Analysis of image plane's Illumination in Image-forming System

To cite this article: Lihua Duan et al 2011 J. Phys.: Conf. Ser. 276 012118

View the article online for updates and enhancements.
Analysis of image plane’s Illumination in Image-forming System

*Journal of Physics: Conference Series*

Lihua Duan¹, Yan’an Zeng, Nanyangsheng Zhang, Zhiguo Wang, Shiliang Yin

Huazhong University of Science and Technology, Wuhan, China

E-mail:victorydlh@163.com

**Abstract.** In the detection of optical radiation, the detecting accuracy is affected by optic power distribution of the detector’s surface to a large extent. In addition, in the image-forming system, the quality of the image is greatly determined by the uniformity of the image’s illumination distribution. However, in the practical optical system, affected by the factors such as field of view, false light and off axis and so on, the distribution of the image’s illumination tends to be non uniform, so it is necessary to discuss the image plane’s illumination in image-forming systems. In order to analyze the characteristics of the image-forming system at a full range, on the basis of photometry, the formulas to calculate the illumination of the imaging plane have been summarized by the numbers. Moreover, the relationship between the horizontal offset of the light source and the illumination of the image has been discussed in detail. After that, the influence of some key factors such as aperture angle, off-axis distance and horizontal offset on illumination of the image has been brought forward. Through numerical simulation, various theoretical curves of those key factors have been given. The results of the numerical simulation show that it is recommended to aggrandize the diameter of the exit pupil to increase the illumination of the image. The angle of view plays a negative role in the illumination distribution of the image, that is, the uniformity of the illumination distribution can be enhanced by compressing the angle of view. Lastly, it is proved that telecentric optical design is an effective way to advance the uniformity of the illumination distribution.

1. **Introduction**

In the detection of optical radiation, the detecting accuracy is affected by optic power distribution of the detector’s surface to a large extent. In addition, in the image-forming system, the quality of the image is greatly determined by the uniformity of the image’s illumination distribution. For example, as to cameras, the film’s exposure is proportional to the amount of illumination. When the non-uniformity of the image’s illumination overpass a certain level, the quality of the picture will be degraded sharply as a result of the uneven photosensibility[1]-[3]. What’s more, in the practical optical system, affected by various factors such as field of view, false light and off axis and so on, the distribution of the image’s illumination tends to be non uniform. Therefore, it is necessary to analyze the image plane’s illumination theoretically.
In this paper, the calculation methods about image plane’s illumination have been summarized, simultaneously, the mathematical model of out-of-focus light sources’ illumination has been addressed. We take the emphasis on the effect of the angle of view, aperture angle, out-of-focus distance and off-axis distance on image plane’s illumination. Finally, the results of the numerical results are given.

2. Mathematical analysis of image plane’s illumination

2.1. Illumination of the central image

When the luminous surface is a Lambertian emitter and the area is small enough to be ignored, forming a clear image through the optical system. The model to calculate the illumination of the central image is given on figure 1, in which $ds$ and $ds'$ are the tiny areas of object and the image near the axis, $y$ and $y'$ are the tiny sizes of the object and the image, $U$ and $U'$ are the angles of the object view and image view, $L$ and $L'$ are the photometric brightness of the object and the image space, respectively[4]-[6].

![Figure 1. The model to calculate illumination of the central image.](image)

The luminous flux of the tiny image emitted from the exit pupil is

$$\Phi' = \pi L ds \sin^2 U$$

(1)

Where $\tau$ denotes luminous flux transmittance of the optical system, the illumination of the centre image is

$$E_0 = \frac{\Phi'}{ds} = \pi L \frac{ds}{ds'} \sin^2 U$$

(2)

In the ideal situation, the aperture angles meet the law of sines, that is

$$ny \sin U = n'y' \sin U'$$

(3)

Where $n$ and $n'$ denotes the refractive indexes respectively, from the formula (2) and (3), we have

$$E_0' = \frac{n'r^2}{n^2} \tau \pi L \sin^2 U'$$

(4)

2.2. Image plane’s illumination of the off-axis view

Figure 2 shows the model of calculating the image’s illumination of off-axis view. $A'$ and $P'$ are the images which formed from object point $A$ and $P$ through optical system, respectively. $\omega'$ is the angle of image view from off-axis image point $P'$. Assuming the exit pupil is filled to the full with the image beam of image point $P'$, in addition, the photometric brightness of object image is even and equivalent to all directions.

Ignoring the vignetting of inclined beams, if diameter of the exit pupil is not very large compared to the distance from the exit pupil to the image of off-axial view, the illumination of point $P'$, the approximated illumination of point $P'$ is

$$E_0' = \frac{l \cos^2 \omega'}{r^2}$$

(5)

Where $I_0$ is the luminous intensity of the centre image, $l'$ is the distance from the exit pupil to the image of off-axial view, $l_0$ is the horizontal distance between the image and exit pupil.
Figure 2. The model to calculate the image’s illumination of off-axis view.

