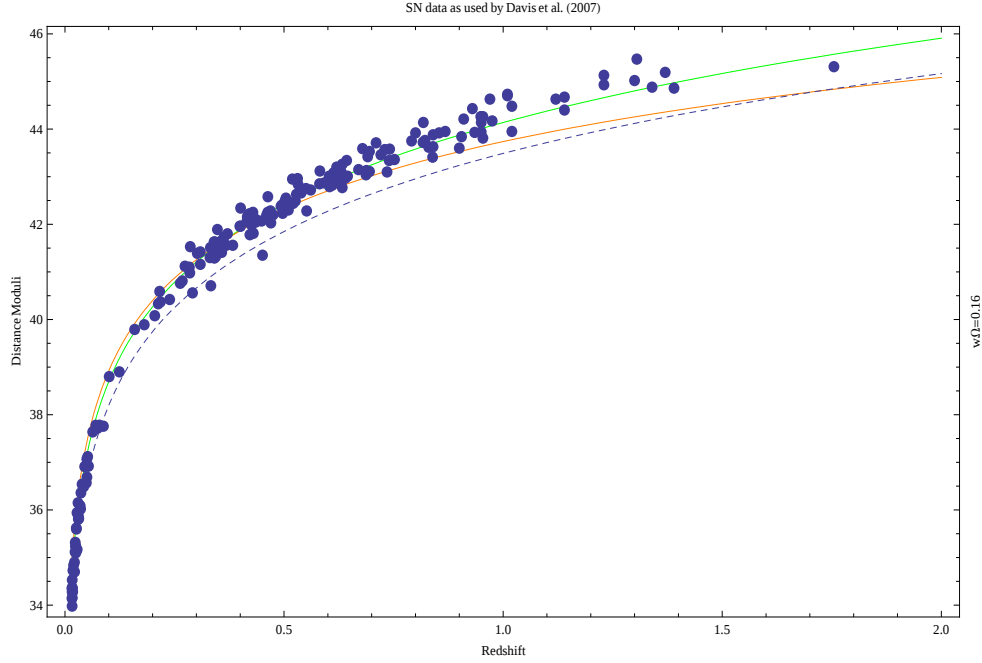


Figure 1. The SNe Ia Hubble diagram of the sample used by Davis et al [68]. Overplotted are the theoretical curves, corresponding to the distance moduli $\mu(z)$ - for $w\Omega_M = 0.16$ - (green solid line), $\tilde{\mu}(z)$ - (also for $w\Omega_M = 0.16$) with $\tilde{d}_L(z)$ being given by Eq. (46) - (orange solid line) and $\tilde{\mu}(z)$ - with $\tilde{d}_L(z)$ being given by Eq. (47) - (dashed line). We observe that, after the thermodynamical content of a collisional-DM fluid is taken into account, the theoretical curve representing the distance modulus, $\mu(z)$ (Eq. (44)), fits the entire dataset quite accurately (green line).

We note that, to terms linear in $w\Omega_M$,

$$\tilde{H}_0 = H_0(1 - 4w\Omega_M), \quad (50)$$

i.e., within the context of the collisionless-DM approach, at the present epoch (where $z = 0$), the Universe expands only as long as $w\Omega_M < 0.25$, and, in any case, at a lower rate than what the collisional-DM treatment implies.

In view of Eq. (36), i.e., at relatively-low values of z , Eq. (49) can be written in terms of $\tilde{z}$, as

$$\begin{aligned} \tilde{H} &= H_0 (1 + \tilde{z})^{\frac{3(1+w\Omega_M)}{2(1-3w\Omega_M)}} (1 - 3w\Omega_M) \\ &\times \frac{1 - 5w\Omega_M + 3w\Omega_M \ln(1 + \tilde{z}) + O(w\Omega_M)^2}{[1 - 2w\Omega_M + 3w\Omega_M \ln(1 + \tilde{z}) + O(w\Omega_M)^2]^2}. \end{aligned} \quad (51)$$

By analogy to Eq. (23), a supporter of the collisionless-DM scenario would define the corresponding deceleration parameter, $\tilde{q}$, as

$$\tilde{q}(\tilde{z}) = \frac{d\tilde{H}/d\tilde{z}}{\tilde{H}(\tilde{z})} (1 + \tilde{z}) - 1, \quad (52)$$

which, by virtue of Eq. (51), yields

$$\tilde{q}(\tilde{z}) = \frac{1}{2} \cdot \left[\frac{1 - 4w\Omega_M + 6w\Omega_M \ln(1 + \tilde{z}) + O(w\Omega_M)^2}{1 - 10w\Omega_M + 6w\Omega_M \ln(1 + \tilde{z}) + O(w\Omega_M)^2} \right]. \quad (53)$$

Now, the condition for accelerated expansion ($\tilde{q} < 0$) is translated to

$$[1 - 4w\Omega_M + 6w\Omega_M \ln(1 + \tilde{z})] \cdot [1 - 10w\Omega_M + 6w\Omega_M \ln(1 + \tilde{z})] < 0, \quad (54)$$

from which, to terms linear in $w\Omega_M$, we obtain

$$\tilde{q}(\tilde{z}) < 0 \Leftrightarrow 1 - 14w\Omega_M + 12w\Omega_M \ln(1 + \tilde{z}) < 0. \quad (55)$$

From Eq. (55) it becomes evident that, from the point of view of someone who insists in adopting the collisionless-DM approach, $\tilde{q}(\tilde{z}) < 0$ at cosmological redshifts

$$\tilde{z} < \tilde{z}_t = e^{\frac{14w\Omega_M - 1}{12w\Omega_M}} - 1. \quad (56)$$

This relation is very interesting: It suggests that, if the Universe matter-content is treated as a collisional-DM fluid with w being larger than a *critical* value, w_c , such that

$$w\Omega_M > w_c\Omega_M = \frac{1}{14} \approx 0.0714 \quad (57)$$

(i.e., $w > w_c \approx 0.238$), then, from the point of view of someone who treats the DM as collisionless, there exists a *transition* value, $\tilde{z}_t$, of $\tilde{z}$, below which, the post-recombination Universe (as being realized by such an observer) is *accelerating*.

In other words, if the Universe evolution is dominated by a collisional-DM fluid with $w > w_c$, then, the apparent acceleration of the cosmic expansion could (very well) be due to a misinterpretation of several cosmologically-relevant parameters, by an observer who (although living in a cosmological model filled with collisional DM) insists in adopting the collisionless-DM approach. At the same time, for such an observer, the cosmologically-distant indicators would appear to be dimmer than expected (cf. Eq. (44)).

We recall here that, the recent observational data concerning the SNe Ia standard candles set the *transition redshift* between accelerated and decelerated expansion at $z_t = 0.46 \pm 0.13$ [6]. In this case, the combination of Eqs. (36) and (56) results in the following non-linear *algebraic* equation, involving the transition value, z_t , of the *truly measured* quantity z ,

$$(1 + z_t) e^{0.25/3w\Omega_M} = 3.2114 (1 + z_t)^{3w\Omega_M}. \quad (58)$$

Eq. (58) can be solved, numerically, with respect to the combination $w\Omega_M$. Accordingly, we verify that the value

$$(w\Omega_M)_t = 0.0932 \pm 0.0060 \quad (59)$$

reproduces (exactly) the above observational result for z_t . By virtue of Eq. (59), $w \simeq \frac{1}{3}$, i.e., compatibility of the collisional-DM approach with the (currently available) observational data, suggests that, the DM itself consists of *relativistic particles*. This means that, on the basis of the collisional-DM approach under study, the matter-content of the *dark sector* consists of "hot" DM. For the time being, the theory of hot dark matter does not appear to be compatible with the large-scale structure of the Universe (see, e.g., [30]), although there are recent studies that could debate this result (see, e.g., [70], [71]). In any case, a hot-DM model looks much less *exotic* than most of the (currently investigated) DE scenarios.

4. Discussion

In the present article, we have examined the possibility that, the extra (dark) energy, needed to flatten the Universe, can be compensated by the energy of the internal motions of a collisional-DM fluid. Accordingly, we ended up with the evolution of a cosmological model which is driven by a perfect fluid of *positive pressure*, and, consequently, the energy of this fluid's internal motions has also been taken into account as a source of the universal gravitational field. Then, we have asked ourselves, whether such a model can accomodate, also, the apparent dimming of the cosmologically-distant indicators and the associated phase of accelerated expansion.

In particular, based on recent observational data, indicating that the DM can be slightly collisional, we have assumed that, the matter-energy content which drives the evolution of the Universe (being modeled by a spatially-flat RW space-time) at every post-recombination epoch, is in the form of a perfect fluid with *positive pressure*. In this way, we have been able to determine the "correct" form of the scale factor, which (under the assumption that the DM is collisional) governs the evolution of the Universe, together with a series of cosmologically-relevant parameters.