Because \( I = I_0 \cos \omega' \) and \( l' = l_0 / \cos \omega' \), thus

\[
E' = I_0 \cos^4 \omega' / l_0^2 = E_0 \cos^4 \omega
\]  \hspace{1cm} (6)

Where \( E_0 \) is the illumination of the image’s central point and \( E_0 = I_0 / l^2 \).

2.3. The relationship between image’s illumination and the offset of light source under ideal optical system

Figure 2 shows the imaging model of ideal optical system. The vertical size of the object is \( y \), forming a reversed image through the optical system. Assuming the optical system is symmetric, that is, the entrance pupil and the exit pupil coincide with the principal planes of the object space and the image space, respectively.

The vertical size of the image is \( y' \), focal distances of the object and the image are \( -f \) and \( -f' \), the object distance and image distance are \( -l \) and \( l' \), respectively. Through Gauss formula, we have

\[
\frac{f'}{l'} + \frac{f}{l} = 1
\]  \hspace{1cm} (7)

When the medium of the object space is the same with the medium of the image space, we have

\[
f' = -f
\]  \hspace{1cm} (8)

Figure 3. The model of the ideal optical system.

The line DE is parallel to the line FH, so \( FH/DE = GH/GE \), that is
When the object moves horizontally along the optical axis, the shifting distance is \(-\Delta x\), the sign is negative if the object moves forward, on the other side, the sign is positive. The object distance changes into \(l + \Delta x\), substituting into the formula (3), combining the formula(4), the image distance\( l'\) changes into

\[
\frac{l'}{l + \Delta x} = \frac{-y'}{y - y''}
\]

After the object move, assuming the aperture angle of image space is \(U''\) and the angle of image view is \(\omega''\), the radius of the exit pupil is \(r\), we have

\[
\sin U'' = \frac{r}{\sqrt{l'^2 + r^2}}
\]

\[
\cos \omega'' = \frac{l'}{\sqrt{l'^2 + y''^2}}
\]

The formula to calculate the illumination of the image’s center after the object moves is

\[
E_0'' = \tau \pi L \sin^2 U''
\]

The formula to calculate the illumination of the off axis changes into

\[
E'' = E_0'' \cos^4 \omega''
\]

3. Numerical results and discussions

In order to discuss the relationship among various parameters, numerical simulations have been performed. In the following calculations, the parameters used are: luminous flux transmittance \(\tau = 0.6\), photometric brightness \(L = IW/(sr \cdot m)\). As to the object on the axis, we can assume a tiny area whose illumination distribution is uniform, ignoring the affect of aberration on the illumination of the image. Figure 4 shows the relation curve between the aperture angle of the image and the illumination of the centre image, it is demonstrated that the illumination of the image increases with the aperture angle of the image. Hence, we can increase the diameter of the exit pupil to increase the illumination of the image.

To analyze the illumination distribution of the off-axis image, we can use \(E'/E_0'\) to illustrate the relative illumination of the image, which can help research the issue about the uniformity of the illumination distribution. Figure 5 shows the effect of the angle of view on \(E'/E_0'\). It is shown that \(E'/E_0'\) decreases as the angle of view increases. \(E'/E_0'\) get close to 1 as the angle of view reaches 0, which means the uniformity of the illumination is the best in this circumstance. In the practical optical design, designers tend to adopt the way of compressing the angle of view to improve the uniformity of the illumination distribution.

\[
\frac{-f}{-l'} = \frac{-y'}{y - y''}
\]
Moreover, we discuss the effect of horizontal offset of the object on image, respective parameter are as follow: the focal distance of the optical system $f' = 10cm$, the initial object distance $l = -14cm$, other parameters are the same as figure 4. Assuming the optical system is ideal and the object is moving on the axis. Figure 6 shows the relation curve of horizontal offset of the object and $E'/E'_0$, it is demonstrated that as the object get farther to the focal plane, $E'/E'_0$ increases. As to telecentric optical system, the uniformity is the best. In addition, when the distance from the object to the focal plane is certain, as the object get closer to the axis, the uniformity decreases, which is an important factor that the illumination distribution is non-uniform.

**Figure 4.** The relation curve between the aperture angle of the image and the illumination of the centre image.

**Figure 5.** The relation curve between the angle of view and $E'/E'_0$.

**Figure 6.** The relation curve between horizontal offset of the object and $E'/E'_0$. 

4. conclusions
As the aperture angle of the image increases, the illumination the centre image increases, so it is recommended to aggrandize the diameter of the exit pupil to increase the illumination of the image. The angle of view plays a negative role in the illumination distribution of the image, that is, we can try to compress the angle of view to improve the uniformity of the illumination distribution. Lastly, it is proved that telecentric optical design is an effective way to enhance the uniformity of the illumination distribution.

References