The outcome is quite promising: In principle, the energy of the internal motions of the collisional-DM fluid can account for the (extra) DE, so that, at the present epoch, $\Omega = 1$ (cf. Eq. (16)), while the post-recombination Universe remains *ever-decelerated* (cf. Eq. (24)).

Accordingly, we have attempted to determine what is realized by someone who, although living in a collisional-DM model, insists in adopting the (traditional) collisionless-DM approach.

To do so, we have applied the technique developed by Kleidis and Spyrou [56]. With the aid of this technique, we have found the (conformal) transformation (cf. Eqs. (28) and (31)), which relates the collisional-DM description of a cosmological model (in terms of which p , $\Pi > 0$ and $\frac{dp}{d\eta} \neq 0$) to the corresponding collisionless-DM (pressureless) approach. Accordingly, we have explored the way that, a supporter of the collisionless-DM scenario interprets the observations carried out in a collisional-DM Universe.

In the collisional-DM Universe there is a *characteristic value*, z_c , of the cosmological redshift (cf. Eq. (41)), above which, the luminosity distance of the various light-emitting sources becomes larger than what is realized by an observer who treats the DM as a pressureless fluid (cf. Eq. (40)). In other words, from the point of view of someone who (although living in a collisional-DM model) insists in adopting the (traditional) collisionless-DM approach, the cosmologically-distant indicators, located at $z > z_c$, seem to lie farther (i.e., they appear to be dimmer) than expected (cf., also, Eq. (44)). The similarity between the characteristic value z_c and the (observationally-determined) *transition redshift*, z_t , that signals the onset of dimming of the SNe Ia standard candles, is obvious.

On the other hand, after the thermodynamical content of a collisional-DM fluid is taken into account, the theoretical curve representing the distance modulus, $\mu(z)$ (now given by Eq. (44)), fits the *Hubble diagram* of an extended sample of SN Ia standard candles quite accurately (green solid line in Fig. 1), in contrast to the corresponding collisionless-DM quantity, $\tilde{\mu}(\tilde{z})$, given either (appropriately) by the combination of Eqs. (43) and (46) (orange solid line in Fig. 1) or (falsely) by the combination of Eqs. (43) and (47) (dashed line in Fig. 1).

At the same time, as far as a supporter of the collisionless-DM scenario is concerned, the Universe appears to be *either accelerating or decelerating*, depending on the value of the cosmological redshift (cf. Eqs. (55) and (56)).

In this case, the quantity w , which, in the collisional-DM approach, parameterizes the various *isothermal flows*, plays also another (more interesting) role: As we have found, for $w\Omega_M \geq 0.0714$, there exists a (theoretically-determined) *transition value*, $\tilde{z}_t$, of the (collisionless-DM-oriented) cosmological redshift, $\tilde{z}$, such that, for $\tilde{z} < \tilde{z}_t$, we have $\tilde{q} < 0$, i.e., from the point of view of someone who adopts the (traditional) collisionless-DM approach, the Universe is *accelerating*. Accordingly, taking into account the observational result that, the transition

redshift between accelerated and decelerated expansion is set at the value $z_t = 0.46 \pm 0.13$ of the *truly* measured quantity z , we have determined the exact value of the combination $w\Omega_M$, for which the collisional-DM approach to the post-recombination Universe is compatible with observations, namely $(w\Omega_M)_t = 0.0932 \pm 0.0060$. According to this result, compatibility of the collisional-DM approach with the observational data currently available, suggests that, the DM itself consists of relativistic particles ($w \simeq \frac{1}{3}$).

In any case, the assumption that the Universe matter-content (basically its DM component) can be slightly collisional, is to be seen as a natural effort to take into account all the (so far neglected) internal physical characteristics of a classical cosmological fluid as sources of the universal gravitational field. As we have shown, under this assumption, one can compensate for the majority of the recent observational data, regarding $\Omega \simeq 1$, as well as the *unexpected dimming* of the SNe Ia standard candles and the *apparent* accelerated expansion of the Universe, without the need of any DE or the cosmological constant, and (certainly) without suffering from the *coincidence problem*, since, in a collisional-DM Universe, a potential *inflection point* (in the d_L versus z diagram) arises naturally. Although speculative, the idea that the DE (needed to flatten the Universe) could be attributed to the internal motions of a collisional-DM fluid, is (at least) intriguing and should be further explored and scrutinized, in the search of conventional alternatives to the DE concept.

References

- [1] de Bernardis P et al. [BOOMERanG Collaboration] 2000 *Nature* **404** 955
- [2] Komatsu E et al. [WMAP Collaboration] 2009 *Astrophys. J. Suppl. Series* **180** 330
- [3] Riess A G et al. [Supernova Search Team Collaboration] 1998 *Astron. J.* **116** 1009
- [4] Perlmutter S et al. [Supernova Cosmology Project Collaboration] 1999 *Astrophys. J.* **517** 565
- [5] Tonry J L et al. [High- z Supernova Search Team] 2003 *Astrophys. J.* **594** 1
- [6] Riess A G et al. [High- z Supernova Search Team] 2004 *Astrophys. J.* **607** 665
- [7] Caldwell R R, Dave R and Steinhardt P J 1998 *Phys. Rev. Lett.* **80** 1528
- [8] Carroll S M 1998 *Phys. Rev. Lett.* **81** 3067
- [9] Esposito-Farese G and Polarski D 2001 *Phys. Rev. D* **63** 063504
- [10] Capozziello S, Carloni S and Troisi A 2003 *Recent Res. Dev. Astron. Astrophys.* **1** 625
- [11] Dvali G R, Gabadadze G and Porratti M 2000 *Phys. Lett. B* **485** 208
- [12] Bousso R and Polchinski J 2000 *JHEP* **0006** 1 - 26
- [13] Kamenshchik A, Moschella U and Pasquier V 2001 *Phys. Lett. B* **487** 7
- [14] Freese K and Lewis M 2002 *Phys. Lett. B* **540**, 1
- [15] Perivolaropoulos L 2003 *Phys. Rev. D* **67** 123516
- [16] Fardon R, Nelson A E and Weiner N 2004 *JCAP* **10** 005
- [17] Peccei R D 2005 *Phys. Rev. D* **71** 023527
- [18] Perivolaropoulos L 2007 *Accelerating Universe: Observational status and theoretical implications in The Invisible Universe: Dark Matter and Dark Energy*, ed. L. Papantonopoulos, *Lecture Notes in Physics* **720** 257 (New York: Springer Verlag)
- [19] Sahni V and Starobinsky A 2000 *IJMP D* **9** 373
- [20] Sahni V 2004 *Dark matter and dark energy in The Physics of the Early Universe*, ed. L. Papantonopoulos, *Lecture Notes in Physics* **653** 141 (Berlin: Springer Verlag)
- [21] Allen S W, Schmidt R W, Ebeling H, Fabian A C and van Speybroeck L 2004 *MNRAS* **353**, 457
- [22] Boughn S and Crittenden R 2004 *Nature* **427** 45
- [23] Eisenstein D J et al. [SDSS Collaboration] 2005 *Astrophys. J.* **633** 560
- [24] Buchert T 2000 *Gen. Relativ. Gravit.* **32** 105
- [25] Kolb E W, Matarrese S and Riotto A 2006 *New J. Phys.* **8** 322
- [26] Celerier M-N 2007 *New Adv. Phys.* **1** 29
- [27] Ellis G F R 2009 *JPhCS* **189** 012011
- [28] Olive K A 2003 *TASI Lectures on Dark Matter* arXiv: 0301505 [astro-ph]
- [29] Kolb E W and Turner M S 1990 *The Early Universe* (Menlo Park: Addison-Wesley)
- [30] Hooper D 2009 *TASI 2008 Lectures on Dark Matter* arXiv: 0901.4090 [hep-ph]
- [31] Chang J et al. [ATIC Collaboration] 2008 *Nature* **456** 362
- [32] Adriani O et al. [PAMELA Collaboration] 2009 *Nature* **458** 607
- [33] Hooper D, Finkbeiner D P and Dobler G 2007 *Phys. Rev. D* **76** 083012

- [34] Barger V, Keung W Y, Marfatia D and Shaughnessy G 2008 *Phys. Lett. B* **672** 141
- [35] Baushev A 2008 *Dark matter annihilation at cosmological redshifts: Possible relic signal from weakly interacting massive particles annihilation* arXiv: 0806.3108 [astro-ph]
- [36] Bergstrom L, Bringmann T and Edsjo J 2008 *Phys. Rev. D* **78** 103520
- [37] Cholis I, Goodenough L, Hooper D, Simet M and Weiner N 2008a *High-energy positrons from annihilating dark matter* arXiv: 0809.1683 [hep-ph]
- [38] Cholis I, Dobler G, Finkbeiner D P, Goodenough L and Weiner N 2008b, *The case for a 700+ GeV WIMP: Cosmic-ray spectra from ATIC and PAMELA* arXiv: 0811.3641 [astro-ph]
- [39] Cirelli M and Strumia A 2008 *Minimal dark-matter predictions and the PAMELA positron excess* arXiv: 0808.3867 [astro-ph]
- [40] Regis M and Ullio P 2008 *Phys. Rev. D* **78** 103505
- [41] Fornasa M, Pieri L, Bertone G and Branchini E 2009 *Anisotropy probe of galactic and extra-galactic dark matter annihilations* arXiv: 0901.2921 [astro-ph]
- [42] Fox P J and Poppitz E 2009 *Phys. Rev. D* **79** 083528
- [43] Zurek K M 2009 *Phys. Rev. D* **79** 115002
- [44] Spergel D N and Steinhardt P J 2000 *Phys. Rev. Lett.* **84** 3760
- [45] Arkani-Hamed N, Finkbeiner D P, Slatyer T R and Weiner N 2009 *Phys. Rev. D* **79** 015014
- [46] Cirelli M, Kadastik M, Raidal M and Strumia A 2009 *Nucl. Phys. B* **813** 1
- [47] Cohen T and Zurek K 2010 *Phys. Rev. Lett.* **104** 101301
- [48] Zimdahl W, Schwarz D J, Balakin A B and Pavon D 2001 *Phys. Rev. D* **64** 3501
- [49] Balakin A B, Pavon D, Schwarz D J and Zimdahl W 2003 *New J. Phys.* **5** 85
- [50] Lima J A S, Silva F E and Santos R C 2008 *Class. Quantum Grav.* **25** 205006
- [51] Basilakos S and Plionis M 2009 *A&A* **507** 47
- [52] Peacock J A 1999 *Cosmological Physics* (Cambridge: Cambridge University Press)
- [53] Fock V 1959 *The theory of space, time and gravitation* (London: Pergamon Press)
- [54] Narlikar J V 1983 *Introduction to Cosmology* (Boston: Jones and Bartlett Publishers Inc.)
- [55] Chandrasekhar S 1965 *Astrophys. J.* **142** 1488
- [56] Kleidis K and Spyrou N K 2000 *Class. Quantum Grav.* **17** 2965
- [57] Spyrou N K and Tsagas C G 2004 *Class. Quantum Grav.* **21** 2435
- [58] Synge J L 1937 *Proc. London Math. Soc.* **43** 376 [reprinted 2002 in *Gen. Relat. Grav.* **34** 2177]
- [59] Lichnerowicz A 1967 *Relativistic Hydrodynamics and Magneto-hydrodynamics* (New York: W. A. Benjamin Inc.)
- [60] Carter B 1979 *Perfect fluid and magnetic field conservation laws in the theory of black-hole accretion rings* in *Active Galactic Nuclei*, ed. C Hazard and S Mitton (Cambridge: Cambridge University Press) p. 273
- [61] Verozub L 2008 *IJMP D* **17** 337
- [62] Riess A G et al. [High-z Supernova Search Team] 2001 *Astrophys. J.* **560** 49
- [63] Riess A G et al. [High-z Supernova Search Team] 2007 *Astrophys. J.* **659** 98
- [64] Astier P et al. [SNLS Collaboration] 2006 *A&A* **447** 31
- [65] Wood-Vasey W M et al. [ESSENCE Supernova Survey Team] 2007 *Astrophys. J.* **666** 694
- [66] Kowalski M et al. [Supernova Cosmology Project Group] 2008 *Astrophys. J.* **686** 749
- [67] Spergel D N et al. [WMAP Collaboration] 2003 *Astrophys. J. Suppl. Series* **148** 175
- [68] Davis T M et al. [ESSENCE Supernova Survey Team] 2007 *Astrophys. J.* **666** 716
- [69] Carroll S M, Press W H and Turner M S 1992 *ARA&A* **30** 499
- [70] Farrar G R and Peebles P J E 2004 *Astrophys. J.* **604** 1
- [71] Gubser S S and Peebles P J E 2004 *Phys. Rev. D* **70** 123510